Math 76

An Incremental Development

Third Edition

Stephen Hake
John Saxon

Saxon Publishers, Inc.

Math 76: An Incremental Development
Third Edition

Printed in the United States of America.

ISBN: 1-56577-153-2

Pre-Press Manager: J. Travis Rose

Production Coordinator: Joan Coleman

Manufacturing Code: 09S0503

Reaching us via the Internet

www.saxonpublishers.com

E-mail: info@saxonpublishers.com

Saxon Publishers, Inc.
2450 John Saxon Blvd.
Norman, OK 73071

Contents

Preface

To The Student

We study mathematics because it is an important part of our daily lives. Our school schedule, our trip to the store, the preparation of our meals, and many of the games we play all involve mathematics. Many of the word problems you will see in this book are drawn from our daily experiences.

Mathematics is even more important in the adult world. In fact, your personal future in the adult world may depend in part upon the mathematics you have learned. This book was written with the hope that more students will learn mathematics and learn it well. For this to happen, you must use this book properly. As you work through the pages of this book, you will find similar problems presented over and over again. **Solving these problems day after day is the secret to success.**

In this book you will find daily lessons and investigations. Each lesson begins with practice of basic number facts and mental math. These exercises will improve your speed, your accuracy, and your ability to do math "in your head." The accompanying pattern and problem-solving activities will give you practice using strategies that can help you solve more complicated problems. Near the end of the lesson is a set of practice problems that focus on the topic of the lesson. Following each lesson is a problem set that reviews the skills you are learning day by day. Investigations are variations of the daily lesson. The investigations included in *Math 76* are activities that may fill an entire class period and contain their own set of questions rather than an integrated problem set.

Work each problem in every practice set, in every problem set, and in every investigation. Do not skip problems. With honest effort you will experience success and true learning which will stay with you and serve you well in the future.

Stephen Hake
Temple City, California

John Saxon
Norman, Oklahoma

Acknowledgments

We thank Shirley McQuade Davis for her ideas on teaching word problem thinking patterns and Dan Gallup for his content editing. We would also like to thank the following people for their contributions in the production of this revision: Edward Burr, Adriana Castaneda, John Chitwood, Chris Cope, Serena Freeberg, Mike Lott, Erin McCain, Emerson Mounger, Tara Nance, Anna Maria Rodriguez, Heather Shaver, Ryan Solomon, Travis Southern, Letha Steinbron, and Julie Webster.

LESSON
1

Adding Whole Numbers and Money • Subtracting Whole Numbers and Money • Fact Families

Facts Practice: 64 Addition Facts (Test A in Test Masters)[†]

Mental Math: Count by 10's from 10 to 100 and from 100 to 0. Count by 100's from 100 to 1000 and from 1000 to 0.

a. 30 + 30 **b.** 300 + 300 **c.** 80 + 40
d. 800 + 400 **e.** 20 + 30 + 40 **f.** 200 + 300 + 400

Problem Solving: Sam thought of a number between ten and twenty. Then he gave a clue: You say the number when you count by twos and when you count by threes, but not when you count by fours. Of what number was Sam thinking?

Adding whole numbers and money

To combine two or more numbers we **add.** The numbers added together are called **addends.** The answer is called the **sum.**

When adding numbers, we add digits that have the same place value.

Example 1 345 + 67

Solution When we add whole numbers on paper, we write the numbers so that the last digits are aligned one above the other. Then we add the digits in each column.

$$\begin{array}{rl} {\scriptstyle 1\,1} & \\ 345 & \text{addend} \\ +\ \ 67 & \text{addend} \\ \hline \mathbf{412} & \text{sum} \end{array}$$

Changing the order of the addends does not change the sum. One way to check an addition answer is to change the order of the addends, and add again.

$$\begin{array}{rl} {\scriptstyle 1\,1} & \\ 67 & \\ +\ 345 & \\ \hline \mathbf{412} & \text{check} \end{array}$$

[†] For instructions on how to use the boxed activities, please consult the preface.

Example 2 \$1.25 + \$12.50 + \$5

Solution When we add money we write the numbers so that the decimal points are aligned. We write \$5 as \$5.00 and add the digits in each column.

$$\begin{array}{r} \$1.25 \\ \$12.50 \\ +\ \$5.00 \\ \hline \mathbf{\$18.75} \end{array}$$

Subtracting whole numbers and money

We find the **difference** between two numbers when we subtract.

$$5 - 3 = 2$$

The difference between 5 and 3 is 2.

When subtracting on paper, we align digits with the same place value. Order matters in subtraction: 2 − 4 is not the same as 4 − 2. Here are two forms we use to show we are subtracting 3 from 5.

$$5 - 3 \qquad \begin{array}{r} 5 \\ -\ 3 \\ \hline \end{array}$$

Example 3 345 − 67

Solution When we subtract whole numbers, we align the last digits. We subtract the bottom number from the top number. We regroup when it is necessary.

$$\begin{array}{r} \overset{2}{\cancel{3}}\,\overset{13}{\cancel{4}}\,{}^{1}5 \\ -\ \ 6\ 7 \\ \hline \mathbf{2\ 7\ 8} \leftarrow \end{array}$$

difference

Example 4 Jim spent \$1.25 for a hamburger. He paid for it with a five-dollar bill. Find how much change he should get back by subtracting \$1.25 from \$5.

