Related Rates W.S.

1. \( \frac{d}{dt}(A = \pi r^2) \)

\[
\frac{dA}{dt} = 2\pi r \frac{dr}{dt}
\]

\[
\frac{dA}{dt} = 2\pi \times 50 \times 1
\]

\[
\frac{dA}{dt} = 100\pi \ cm^2/s
\]
\[ x^2 + y^2 = 100 \]

\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \]

\[ (6)(-1) + (8) \frac{dy}{dt} = 0 \]

\[ \frac{dy}{dt} = \frac{6}{8} = \frac{3}{4} \text{ PT MIN} \]
Related Rates Recipe:

1. Make a drawing - label parts that change with a variable; label parts that do not change with a number.

2. Create an equation/formula relating the quantities in the drawing.

3. Differentiate with respect to $t$ (implicitly.)

4. Plug in values - you may have to freeze the drawing to find missing parts.

5. Solve for the unknown quantity (most often its a rate.)

6. Does the answer make sense? Specifically, the sign of the answer.
\[ \tan \theta = \frac{30}{x} = 30x^{-1} \]
\[ \sec^2 \theta \frac{d\theta}{dt} = -30 \cdot \frac{dx}{x^2} \cdot \frac{dt}{dt} \]
\[ \frac{25}{16} \left( -\frac{2^\circ}{180^\circ} \cdot \frac{\pi}{180^\circ} \right) = -\frac{30}{40^2} \cdot \frac{dx}{dt} \]
\[ \frac{25}{16} \left( \frac{\pi}{90} \right) = \frac{dx}{dt} \]
\[ \frac{dx}{dt} = \frac{\frac{25\pi}{27}}{1} \cdot \frac{\text{HR}}{\text{hr}} \approx 2.909 \text{ hr}^{-1} \]