Section 10.5

Polar Graphs Part 1
**Lines:**

**Horizontal:** \( y = b \Rightarrow r \sin \theta = b \Rightarrow r = \frac{b}{\sin \theta} \) or \( r = b \csc \theta \)

**Vertical:** \( x = a \Rightarrow r \cos \theta = a \Rightarrow r = \frac{a}{\cos \theta} \) or \( r = a \sec \theta \)

**Through Origin:**

\( \theta = \theta_0 \) (constant) \( \Rightarrow \theta = \pi/3 \)
Circles:

Centered at Origin: \( r = a \)  \( C(0, 0) \) RADIUS \( = a \)

Through Origin:

\( r = \pm 2a \cos \theta \) or \( r = \pm 2a \sin \theta \)

\( r = 4 \cos \theta \) or \( r = -3 \cos \theta \)

\( r = \sin \theta \) or \( r = -2 \sin \theta \)

DIAm = 4
ON X-AXIS
C(2,0)
THRU (O,0)

DIAm = 0
ON X-AXIS
C(-1.5,0)
THRU (0,0)

DIAm = 1
ON POS. Y-AXIS
C(0,1/2)
THRU (0,0)

DIAm = 2
ON NEG Y-AXIS
C(0,-1)
THRU (0,0)
Cardioids:

\[ r = a \pm a \cos \theta \]

Centered on y-axis
Cusp at (0,0)

Example:
\[ r = -2 \pm 2 \cos \theta \]

\[ r = a \pm a \sin \theta \]

Centered on y-axis
Cusp at (0,0)

Example:
\[ r = -5 \pm 5 \sin \theta \]
Limaçons (part 1):

\[ r = a \pm b \cos \theta \]
\[ r = a \pm b \sin \theta \]

Ex: \[ r = 1 + 3 \sin \theta \]

\[ \theta = \frac{3\pi}{2} \]
\[ \theta = \frac{3\pi}{2} \]
\[ \theta = 194.4^\circ \]
\[ \sin \theta = -\frac{\sqrt{3}}{2} \]
\[ \theta \approx 194^\circ \]

Ex: \[ r = -2 + 3 \cos \theta \]

\[ \theta = \frac{3\pi}{2} \]
\[ \theta = 312^\circ \]
\[ \theta = 48^\circ \]
Limaçons (part 2): Dimpled Limaçon

\[ r = 4 - 3 \sin \theta \]

\[ 4 - 3 \sin \theta = 0 \]
\[ \sin \theta = \frac{4}{3} \]
\[ \phi \]

\[ r = 3 + 2 \cos \theta \]

\[ \theta = \pi \]
\[ \theta = \frac{3\pi}{2} \]
\[ \theta = 0 \]
Limaçons (part 3): Convex Limaçon

\[ r = 4 + \sin \theta \]
Limaçons (summary):

\[ r = a \pm b \sin \theta \]
\[ r = a \pm b \cos \theta \]

**INNER LOOP**

\[ \left| \frac{a}{b} \right| < 1 \]
\[ r = 3 - 4 \cos \theta \]
\[ \left| \frac{3}{4} \right| \]

**DIPPED**

\[ 1 < \left| \frac{a}{b} \right| < 2 \]
\[ r = 4 + 3 \cos \theta \]
\[ \left| \frac{4}{3} \right| \]

**CONVEX**

\[ \left| \frac{a}{b} \right| \geq 2 \]
\[ r = 2 + \sin \theta \]
\[ \left| 2 \right| \]
Classwork:

Homework:

Anton Handout # 21 – 40