Solution Order matters when we subtract. The starting amount is put on top. We write \$5 as \$5.00. We line up the decimal points to line up place values. Then we subtract. Jim should get back \$3.75.

$$\begin{array}{r} \$\overset{4}{\cancel{5}}.\overset{9}{\cancel{0}}{}^{1}0 \\ -\ \$1.2\ 5 \\ \hline \mathbf{\$3.7\ 5} \end{array}$$

We may check the answer to a subtraction problem by adding. If we add the answer (difference) to the amount subtracted, the total should equal the starting amount. We do not need to rewrite the problem. We just add the two bottom numbers to see if their sum equals the top number.

Subtract Down: To find the difference	↓	$\begin{array}{r} \$5.00 \\ - \$1.25 \\ \hline \$3.75 \end{array}$	↑	**Add Up:** To check the answer

Fact families

The numbers 4, 5, and 9 are a fact family. They can be arranged to form two addition facts and two subtraction facts.

$$\begin{array}{r} 4 \\ +\ 5 \\ \hline 9 \end{array} \qquad \begin{array}{r} 5 \\ +\ 4 \\ \hline 9 \end{array} \qquad \begin{array}{r} 9 \\ -\ 5 \\ \hline 4 \end{array} \qquad \begin{array}{r} 9 \\ -\ 4 \\ \hline 5 \end{array}$$

Example 5 Rearrange the numbers in this addition fact to form another addition fact and two subtraction facts.

$$11 + 14 = 25$$

Solution We form another addition fact by reversing the addends.

$$\mathbf{14 + 11 = 25}$$

We form two subtraction facts by making the sum, 25, the first number of the two subtraction facts. Each of the other numbers may be subtracted from 25.

$$\mathbf{25 - 11 = 14}$$

$$\mathbf{25 - 14 = 11}$$

Example 6 Rearrange the numbers in this subtraction fact to make another subtraction fact and two addition facts.

$$\begin{array}{r} 11 \\ -\ \ 6 \\ \hline 5 \end{array}$$

Solution We may not reverse the first two numbers of a subtraction problem, but we may reverse the last two numbers.

$$\begin{tikzcd} 11 & 11 \\ \underline{-\ 6} \arrow[dr] & \underline{-\ 5} \\ 5 \arrow[ur] & 6 \end{tikzcd}$$

We make 11 the sum of the two addition facts.

$$\begin{array}{r} 5 \\ \underline{+\ 6} \\ 11 \end{array} \qquad \begin{array}{r} 6 \\ \underline{+\ 5} \\ 11 \end{array}$$

Practice

a. 3675 + 426 + 1357

b. \$6.25 + \$8.23 + \$12

c. 5374 − 168

d. \$6 − \$1.35

e. Arrange the numbers 6, 8, and 14 to make two addition facts and two subtraction facts.

f. Rearrange the numbers in this subtraction fact to make another subtraction fact and two addition facts.

$$25 - 10 = 15$$

Problem set 1

1. What is the sum of 25 and 40?

2. Johnny had 137 apple seeds in one pocket and 89 in another. He found 9 more seeds in his cuff. Find how many seeds he had in all by adding 137, 89, and 9.

3. What is the difference when 93 is subtracted from 387?

4. John paid \$5 for a movie ticket that cost \$3.75. To find out how much money John should get back, subtract \$3.75 from \$5.

5. Monica had \$5.22 and earned \$1.15 more. To find how much money Monica had in all, add \$1.15 to \$5.22.

6. The hamburger cost $1.25, the fries cost $0.70, and the drink cost $0.60. To find the total price of the lunch, add $1.25, $0.70, and $0.60.

7. $\begin{array}{r} 63 \\ 47 \\ +\ 50 \\ \hline \end{array}$ **8.** $\begin{array}{r} 632 \\ 57 \\ +\ 198 \\ \hline \end{array}$ **9.** $\begin{array}{r} 78 \\ 9 \\ +\ 967 \\ \hline \end{array}$ **10.** $\begin{array}{r} 432 \\ 579 \\ +\ 3604 \\ \hline \end{array}$

11. 345 − 67

12. 678 − 416

13. 3764 − 96

14. 875 + 1086 + 980

15. 10 + 156 + 8 + 27

16. $\begin{array}{r} \$3.47 \\ -\ \$0.92 \\ \hline \end{array}$ **17.** $\begin{array}{r} \$24.15 \\ -\ \ \$1.45 \\ \hline \end{array}$ **18.** $\begin{array}{r} \$0.75 \\ +\ \$0.75 \\ \hline \end{array}$ **19.** $\begin{array}{r} \$0.12 \\ \$0.46 \\ +\ \$0.50 \\ \hline \end{array}$

20. What is the name for the answer when we add?

21. What is the name for the answer when we subtract?

22. The numbers 5, 6, and 11 are a fact family. Make two addition facts and two subtraction facts with these three numbers.

