ACCEL. PRE-CALCULUS/TRIG 3

Unit 3 Review

Graphing
- Know 2^x, 3^x, and 4^x parent points and how to fill in the 1st table
- Study all of the transformations and how to fill in the 2nd table

1) \( y = 2^{x-2} + 3 \)

\[
\begin{array}{c|c}
\hline
x & y \\
\hline
\end{array}
\quad \begin{array}{c|c}
\hline
x & y \\
\hline
\end{array}
\]

Asymptote: ________

2) \( y = -3^{x-3} \)

\[
\begin{array}{c|c}
\hline
x & y \\
\hline
\end{array}
\quad \begin{array}{c|c}
\hline
x & y \\
\hline
\end{array}
\]

Asymptote: ________

3) \( y = -\left(\frac{1}{3}\right)^{x+2} - 4 \)

\[
\begin{array}{c|c}
\hline
x & y \\
\hline
\end{array}
\quad \begin{array}{c|c}
\hline
x & y \\
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\end{array}
\]

Asymptote: ________
4) \( y = \log_2(x + 3) - 2 \)  
\[
\begin{array}{c|c}
 x & y \\
\hline
 & \\
 & \\
 & \\
\end{array}
\]
\[
\begin{array}{c|c}
 x & y \\
\hline
 & \\
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\end{array}
\]

Asymptote: ________

5) \( y = -\log_3(x) + 3 \)  
\[
\begin{array}{c|c}
 x & y \\
\hline
 & \\
 & \\
 & \\
\end{array}
\]
\[
\begin{array}{c|c}
 x & y \\
\hline
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\end{array}
\]

Asymptote: ________

6) \( y = \log_4(-x + 1) \)  
\[
\begin{array}{c|c}
 x & y \\
\hline
 & \\
 & \\
 & \\
\end{array}
\]
\[
\begin{array}{c|c}
 x & y \\
\hline
 & \\
 & \\
 & \\
\end{array}
\]

Asymptote: ________
Complete the following problems on a separate paper.

Present Value of an Annuity: \[ P_n = P \left( \frac{1-(1+i)^{-n}}{i} \right) \]

Future Value of an Annuity: \[ F_n = P \left( \frac{(1+i)^n - 1}{i} \right) \]

Sam bought a house for $375,000. He paid a 15% down payment. He signed up for a 30 year mortgage with an interest rate of 4.65%.
1) a) How much was his down payment? b) How much of the house is financed?
2) What is his monthly payment for principal and interest?
3) How much will the house cost him in total?
4) How much interest will he pay over the life of the loan?

Sarah contributes $2160 to her individual retirement account (IRA) each year for 35 years. She hopes to earn an average annual percentage rate of 2.3% over the 35 years.
5) How much will she contribute over the 35 years?
6) a) How much will the IRA be worth after 35 years? b) How much interest will she earn?

Compound interest. \[ A = P(1 + \frac{r}{n})^{nt} \]
7) $2000, 6.25%, 7 years, compounded weekly
8) $4500, 10%, 5 years 3 months, compounded quarterly
9) $7000, 3.75%, 40 months, compounded annually
10) $1625, 6%, 22 years, compounded continuously
11) Determine the principle \( P \) that must be invested at a 5.5% interest rate compounded continuously, so that 400,000 will be available in 30 years?

Re-write in exponential form.
12) \( \log_7 \frac{1}{49} = -2 \) 13) \( \log_2 243 = 5 \) 14) \( \log_3 = \frac{1}{z} \) 15) \( \ln 403.4 = 6 \)

Re-write in logarithmic form.
16) \( 81^\frac{1}{2} = 3 \) 17) \( 6^{-2} = \frac{1}{36} \) 18) \( 7^0 = 1 \) 19) \( e^x = 54 \)

Evaluate. (no calculator!)
20) \( \log_2 \left( \frac{9}{4} \right) \) 21) \( 6^{\log_6 7} \) 22) \( \log 1 + \log 100 \) 23) \( \ln \frac{1}{e^4} \)

Expand.
24) \( \log_4 \left( \frac{x^2y^3}{z} \right) \) 25) \( \ln(3b^2) \) 26) \( \log_7 \left( \frac{x^3y^3}{z^4} \right) \)

