AP Calculus BC
Chapter 10 Part 2 – AP Exam Problems

All problems are NON CALCULATOR unless otherwise indicated.

1. The area of the region enclosed by the polar curve \( r = 2\sin(2\theta) \) for \( 0 \leq \theta \leq \frac{\pi}{2} \) is

   A) 0  B) \( \frac{1}{2} \)  C) 1  D) \( \frac{\pi}{2} \)  E) \( \frac{\pi}{4} \)

2. Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve \( r = 4\cos(3\theta) \) shown in the figure above?

   A) \( 16 \int_{-\pi/3}^{\pi/3} \cos(3\theta) \, d\theta \)  D) \( 16 \int_{-\pi/6}^{\pi/6} \cos^2(3\theta) \, d\theta \)
   
   B) \( 8 \int_{-\pi/6}^{\pi/6} \cos(3\theta) \, d\theta \)  E) \( 8 \int_{-\pi/6}^{\pi/6} \cos^2(3\theta) \, d\theta \)
   
   C) \( 8 \int_{-\pi/3}^{\pi/3} \cos^2(3\theta) \, d\theta \)

3. Which of the following is equal to the area of the region inside the polar curve \( r = 2\cos\theta \) and outside the polar curve \( r = \cos\theta \)?

   A) \( 3 \int_{0}^{\pi/2} \cos^2 \theta \, d\theta \)  B) \( 3 \int_{0}^{\pi} \cos^2 \theta \, d\theta \)  C) \( \frac{3}{2} \int_{0}^{\pi} \cos^2 \theta \, d\theta \)
   
   D) \( 3 \int_{0}^{\pi/2} \cos \theta \, d\theta \)  E) \( 3 \int_{0}^{\pi} \cos \theta \, d\theta \)

4. The area of the region inside the polar curve \( r = 4\sin \theta \) and outside the polar curve \( r = 2 \) is given by

   A) \( \frac{1}{2} \int_{0}^{\pi} (4\sin \theta - 2)^2 \, d\theta \)  D) \( \frac{1}{2} \int_{-\pi/6}^{\pi/6} (16\sin^2 \theta - 4) \, d\theta \)
   
   B) \( \frac{1}{2} \int_{-\pi/6}^{\pi/6} (4\sin \theta - 2)^2 \, d\theta \)  E) \( \frac{1}{2} \int_{0}^{\pi} (16\sin^2 \theta - 4) \, d\theta \)
   
   C) \( \frac{1}{2} \int_{-\pi/6}^{\pi/6} (4\sin \theta - 2)^2 \, d\theta \)
5. Which of the following expressions gives the total area enclosed by the polar curve \( r = \sin^2 \theta \) shown in the figure to the right?

(A) \( \frac{1}{2} \int_{0}^{\pi} \sin^2 \theta d\theta \)
(B) \( \int_{0}^{\pi} \sin^2 \theta d\theta \)
(C) \( \frac{1}{2} \int_{0}^{\pi} \sin^4 \theta d\theta \)
(D) \( \int_{0}^{\pi} \sin^4 \theta d\theta \)
(E) \( 2 \int_{0}^{\pi} \sin^4 \theta d\theta \)

6. (1984 BC5) Consider the curves \( r = 3\cos \theta \) and \( r = 1 + \cos \theta \).

(a) Sketch the curves on a set of \( x \) and \( y \)-axes.
(b) Find the area of the region inside the curve \( r = 3\cos \theta \) and outside the curve \( r = 1 + \cos \theta \) by setting up and evaluating a definite integral. Your work must include an antiderivative.

7. (1990 BC4) Let \( R \) be the region inside the graph of the polar curve \( r = 2 \) and outside the graph of the polar curve \( r = 2(1 - \sin \theta) \).

(a) Sketch the two polar curves on a set of \( x \) and \( y \) axes and shade the region \( R \).
(b) Find the area of \( R \).

8. (1993 BC4) Consider the polar curve \( r = 2\sin(3\theta) \) for \( 0 \leq \theta \leq \pi \).

(a) Sketch the curve on a set of \( x \) and \( y \)-axes.
(b) Find the area of the region inside the curve.
(c) Find the slope of the curve at the point where \( \theta = \frac{\pi}{4} \).
9. (2003B BC2) The figure shows the graphs of the circles \( x^2 + y^2 = 2 \) and \( (x - 1)^2 + y^2 = 1 \). The graphs intersect at the points (1, 1) and (1, -1). Let \( R \) be the shaded region in the first quadrant bounded by the two circles and the \( x \)-axis.

(a) Set up an expression involving one or more integrals with respect to \( x \) that represents the area of \( R \).

(b) Set up an expression involving one or more integrals with respect to \( y \) that represents the area of \( R \).