23. Rearrange the numbers in this addition fact to make another addition fact and two subtraction facts.

$$27 + 16 = 43$$

24. Rearrange the numbers in this subtraction fact to make another subtraction fact and two addition facts.

$$50 - 21 = 29$$

25. Describe a way to check the correctness of a subtraction answer.

LESSON
2

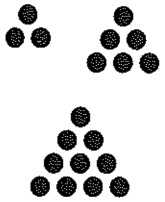
Multiplying Whole Numbers and Money • Dividing Whole Numbers and Money • Fact Families

Facts Practice: 64 Addition Facts (Test A in Test Masters)

Mental Math: Count by 5's from 5 to 100 and from 100 to 0. Count by 2's from 2 to 20 and from 20 to 2.

a. 500 + 40
b. 60 + 200
c. 30 + 200 + 40
d. 70 + 300 + 400
e. 400 + 50 + 30
f. 60 + 20 + 400

Problem Solving: Robert made three triangle patterns using 3 coins, 6 coins, and 10 coins. If he continues the patterns, how many coins will he need to make each of the next two triangle patterns?



Multiplying whole numbers and money

When we add the same number several times, we get the sum. We can get the same answer by multiplying.

$$\underbrace{67 + 67 + 67 + 67 + 67}_{\text{Five 67's equal 335.}} = 335$$

$$5 \times 67 = 335$$

Numbers that are multiplied together are called **factors.** The answer is called the **product.**

When we multiply by a two-digit number on paper we multiply twice. To multiply 28 by 14, we first multiply 28 by 4. Then we multiply 28 by 10. For each multiplication, we write a partial product. We add the partial products to find the final product.

28	factor
× 14	factor
112	partial product (4 × 28)
280	partial product (10 × 28)
392	product (14 × 28)

When multiplying dollars and cents by a whole number the answer will have cents' places, that is, two places after the decimal point.

$$\begin{array}{r} \$1.35 \\ \times \quad\quad 6 \\ \hline \$8.10 \end{array}$$

Example 1 Find the cost of two dozen pencils at 35¢ each.

Solution Two dozen is two 12's, which is 24. To find the cost of 24 pencils we multiply 35¢ by 24.

$$\begin{array}{r} 35¢ \\ \times \;\; 24 \\ \hline 140 \\ 700 \\ \hline 840¢ \end{array}$$

The cost of two dozen pencils is 840¢, which is **$8.40.**

Reversing the order of the factors does not change the product.

$$4 \times 2 = 2 \times 4$$

One way to check multiplication is to reverse the order of factors and multiply again.

$$\begin{array}{rrl} 23 & 14 & \text{factors reversed} \\ \times \; 14 & \times \; 23 & \\ \hline 92 & 42 & \\ 230 & 280 & \\ \hline 322 & 322 & \text{check} \end{array}$$

When one of the two factors of a multiplication is one, the product equals the other factor. When one of the two factors is zero, the product is zero.

$$\text{Any number} \times 0 = 0$$

Example 2 Multiply:

$$\begin{array}{r} 400 \\ \times\ 874 \\ \hline \end{array}$$

Solution Reversing the order of the factors may make a multiplication problem easier. Writing trailing zeros so that they "hang out" to the right simplifies the multiplication.

$$\begin{array}{r} \overset{2\,1}{874} \\ \times \quad 400 \\ \downarrow\downarrow \\ \hline \mathbf{349{,}600} \end{array}$$

Dividing whole numbers and money

When a number is to be separated into a certain number of equal parts, we divide. We can indicate division in several ways. Here are three ways to show that 24 is to be divided by 2.

$$24 \div 2 \qquad 2\overline{)24} \qquad \frac{24}{2}$$

The answer to a division problem is the **quotient.** The number that is divided is the **dividend.** The number by which the dividend is divided is the **divisor.** We show these terms in the three division forms in this table.

dividend ÷ divisor = quotient
$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$
$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$

When the dividend is zero, the quotient is zero. The divisor may not be zero. When the dividend and divisor are equal (and not zero), the quotient is one.

Example 3 $3456 \div 7$

Solution We show both the long division and short division methods.

LONG DIVISION:

$$\begin{array}{r} \mathbf{493 \text{ r } 5} \\ 7\overline{)3456} \\ \underline{28} \\ 65 \\ \underline{63} \\ 26 \\ \underline{21} \\ 5 \end{array}$$

SHORT DIVISION:

$$\begin{array}{r} \mathbf{4\;9\;3 \text{ r } 5} \\ 7\overline{)3\;4^{6}5^{2}6} \end{array}$$

Using the short division method we perform the multiplication and subtraction steps mentally, recording only the result of each subtraction.