Condense.
27) \( 3 \ln x + 2 \ln y - 4 \ln z \) 28) \( 2 \log_2 x - \frac{1}{2} \log_2(x^2 + 1) \) 29) \( \frac{3}{2} \left[ \ln(x^2 + 1) - \ln(x + 1) - \ln(x - 1) \right] \)

Solve. (when necessary, round to 2 decimal places)
30) \( 12 \log_{10} 25 = x \) 31) \( 5^x = \frac{1}{25} \) 32) \( \ln x = \frac{1}{2} \)
33) \( \log_4 x = 2 \) 34) \( \ln x - 5 \ln 2 = \ln 4 \) 35) \( 3e^{5x} = 132 \)
36) \( 4 \log_2 x + \log_2 5 = \log_2 405 \) 37) \( 6x^{2-2} = 48 \) 38) \( \log_{\frac{3}{2}} \left( \frac{3}{10} \right) = x \)
39) \( 2^x + 13 = 35 \) 40) \( \log_e 18.9 = 0.4 \) 41) \( 2^{x+5} = 3^{x-2} \)
42) \( \ln x + \ln(x-2) = 1 \) 43) \( e^{2x} - 8e^x + 7 = 0 \) 44) \( x^{0.4} = 15 \)

**Look over all of the application problems we did this chapter**
Unit 3 Review

Graphing

- Know $2^x$, $3^x$, and $4^x$ parent points and how to fill in the 1st table
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1) $y = 2^{x-2} + 3$

   **Parent:** $y = 2^x$

   **Asymptote:** $y = 3$

   **Movements:** right 2, up 3

<table>
<thead>
<tr>
<th>x</th>
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<tbody>
<tr>
<td>+2</td>
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2) $y = -3^{x-3}$

   **Parent:** $y = 3^x$

   **Asymptote:** $y = 0$

   **Movements:** reflects x-axis, right 3

<table>
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<tr>
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<tr>
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<td>4</td>
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<td>5</td>
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3) $y = -\left(\frac{1}{3}\right)^{x+2} - 4$

   **Parent:** $y = 3^x$

   **Asymptote:** $y = -4$

   **Movements:** reflects x and y-axis, left 2, down 4

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
<tr>
<td>-2</td>
<td>-4</td>
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<td>0</td>
<td>-1</td>
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<tr>
<td>-1</td>
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<table>
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<tr>
<th>x</th>
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<tbody>
<tr>
<td>-2</td>
<td>-5</td>
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<tr>
<td>-3</td>
<td>-7</td>
</tr>
<tr>
<td>-4</td>
<td>-13</td>
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</tbody>
</table>
4) \( y = \log_2(x + 3) - 2 \)

\[
\begin{array}{c|c}
 x & y \\
 \hline
 -3 & -2 \\
 1 & 0 \\
 2 & 1 \\
 4 & 2 \\
\end{array}
\]

movements: left 3, down 2

Asymptote: \( x = -3 \)

5) \( y = -\log_3(x) + 3 \)

\[
\begin{array}{c|c}
 x & y +3 \\
 \hline
 1 & 0 \\
 3 & -1 \\
 9 & -2 \\
\end{array}
\]

movements: reflects x-axis, up 3

Asymptote: \( x = 0 \)

6) \( y = \log_4(-x+1) \)

\[
\begin{array}{c|c}
 x & y \\
 \hline
 0 & 0 \\
 -3 & 1 \\
 -15 & 2 \\
\end{array}
\]

movements: reflects y-axis, right 1

Asymptote: \( x = 1 \)
Complete the following problems on a separate paper.

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Evaluate. (no calculator!)
20) \( \log_2 \frac{9}{4} \)  
21) \( 6^{\log_6 7} \)  
22) \( \log 1 + \log 100 \)  
23) \( \ln \frac{1}{e^4} \)

Expand.
24) \( \log_4 \left( \frac{xy^3}{x} \right) \)  
25) \( \ln(3b^2) \)  
26) \( \log_7 \frac{x^{2y}}{z^4} \)

Condense.
27) \( 3 \ln x + 2 \ln y - 4 \ln z \)  
28) \( 2 \log_2 3 - \frac{1}{2} \log_2 (x^2 + 1) \)  
29) \( \frac{3}{2} [\ln(x^2 + 1) - \ln(x + 1) - \ln(x - 1)] \)

