Annuities

Section 3.5
Future Value of an Annuity

The value of a group of payments at a specified date in the future is called an *annuity*.

It differs from compounded interest because repeated payments are made into the account at regular intervals.

*Future value* measures how much you would have in the future given a specified rate of return (interest rate.)
Future Value of an Annuity:

\[ F_n = P\left[\frac{(1+i)^n - 1}{i}\right] \]

\( P = \) payment amount/time period (month)

\( n = \) total # payments ( #years \( \times \) # payments/year)

\( i = \) APR/# time periods/year

\( F_n = \) value of annuity after n years
Example 1:

You plan on depositing $80 per month into an annuity fund yielding 3.7% APR.

a. How much will be in the annuity after 12 years?

\[
F_n = p \left[ \frac{(1+i)^n - 1}{i} \right]
\]

\[
\begin{align*}
\text{F}_n &= F_n = 80 \left[ \frac{(1+0.037/12)^{12 \times 12} - 1}{0.037/12} \right] \\
&= $14,474.36
\end{align*}
\]

Total deposits: \( 80(144) = \$11,520 \)

Total interest: \( $14,474.36 - 11,520 = \$2,954.36 \)
Example 1:

b. What would your monthly payments need to be to have $20,000 after 12 years?

\[ F_n = p \left[ \frac{(1+i)^n - 1}{i} \right] \]

- \( n = 144 \)
- \( i = \frac{0.037}{12} \)

\[ 20,000 = p \left[ \frac{(1+i)^n - 1}{i} \right] \]

\[ p = \$110.54 \]
Present Value of an Annuity

For our purposes, \textit{present value} will apply to situations where you are repaying a loan over a given amount of time.

\[ p = \text{payment amount/time period (month)} \]

\[ n = \text{total \# payments} \]

\[ i = \text{APR/\# payments/year} \]

\[ P_n = \text{total loan amount (present value)} \]

\[ P_n = p \left[ \frac{1 - (1+i)^{-n}}{i} \right] \]
Example 2:

You plan on getting a mortgage and want to pay $1200/month for 30 years at 4.75%.

a. How large of a mortgage (loan) can you afford?

\[ n = 30(12) = 360 \]
\[ i = \frac{0.0475}{12} \]

\[ P_n = 1200 \left( \frac{1 - (1+i)^{-n}}{i} \right) = \boxed{230,040.47} \]

DO YOU PAY?

\$1200(360) \rightarrow \$432,000 - 230,040.47 = \boxed{201,959.53} \]

HOW MUCH INTEREST
Example 2:

You plan on getting a mortgage and want to pay $1200/month for 30 years at 4.75%.

b. If you wanted to get a $300,000 mortgage, what would your payment be?

\[ n = 360 \]
\[ i = \frac{0.0475}{12} \]

\[ 300,000 = P \left( \frac{1 - (1+i)^{-n}}{i} \right) \]

\[ P = \$1364.94 \]
Example 1 (Worksheet):
Example 2 (Worksheet):
Homework:

Annuities Worksheet