To check our work we multiply the quotient by the divisor, then add the remainder to this answer. The result should be the dividend. For this example we multiply 493 by 7. Then we add 5.

$$\begin{array}{r} {\scriptstyle 6\,2} \\ 493 \\ \underline{\times \quad 7} \\ 3451 \\ \underline{+ \quad\;\; 5} \\ 3456 \end{array}$$

When dividing dollars and cents, there will be cents in the answer. Notice that the decimal point is directly up from the decimal point in the division box, separating the dollars from the cents.

$$\begin{array}{r} \$1.60 \\ 3\overline{)\$4.80} \\ \underline{3} \\ 1\;8 \\ \underline{1\;8} \\ 00 \\ \underline{0} \\ 0 \end{array}$$

Fact families There are multiplication and division fact families just as there are addition and subtraction fact families. The numbers 5, 6, and 30 are a fact family. We can form two multiplication facts and two division facts with these numbers.

$$5 \times 6 = 30 \qquad 30 \div 5 = 6$$

$$6 \times 5 = 30 \qquad 30 \div 6 = 5$$

Example 4 Rearrange the numbers in this multiplication fact to make another multiplication fact and two division facts.

$$5 \times 12 = 60$$

Solution By reversing the factors we make another multiplication fact.

$$\mathbf{12 \times 5 = 60}$$

By making 60 the dividend we can form two division facts.

$$\mathbf{60 \div 5 = 12}$$

$$\mathbf{60 \div 12 = 5}$$

Practice*[†]

a. 20 × 37¢

b. 37 × 0

c. 407 × 37

d. $5\overline{)\$8.40}$

e. 200 ÷ 12

f. $\frac{234}{3}$

g. Which numbers are the divisors in problems (d), (e), and (f)?

h. Use the numbers 8, 9, and 72 to make two multiplication facts and two division facts.

[†] All lessons with practice sets starred with an asterisk (*) have supplemental practice sets in the appendix. These sets may be used as needed for additional practice.

Problem set 2

†**1.** *(2)* If the factors are 7 and 11, what is the product?

2. *(1)* What is the difference between 97 and 79?

3. *(1)* If the addends are 170 and 130, then what is the sum?

4. *(2)* If 36 is the dividend and 4 is the divisor, what is the quotient?

5. *(1)* Find the sum of 386, 98, and 1734.

6. *(1)* Jim spent $2.25 for a hamburger. He paid for it with a five-dollar bill. Find how much change he should get back by subtracting $2.25 from $5.

7. *(1)* Luke wants to buy a $70.00 radio for his car. He has $47.50. Find how much more money he needs by subtracting $47.50 from $70.00.

8. *(2)* Each energy bar costs 75¢. Find the cost of one dozen energy bars by multiplying 75¢ by 12.

9. *(1)* $\begin{array}{r} 312 \\ -\ \ 86 \\ \hline \end{array}$

10. *(1)* $\begin{array}{r} 4106 \\ +\ 1398 \\ \hline \end{array}$

11. *(1)* $\begin{array}{r} 4000 \\ -\ 1357 \\ \hline \end{array}$

12. *(1)* $\begin{array}{r} \$10.00 \\ -\ \ \$2.83 \\ \hline \end{array}$

13. *(2)* $\begin{array}{r} 405 \\ \times\ \ \ 8 \\ \hline \end{array}$

14. *(2)* $\begin{array}{r} 25 \\ \times\ 25 \\ \hline \end{array}$

15. *(2)* $\frac{288}{6}$

16. *(2)* $\frac{225}{15}$

17. *(2)* $1.25 × 8

18. *(2)* 400 × 50

19. *(2)* 1000 ÷ 8

20. *(2)* $45.00 ÷ 20

---

†The italicized numbers within parentheses underneath each problem number are called *lesson reference numbers.* These numbers refer to the lesson(s) in which the major concept of that particular problem is introduced. A lesson reference number of *N.R.* means "no reference." If additional assistance is needed, reference should be made to the discussion, examples, practice, or problem set of that lesson.

21. (2) Use the numbers 6, 8, and 48 to make two multiplication facts and two division facts.

22. (2) Rearrange the numbers in this division fact to make another division fact and two multiplication facts.

$$4\overline{)36}^{\,9}$$

23. (1) Rearrange the numbers in this addition fact to make another addition fact and two subtraction facts.

$$12 + 24 = 36$$

24. (1) Find the sum of 9 and 6 and the difference between 9 and 6.

25. (2) The divisor, dividend, and quotient are in these positions when we use a division sign.

Dividend ÷ divisor = quotient

On your paper, draw a division box and show the position of the divisor, dividend, and quotient.

26. (2) Multiply to find the answer to this addition problem:

39¢ + 39¢ + 39¢ + 39¢ + 39¢ + 39¢

27. (2) 365 × 0

28. (2) 0 ÷ 50

29. (2) 365 ÷ 365

30. (2) Describe a way to check the correctness of a division answer that has no remainder.

LESSON 3 Missing Numbers in Addition • Missing Numbers in Subtraction

Facts Practice: 100 Addition Facts (Test B in Test Masters)

Mental Math: Count by 5's from 5 to 100 and from 100 to 0. Count by 50's from 50 to 1000 and from 1000 to 0.

a. 3000 + 4000 **b.** 600 + 2000 **c.** 20 + 3000

d. 600 + 300 + 20 **e.** 4000 + 300 + 200 **f.** 70 + 300 + 4000

Problem Solving: Shoes in a typical shoe box cannot "get out" because they are closed in by a number of flat surfaces. How many surfaces enclose a pair of shoes in a closed shoe box?

Missing numbers in addition

Below is an addition fact with three numbers. If one of the addends is missing we can use the other addend and the sum to figure out the missing number.

$$\begin{array}{r} 4 \\ +\ 3 \\ \hline 7 \end{array}$$

Cover the 4 with your finger. How can you use the 7 and the 3 to figure out that the number under your finger is 4?