Solve. (when necessary, round to 2 decimal places)
30) \( 12 \log_{10} 15 = x \)  
31) \( 5^x = \frac{1}{25} \)  
32) \( \ln x = \frac{1}{2} \)
33) \( \log_4 x = 2 \)  
34) \( \ln x - 5 \ln 2 = \ln 4 \)  
35) \( 3e^{3x} = 132 \)
36) \( 4 \log_2 x + \log_2 5 = \log_2 405 \)  
37) \( 6^{x^2 - 2} = 48 \)  
38) \( \log_2 \sqrt{10} = x \)
39) \( 2^x + 13 = 35 \)  
40) \( \log_5 18.9 = 0.4 \)  
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43) \( e^{2x} - 8e^x + 7 = 0 \)  
44) \( x^{0.4} = 15 \)

**Look over all of the application problems we did this chapter**
1. a) \( 375,000 \times 0.15 = \$56,250 \)
   
   b) \( 375,000 - 56,250 = \$318,750 = P_n \)

2. \( 318,750 = P \left[ 1 - \left( \frac{1 + \frac{0.0465}{12}}{\frac{0.0465}{12}} \right)^{360} \right] \)  
   \( P = \$1643.59 \)

3. \( P \times n + \text{down payment} \)
   \( 1643.59 \times 360 + 56250 = \$647,942.40 \)

4. \( P \times n - P_n \)
   \( 1643.59 \times 360 - 318750 = \$272,942.40 \)

5. \( 2160 \times 35 = \$75,600 \)

6. a) \( F_n = 2160 \left[ \left( 1 + \frac{0.023}{35} \right)^{35} - 1 \right] = \$114,235.37 \)
   
   b) \( F_n - P \times n \)
   \( 114235.37 - 75,600 = \$38,635.37 \)

7. \( A = 2000 \left( 1 + \frac{0.0625}{52} \right)^{52 \times 7} = \$3,096.85 \)

8. \( A = 4500 \left( 1 + \frac{0.25}{4} \right)^{4 \times 5.25} = \$7,558.12 \)
9. \( A = 7000 \left(1 + \frac{0.0375}{1}\right)^{1 \times 10} = \$7,913.92 \)

10. \( A = 1025 e^{0.06 \times 22} = \$6,083.06 \)

11. \( \frac{400000 = P e^{0.055 \times 30}}{e^{0.055 \times 30}} \rightarrow P = \$760,819.96 \)

12. \( 7^{-2} = \frac{1}{49} \)

13. \( 3^5 = 243 \)

14. \( 9^{\frac{1}{2}} = 3 \)

15. \( e^6 = 403.4 \)

16. \( \log_{81} 3 = \frac{1}{4} \)

17. \( \log_{6} \frac{1}{36} = -2 \)

18. \( \log_{7} 1 = 0 \)

19. \( \ln 54 = x \)

20. \( \log_{\frac{4}{3}} \left(\frac{3}{4}\right) = \frac{9}{4} \)

21. \( 6^\log_{6} 7 = 7 \)

22. \( \log 1 + \log 100 = 10^x = 10, 10^x = 100 \)

23. \( \ln \frac{1}{e^4} = \ln e^{-4} = 2 \)

24. \( \log_4 \left(\frac{x^3}{4^2}\right) = \log_4 x + 3 \log_4 y - 2 \log_4 z \)

25. \( \ln (3b^2) = \ln 3 + \ln b^2 = \ln 3 + 2 \ln b \)

26. \( \log_7 \left(\frac{x^5 y^3}{z^4}\right) = \log_7 \left(\frac{x^5 y^3}{z^4}\right) = \log_7 x^5 + \frac{1}{3} \log_7 y^3 - \log_7 z^4 = 5 \log_7 x + \frac{1}{3} \log_7 y - 4 \log_7 z \)
27. \[ 3 \ln x + 2 \ln y - 4 \ln z = \ln x^3 + \ln y^2 - \ln z^4 = \ln \frac{x^3}{z^4} \]

28. \[ 2 \log_2 3 - \frac{1}{2} \log_2 (x^2 + 1) = \log_2 3^2 - \log_2 (x^2 + 1)^{\frac{1}{2}} = \log_2 \frac{9}{x^2 + 1} \text{ or} \]
\[ \log_2 \frac{9}{x^2 + 1} \cdot \frac{x^2 + 1}{x^2 + 1} = \log_2 \frac{9}{x^2 + 1} \]

