Arc Length Formulas:

\[
L = \int_{x_1}^{x_2} \sqrt{1 + (f'(x))^2} \, dx
\]

\[
L = \int_{y_1}^{y_2} \sqrt{1 + [f'(y)]^2} \, dy
\]

* \( f(x) \) and \( f(y) \) must be differentiable
Derivation of Arc Length Formula:

\[
\begin{align*}
\frac{dl}{dx} &= \sqrt{(dx)^2 + (dy)^2} \\
L &= \int_{x_1}^{x_2} \sqrt{(dx)^2 + (dy)^2} \\
&= \int \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2} \, dx \\
&= \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx
\end{align*}
\]
\[ y = x^{\frac{3}{2}} \text{ from } (1,1) \text{ to } (2, 2\sqrt{2}) \]

\[ y' = \frac{3}{2} x^{\frac{1}{2}} \]

\[ L = \int_{1}^{2} \sqrt{1 + \left(\frac{3}{2} x^{\frac{1}{2}}\right)^2} \, dx \]

\[ = \int_{1}^{2} \sqrt{1 + \frac{9}{4} x} \, dx \]

\[ u = 1 + \frac{9}{4} x \]

\[ du = \frac{9}{4} \, dx \]

\[ = \frac{4}{9} \int_{\frac{\sqrt{13}}{4}}^{\frac{\sqrt{22}}{4}} u^{\frac{1}{2}} \, du \]

\[ \frac{4}{9} \cdot \frac{2}{3} u^{\frac{3}{2}} \bigg|_{\frac{\sqrt{13}}{4}}^{\frac{\sqrt{22}}{4}} \]

\[ = \frac{8}{27} \left( \left(\frac{\sqrt{22}}{4}\right)^{\frac{3}{2}} - \left(\frac{\sqrt{13}}{4}\right)^{\frac{3}{2}} \right) \]

\[ = \frac{1}{27} \left( \frac{\sqrt{22}}{2} \right) \sqrt{13} - 13 \sqrt{13} \]
b) \[ y = x^{\frac{3}{2}} \Rightarrow x = y^{\frac{2}{3}} \Rightarrow x' = \frac{2}{3} y^{-\frac{1}{3}} \]

\[ \frac{2}{3} y^{\frac{2}{3}} \sqrt{y^{\frac{2}{3}} + \frac{4}{9}} \, dy \]

\[ \frac{3}{2} \int_{0}^{\frac{2}{3}a} u^{\frac{11}{2}} \, du \]

\[ \int_{0}^{\frac{2}{3}a} \left( \frac{22}{9} \right) \left( \frac{3}{2} \right) \, du \]

\[ = \frac{1}{27} \left( 22 \cdot 22 - 13 \cdot 19 \right) \]
Question 5

Let $R$ be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of $R$.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of $R$.

\[ u = x^2, \quad \frac{du}{dx} = 2x, \quad x^2 = \frac{u}{2} \]

\[ \int_0^1 xe^{x^2} + 2x \, dx \]

\[ \int_0^1 e^{x^2} \, dx + \int_0^1 2x \, dx \]

\[ \frac{1}{2} \left[ \int_0^1 e^{x^2} \, dx + x^2 \right]_0^1 \]

\[ \frac{1}{a} (e-1) + 1 \]

\[ \frac{1}{a} e - \frac{1}{2}a + 1 \]

\[ \frac{1}{a} e + \frac{1}{2}a \]

\[ \frac{1}{a} (e^a) = \frac{e+1}{a} \]
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(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of \( R \).

\[ V = \pi \int_0^1 (r_1^2 - r_2^2) \, dx \]
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\[ y_1 = xe^{x^2} \]
\[ \frac{dy_1}{dx} = xe^{x^2} + x^2 e^{x^2} \]
\[ = 2x^2 e^{x^2} + x e^{x^2} \]

\[ L = e + 2 + \sqrt{5} + \int_{0}^{1} \sqrt{1 + (\frac{dy_1}{dx})^2} \, dx \]
HW: CHAPTER 7 AP PACKET

# 15, 30, 61-69 odd, 70, 71