Now cover the 3 instead of the 4. How can you use the other two numbers to figure out that the number under your finger is 3?

Notice that we can find a missing addend by subtracting the known addend from the sum. We will use a letter to stand for a missing number. The letter may be lowercase or uppercase.

Example 1 Find the number for *m*:

$$\begin{array}{r} 12 \\ +\ m \\ \hline 31 \end{array}$$

Solution One of the addends is missing. The known addend is 12. The sum is 31. If we subtract 12 from 31 we find that the missing addend is **19.** We check our answer by using 19 in place of m in the original problem.

$$\begin{array}{r} \overset{2}{\cancel{3}}\,{}^{1}1 \\ -\ 1\,2 \\ \hline 1\,9 \end{array} \xrightarrow{\text{Use 19 in place of } m.} \begin{array}{r} \overset{1}{1}2 \\ +\ 19 \\ \hline 31 \end{array} \quad \text{check}$$

Example 2 Find the number for n:

$$36 + 17 + 5 + n = 64$$

Solution First we add all the known addends.

$$\begin{array}{rl} \underbrace{36 + 17 + 5} & + \ n = 64 \\ 58 \qquad & + \ n = 64 \end{array}$$

Then we find n by subtracting 58 from 64.

$$64 - 58 = 6 \quad \text{So } n \text{ is } \mathbf{6}.$$

We check our work by using 6 in place of n in the original problem to be sure the sum is 64.

$$36 + 17 + 5 + 6 = 64 \quad \text{The answer checks.}$$

Missing numbers in subtraction

Below is a subtraction fact. Cover the 8 with your finger and describe how to use the other two numbers to figure out that the number under your finger is 8.

$$\begin{array}{r} 8 \\ -\ 3 \\ \hline 5 \end{array}$$

Now cover the 3 instead of the 8. Describe how to use the other two numbers to figure out that the covered number is 3.

Example 3 Find the number for W:

$$\begin{array}{r} W \\ -\ 16 \\ \hline 24 \end{array}$$

Solution We can find the first number of a subtraction problem by adding the other two numbers. We add 16 and 24 and get **40.** We check our answer by using 40 in place of *W*.

$$\begin{array}{r} {}^{1} \\ 16 \\ +\ 24 \\ \hline 40 \end{array} \xrightarrow{\text{Use 40 in place of } W.} \begin{array}{r} {}^{3}\ \ \ \\ \cancel{4}{}^{1}0 \\ -\ 1\,6 \\ \hline 2\,4 \end{array} \ \text{check}$$

Example 4 Find the number for *y*:

$$236 - y = 152$$

Solution One way to figure out how to find a missing number is to think of a simpler problem that is similar. Here is a simpler subtraction fact.

$$5 - 3 = 2$$

In the problem, *y* is in the same position as the 3 in the simpler subtraction fact. Just as we can find 3 by subtracting 2 from 5, so we can find *y* by subtracting 152 from 236.

$$\begin{array}{r} {}^{1}\ \ \ \ \ \ \\ \cancel{2}{}^{1}3\,6 \\ -\ 1\,5\,2 \\ \hline 8\,4 \end{array}$$

We find that *y* is **84.** Now we check our answer by using 84 in place of *y* in the original problem.

$$\begin{array}{rl} {}^{1}\ \ \ \ \ \ & \\ \cancel{2}{}^{1}3\,6 & \\ -\ \ \ 8\,4 & \leftarrow \text{Use 84 in place of } y. \\ \hline 1\,5\,2 & \leftarrow \text{The answer checks.} \end{array}$$

To summarize, we find the first number of a subtraction problem by adding the other two numbers. We find the second number by subtracting the difference from the first number.

Practice* Find the missing number in each problem:

a. $\begin{array}{r} A \\ +\ 12 \\ \hline 45 \end{array}$ **b.** $\begin{array}{r} 32 \\ +\ B \\ \hline 60 \end{array}$ **c.** $\begin{array}{r} C \\ -\ 15 \\ \hline 24 \end{array}$ **d.** $\begin{array}{r} 38 \\ -\ D \\ \hline 29 \end{array}$

e. $e + 24 = 52$ **f.** $29 + f = 70$

g. $g - 67 = 43$ **h.** $80 - h = 36$

i. $36 + 14 + n + 8 = 75$

Problem set 3

1. (2) If the two factors are 25 and 12, then what is the product?

2. (1) If the addends are 25 and 12, then what is the sum?

3. (1) What is the difference of 25 and 12?

4. (2) Each of the 31 students brought 75 aluminum cans to class. Find how many cans the class collected by multiplying 31 and 75.

5. (2) Find the total price of one dozen pepperoni pizzas at $7.85 each by multiplying $7.85 by 12.

6. (1) The basketball team scored 63 of its 102 points in the first half of the game. Find how many points the team scored in the second half by subtracting 63 points from 102 points.