29. \[ \frac{3}{2} \left[ \ln (x^2 + 1) - \ln (x+1) - \ln (x-1) \right] = \frac{3}{2} \left[ \ln \frac{x^2 + 1}{(x+1)(x-1)} \right] \]
\[ = \frac{3}{2} \left[ \ln \frac{x^2 + 1}{x^2 - 1} \right] = \ln \left( \frac{x^2 + 1}{x^2 - 1} \right)^{\frac{3}{2}} = \ln \frac{(x^2 + 1)^3}{(x^2 - 1)^3} = \ln \frac{(x^2 + 1)(x^4 - 1)}{(x^2 - 1)(x^2 - 1)(x^2 - 1)} \text{ or} \]
\[ \ln \frac{(x^2 + 1)}{(x^2 - 1)} \cdot \frac{x^2 + 1}{x^2 - 1} = \ln \frac{(x^2 + 1)(x^4 - 1)}{(x^2 - 1)(x^2 - 1)(x^2 - 1)} = \ln \frac{(x^2 + 1)(x^4 - 1)}{x^4 - 2x^2 + 1} \]

30. \[ 12 \log_2 5 = x \]
\[ x = 5 \]

31. \[ 5^x = \frac{1}{25} \]
\[ x = -2 \]

32. \[ \ln x = \frac{1}{2} \]
\[ x = e^{\frac{1}{2}} = \sqrt{e} = 1.65 \]

33. \[ \log_4 x = 2 \]
\[ x = 4^2 = 16 \]

34. \[ \ln x - 5 \ln 2 = \ln 4 \]
\[ \ln \frac{x}{2^5} = \ln 4 \]
\[ \frac{x}{32} = 4 \]
\[ x = 128 \]

35. \[ 3e^{-5x} = 132 \]
\[ \frac{3e^{-5x}}{3} = 44 \]
\[ \ln 44 = \frac{7x}{-5} \]
\[ x = -0.76 \]
31. \[4 \log_2 x + \log_2 5 = \log_2 405\]
   \[\log_2 5x^4 = \log_2 405\]
   \[5x^4 = 405\]
   \[x^4 = 81\]
   \[x = \pm 3\]
   \[x = 3\]

37. \[6x^2 - 2 = 48\]
   \[\log_6 48 = x^2 - 2\]
   \[\log 6 = x^2 - 2\]
   \[2.160 \approx x^2 - 2\]
   \[4.16 \approx x^2\]
   \[x = \pm 2.04\]

38. \[\log_{3\frac{1}{3}} 10 = x\]
   \[\log 10^{\frac{1}{3}} = x\]
   \[x = \frac{1}{3}\]

39. \[2^x + 13 = 35\]
   \[2^x = 22\]
   \[\log_2 22 = x\]
   \[x = \log_2 22\]
   \[\log 2\]
   \[x = 4.416\]

40. \[\log_4 18.9 = 0.4\]
   \[x^{0.4} = 18.9\]
   \[x^{\frac{5}{2}} = 18.9^{\frac{5}{2}}\]
   \[x = 1552.94\]

41. \[2^{x+5} = 3^{x-2}\]
   \[\log_2 2^{x+5} = \log_3 3^{x-2}\]
   \[(x+5) \log_2 = (x-2) \log 3\]
   \[x \log 2 + 5 \log 2 = x \log 3 - 2 \log 3\]
   \[x \log 2 - x \log 3 = -5 \log 2 - 2 \log 3\]
   \[x(\log 2 - \log 3) = (5 \log 2 - 2 \log 3)\]
   \[\log 2 - \log 3 = \frac{5 \log 2 - 2 \log 3}{x}\]
   \[x = 13.97\]
\[42 \ln x + \ln (x-2) = 1\]
\[\ln (x^2 - 2x) = 1\]
\[e^1 = x^2 - 2x\]
\[0 = x^2 - 2x - e\]
\[x = \frac{2 \pm \sqrt{4 + 4e}}{2} = \frac{1 \pm \sqrt{1 + e}}{2}\]
\[x = 2.93, -0.98\]

\[43 \quad e^{2x} - 8e^x + 7 = 0\]
\[(e^x - 7)(e^x - 1) = 0\]
\[e^x = 7, \quad e^x = 1\]
\[x = \ln 7, \quad x = \ln 1\]
\[x = 1.95, 0\]

\[44 \quad x^{0.4} = 15\]
\[x^{\frac{5}{2}} = 15^{\frac{5}{2}}\]
\[x = 871.42\]