(c) The polar equations of the circles are \( r = \sqrt{2} \) and \( r = 2 \cos \theta \), respectively. Set up an expression involving one or more integrals with respect to the polar angle \( \theta \) that represents the area of \( R \).
10. (2005 BC2) The curve above is drawn in the $xy$ – plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \leq \theta \leq \pi$, where $r$ is measured in meters and $\theta$ is measured in radians. The derivative of $r$ with respect to $\theta$ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

(a) Find the area bounded by the curve and the $x$ – axis.

(b) Find the angle $\theta$ that corresponds to the point on the curve with $x$ – coordinate $-2$.

(c) For $\frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about $r$? What does this fact say about the curve?

(d) Find the value of $\theta$ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.
11. (2007 BC3) The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos \theta$ are shown in the figure below. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

(a) Let $R$ be the region that is inside both graphs. Find the area of $R$.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos \theta$ has position $(x(t), y(t))$ at time $t$, with $\theta = 0$ when $t = 0$. The particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.
12. (2009B BC4) The graph of the polar curve \( r = 1 - 2\cos \theta \) for \( 0 \leq \theta \leq \pi \) is shown below. Let \( S \) be the shaded region in the third quadrant bounded by the curve and the \( x \)-axis.

(a) Write an integral expression for the area of \( S \).

(b) Write expression for \( \frac{dx}{d\theta} \) and \( \frac{dy}{d\theta} \) in terms of \( \theta \).

(c) Write an equation in terms of \( x \) and \( y \) for the line tangent to the graph of the polar curve at the point where \( \theta = \frac{\pi}{2} \). Show the computations that lead to your answer.

13. (2011B BC2) The polar curve \( r \) is given by \( r(\theta) = 3\theta + \sin \theta \), where \( 0 \leq \theta \leq 2\pi \).

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of \( r \).

(b) For \( \frac{\pi}{2} \leq \theta \leq \pi \), there is one point \( P \) on the polar curve \( r \) with \( x \)-coordinate \(-3\). Find the angle \( \theta \) that corresponds to point \( P \). Find the \( y \)-coordinate of point \( P \). Show the work that leads to your answers.

(c) A particle is traveling along the polar curve \( r \) so that its position at time \( t \) is \( (x(t), y(t)) \) and such that \( \frac{d\theta}{dt} = 2 \). Find \( \frac{dy}{dt} \) at the instant that \( \theta = \frac{2\pi}{3} \), and interpret the meaning of your answer in the context of the problem.
14. (2013 BC2) The graphs of the polar curves \( r = 3 \) and \( r = 4 - 2\sin \theta \) are shown in the figure below. The curves intersect when \( \theta = \frac{\pi}{6} \) and \( \theta = \frac{5\pi}{6} \).

(a) Let \( S \) be the shaded region that is inside the graph of \( r = 3 \) and also inside the graph of \( r = 4 - 2\sin \theta \). Find the area of \( S \).

(b) A particle moves along the polar curve \( r = 4 - 2\sin \theta \) so that at time \( t \) seconds, \( \theta = t^2 \). Find the time \( t \) in the interval \( 1 \leq t \leq 2 \) for which the \( x \)-coordinate of the particle’s position is \(-1\).

(c) For the particle described in part (b), find the position vector in terms of \( t \). Find the velocity at time \( t = 1.5 \).
15. (2014 BC2) The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure below for $0 \leq \theta \leq \pi$.

(a) Let $R$ be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of $R$.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{6}$.

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to $\theta$ when $\theta = \frac{\pi}{3}$.

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.
16. (2017 BC2) The figure shows the polar curves \( r = f(\theta) = 1 + \sin \theta \cos (2\theta) \) and \( r = g(\theta) = 2\cos \theta \) for \( 0 \leq \theta \leq \frac{\pi}{2} \). Let \( R \) be the region in the first quadrant bounded by the curve \( r = f(\theta) \) and the \( x \)-axis. Let \( S \) be the region in the first quadrant bounded by the curve \( r = f(\theta) \), the curve \( r = g(\theta) \), and the \( x \)-axis.

(a) Find the area of \( R \).

(b) The ray \( \theta = k \), where \( 0 < k < \frac{\pi}{2} \), divides \( S \) into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of \( k \).

(c) For each \( \theta \), \( 0 \leq \theta \leq \frac{\pi}{2} \), let \( w(\theta) \) be the distance between the points with polar coordinates \((f(\theta), \theta)\) and \((g(\theta), \theta)\). Write an expression for \( w(\theta) \). Find \( w_A \), the average value of \( w(\theta) \) over the interval \( 0 \leq \theta \leq \frac{\pi}{2} \).

(d) Using the information from part (c), find the value of \( \theta \) for which \( w(\theta) = w_A \). Is the function \( w(\theta) \) increasing or decreasing at that value of \( \theta \)? Give a reason for your answer.

**Answers**

1. D 1985 BC #24 41%
2. E 1988 BC #23 55%
3. A 1997 BC #21 22%
4. D 1998 BC #19 37%
5. D 2008 BC #26 38%