7. (2) $\begin{array}{r} \$3.68 \\ \times\ \ \ 9 \\ \hline \end{array}$ **8.** (2) $\begin{array}{r} 407 \\ \times\ 80 \\ \hline \end{array}$ **9.** (2) $\begin{array}{r} 28¢ \\ \times\ 14 \\ \hline \end{array}$ **10.** (2) $\begin{array}{r} 370 \\ \times\ 140 \\ \hline \end{array}$

11. (2) 100×100 **12.** (2) $144 \div 12$ **13.** (2) 12×5

14. (1) $\begin{array}{r} 3627 \\ 598 \\ +\ 4881 \\ \hline \end{array}$ **15.** (1) $\begin{array}{r} 5010 \\ -\ 1376 \\ \hline \end{array}$ **16.** (1) $\begin{array}{r} \$10.00 \\ -\ \ \$0.26 \\ \hline \end{array}$

Find the missing number in each problem:

17. (3)
$$\begin{array}{r} A \\ +\ 16 \\ \hline 48 \end{array}$$

18. (3)
$$\begin{array}{r} 23 \\ +\ \ B \\ \hline 52 \end{array}$$

19. (3)
$$\begin{array}{r} C \\ -\ 17 \\ \hline 31 \end{array}$$

20. (3)
$$\begin{array}{r} 42 \\ -\ \ D \\ \hline 25 \end{array}$$

21. (3) $x + 38 = 75$

22. (3) $x - 38 = 75$

23. (3) $75 - y = 38$

24. (3) $6 + 8 + w + 5 = 32$

25. (1) Rearrange the numbers in this addition fact to make another addition fact and two subtraction facts.

$$24 + 48 = 72$$

26. (2) Rearrange the numbers in this multiplication fact to make another multiplication fact and two division facts.

$$6 \times 15 = 90$$

27. (2) Find the quotient when the divisor is 20 and the dividend is 200.

28. (2) Multiply to find the answer to this addition problem:

$$15 + 15 + 15 + 15 + 15 + 15 + 15 + 15$$

29. (2) $144 \div 144$

30. (3) Describe how to find a missing addend in an addition problem.

LESSON
4

Missing Numbers in Multiplication • Missing Numbers in Division

Facts Practice: 64 Addition Facts (Test A in Test Masters)

Mental Math: Count up and down by 5's between 5 and 100. Count up and down by 50's between 50 and 1000.

a. 600 + 2000 + 300 + 20
b. 3000 + 20 + 400 + 5000
c. 7000 + 200 + 40 + 500
d. 700 + 2000 + 50 + 100
e. 60 + 400 + 30 + 1000
f. 900 + 8000 + 100 + 50

Problem Solving: The digits 1, 3, and 5 can be arranged to make six different three-digit numbers. Two of the six numbers are 135 and 153. What are the other four numbers?

Missing numbers in multiplication

This multiplication fact has three numbers. If one of the factors is missing, we can use the other factor and the product to figure out the missing factor.

$$\begin{array}{r} 4 \\ \times\ 3 \\ \hline 12 \end{array}$$

Cover up the factors in this multiplication fact one at a time. Describe how you can use the two numbers to find the number that is covered. Notice that we can find a missing factor by dividing the product by the known factor.

Example 1 Find the missing number: $\begin{array}{r} A \\ \times\ 6 \\ \hline 72 \end{array}$

Solution The missing number is a factor. The product is 72. The factor that we know is 6. Dividing 72 by 6 we find that the missing factor is **12.** We check our work by using 12 in the original problem.

$$\begin{array}{r} 12 \\ 6\overline{)72} \end{array} \longrightarrow \begin{array}{r} 12 \\ \times\ 6 \\ \hline 72 \end{array} \text{ check}$$

Example 2 Find the missing number: $6w = 84$

Solution When a number and a letter are written side by side it means that they are to be multiplied. In this problem, $6w$ means 6 times w. We divide 84 by 6 and find that the missing factor is **14.** We check the answer by multiplying.

$$\begin{array}{r} 14 \\ 6\overline{)8^{2}4} \end{array} \longrightarrow \begin{array}{r} {}^{2} \\ 14 \\ \times\ 6 \\ \hline 84 \end{array} \text{ check}$$

Missing numbers in division

This division fact has three numbers. If we know two of the numbers we can figure out the third number. Cover each of the numbers with your finger and describe how to use the other two numbers to find the covered number.

$$\begin{array}{r} 4 \\ 6\overline{)24} \end{array}$$

Notice that we can find the dividend, the number inside the division box, by multiplying the other two numbers. We can find either the divisor or quotient, the numbers outside of the box, by dividing.

Example 3 Find the missing number:

$$\frac{k}{6} = 15$$

Solution The letter k is in the position of the dividend. If we rewrite this problem with a division box it looks like this.

$$\begin{array}{r} 15 \\ 6\overline{)k} \end{array}$$

We find a missing dividend by multiplying the divisor and quotient. We multiply 15 by 6 and find that the missing number is **90.** Then we check our work.

$$\begin{array}{r} {}^{3} \\ 15 \\ \times\ 6 \\ \hline 90 \end{array} \longrightarrow \begin{array}{r} 15 \\ 6\overline{)9^{3}0} \end{array} \text{ check}$$

Example 4 Find the missing number: $126 \div m = 7$

Solution The letter m is in the position of the divisor. If we rewrite the problem with a division box it looks like this.

$$\begin{array}{r} 7 \\ m\overline{)126} \end{array}$$

We can find m by dividing 126 by 7.

$$\begin{array}{r} 1\,8 \\ 7\overline{)1\,2^{5}6} \end{array}$$

We find that m is **18.** We will check our answer by multiplying.

$$\begin{array}{r} {}^{5} \\ 18 \\ \times\ 7 \\ \hline 126 \end{array}$$

Practice* Find each missing number:

a. $\begin{array}{r} A \\ \times\ 7 \\ \hline 91 \end{array}$

b. $\begin{array}{r} 20 \\ \times\ B \\ \hline 440 \end{array}$

c. $\begin{array}{r} 15 \\ 7\overline{)C} \end{array}$

d. $\begin{array}{r} 8 \\ D\overline{)144} \end{array}$

e. $7w = 84$

f. $112 = 8m$

g. $\dfrac{360}{x} = 30$

h. $\dfrac{n}{5} = 60$

Problem set 4

1. (2) Five dozen carrot sticks are to be divided evenly among 15 children. Find how many carrot sticks each child should receive by dividing 60 by 15.

2. (2) Matt separated 100 pennies into 4 equal piles. Find how many pennies are in each pile by dividing 100 by 4.

3. (2) Sandra put 100 pennies into stacks of 5 pennies each. Find how many stacks are formed by dividing 100 by 5.

4. *(2)* Find the number of 14-player soccer teams that can be formed if 294 players sign up for soccer by dividing 294 by 14.

5. *(1)* Tom is reading a 280-page book. He has just finished page 156. Find how many pages he still has to read by subtracting 156 from 280.

6. *(2)* Each month Bill earns \$0.75 per customer for delivering newspapers. Find how much money he would earn in a month in which he had 42 customers by multiplying \$0.75 by 42.

Find the missing number:

7. *(4)*
$$\begin{array}{r} J \\ \times\ 5 \\ \hline 60 \end{array}$$

8. *(3)*
$$\begin{array}{r} 27 \\ +\ K \\ \hline 72 \end{array}$$

9. *(3)*
$$\begin{array}{r} L \\ +\ 36 \\ \hline 37 \end{array}$$

10. *(3)*
$$\begin{array}{r} 64 \\ -\ M \\ \hline 46 \end{array}$$

11. *(3)* $n - 48 = 84$

12. *(4)* $7p = 91$

13. *(4)* $q \div 7 = 0$

14. *(4)* $144 \div r = 6$

15. *(2)* $6\overline{)\$12.36}$

16. *(2)* $\dfrac{5760}{8}$

17. *(2)* $526 \div 18$

18. *(1)* $563 + 563 + 563 + 563$

19. *(2)* $\$3.75 \times 16$

20. *(1)* $\$3 + \$2.86 + \$0.98$

21. *(1)* $\$10 - \$6.43$

22. *(4)* If the divisor is 3 and the quotient is 12, then what is the dividend?

23. *(4)* If the product is 100 and one factor is 5, then what is the other factor?

24. *(1)* Rearrange the numbers in this subtraction fact to make another subtraction fact and two addition facts.

$$17 - 9 = 8$$

25. (2) Rearrange the numbers in this division fact to make another division fact and two multiplication facts.

$$72 \div 8 = 9$$

26. (3) $w + 6 + 8 + 10 = 40$

27. (2) Find the answer to this addition problem by multiplying:

$$23¢ + 23¢ + 23¢ + 23¢ + 23¢ + 23¢ + 23¢$$

28. (4) $25m = 25$

29. (4) $15n = 0$

30. (4) Describe how to find a missing factor in a multiplication problem.

LESSON 5

Order of Operations, Part 1

Facts Practice: 100 Addition Facts (Test B in Test Masters)

Mental Math: Count by 25's from 25 to 1000.

a. 560 + 200 **b.** 840 + 30 **c.** 5200 + 2000

d. 650 + 140 **e.** 3800 + 2000 **f.** 440 + 200

Problem Solving: Copy this addition problem and fill in the five missing digits.

$$\begin{array}{r} 1\_5 \\ +\ \_3\_ \\ \hline \_\_93 \end{array}$$

When there is more than one addition or subtraction step within a problem, we take the steps in order from left to right. In this problem we first subtract 4 from 9. Then we add 3.

$$9 - 4 + 3 = 8$$

If a different order of steps is desired, parentheses are used to show which step should be taken first. In the problem below we first add 4 and 3, which equals 7. Then we subtract 7 from 9.

$$9 - (4 + 3) = 2$$

Example 1 (a) $18 - 6 - 3$ (b) $18 - (6 - 3)$

Solution (a) We subtract in order from left to right.

$\underbrace{18 - 6} - 3$ First subtract 6 from 18.
$12 - 3$ Then subtract 3 from 12.
9 The answer is 9.

(b) We subtract within the parentheses first.

$18 - \underbrace{(6 - 3)}$ First subtract 3 from 6.
$18 - 3$ Then subtract 3 from 18.
15 The answer is 15.

When there is more than one multiplication or division step within a problem, we take the steps in order from left to right.

$$24 \div 6 \times 2 = 8$$

If there are parentheses we first do the work within the parentheses. In this problem we first multiply 6 by 2 and get 12. Then we divide 24 by 12.

$$24 \div (6 \times 2) = 2$$

Example 2 (a) $18 \div 6 \div 3$ (b) $18 \div (6 \div 3)$

Solution (a) We take the steps in order from left to right.

$\underbrace{18 \div 6} \div 3$ First divide 18 by 6.
$3 \div 3$ Then divide 3 by 3.
1 The answer is 1.

(b) We divide within the parentheses first.

$$18 \div \underbrace{(6 \div 3)}_{2}$$ First divide 6 by 3.

$$18 \div 2$$ Then divide 18 by 2.

$$\mathbf{9}$$ The answer is 9.

Example 3 $\dfrac{5 + 7}{1 + 2}$

Solution Before dividing we add above the bar and below the bar. Then we divide 12 by 3. The quotient is 4.

$$\frac{5 + 7}{1 + 2} = \frac{12}{3} = \mathbf{4}$$

Practice

a. $16 - 3 + 4$

b. $16 - (3 + 4)$

c. $24 \div (4 \times 3)$

d. $24 \div 4 \times 3$

e. $24 \div 6 \div 2$

f. $24 \div (6 \div 2)$

g. $\dfrac{6 + 9}{3}$

h. $\dfrac{12 + 8}{12 - 8}$

Problem set 5

1. (1) Jack paid $5 for a hamburger that cost $1.25 and a drink that cost $0.60. How much change should he get back?

2. (1) In one day the elephant ate 82 pounds of straw, 8 pounds of apples, and 12 pounds of peanuts. How many pounds of food did it eat in all?

3. (1) What is the difference of 110 and 25?

4. (2) What is the total price of one dozen apples that cost 25¢ each?

5. (3) What number must be added to 149 to total 516?

6. *(2)* Judy plans to read a 235-page book in 5 days. Describe how to find the average number of pages she needs to read each day.

7. *(5)* $5 + (3 \times 4)$

8. *(5)* $(5 + 3) \times 4$

9. *(5)* $800 - (450 - 125)$

10. *(5)* $600 \div (20 \div 5)$

11. *(5)* $800 - 450 - 125$

12. *(5)* $600 \div 20 \div 5$

13. *(5)* $144 \div (8 \times 6)$

14. *(5)* $144 \div 8 \times 6$

15. *(5)* $\$5 - (\$1.25 + \$0.60)$

16. *(2)* Use the numbers 63, 7, and 9 to make two multiplication facts and two division facts.

17. *(4)* If the quotient is 12 and the dividend is 288, then what number is the divisor?

18. *(2)* $25\overline{)\$10.00}$

19. *(2)* $\begin{array}{r} 378 \\ \times\ 64 \\ \hline \end{array}$

20. *(2)* $\begin{array}{r} 506 \\ \times\ 370 \\ \hline \end{array}$

21. *(1)* $\begin{array}{r} \$10.10 \\ -\ \$9.89 \\ \hline \end{array}$

22. *(3)* $n - 63 = 36$

23. *(3)* $63 - p = 36$

24. *(3)* $56 + m = 432$

25. *(4)* $8w = 480$

26. *(3)* $5 + 12 + 27 + y = 50$

27. *(4)* $36 \div a = 4$

28. *(4)* $x \div 4 = 8$

29. *(1)* Use the numbers 7, 11, and 18 to make two addition facts and two subtraction facts.

30. *(5)* $3 \times 4 \times 5$

LESSON 6

Fractional Parts

Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

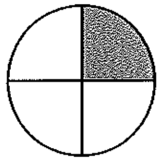
Mental Math: Count up and down by 25's between 25 and 1000.

a. 2500 + 400	**b.** 6000 + 2400	**c.** 370 + 400
d. 9500 + 240	**e.** 360 + 1200	**f.** 480 + 2500

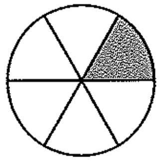
Problem Solving: Alex had seven coins in his pocket totaling exactly one dollar. Name a possible collection of coins in his pocket. How many different collections of coins are possible?

When we first began to learn about numbers as young children we counted objects. When we count we are using whole numbers. As we grew older we discovered that there are parts of wholes—like parts of a candy bar—which cannot be named with whole numbers. We can name these parts with **fractions.** A common fraction is written with two numbers and a fraction bar. The "bottom" number is the **denominator.** The denominator shows the number of equal parts in the whole. The "top" number, the **numerator,** shows the number of the parts being described.

We see that this whole circle has been divided into **4** equal parts; **1** part is shaded. The fraction of the circle which is shaded is 1 out of 4 parts. We call this part one fourth and write it as $\frac{1}{4}$.

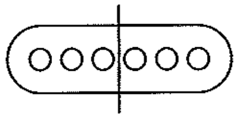


Example 1 What fraction of the circle is shaded?



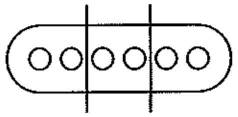
Solution The circle has been divided into 6 equal parts. We use 6 for the bottom of the fraction. One of the parts is shaded, so we use 1 for the top of the fraction. The fraction of the circle which is shaded is one sixth, which we write as $\frac{1}{6}$.

We can also use a fraction to name a part of a group. There are 6 members in this group. We can divide this group in half by dividing it into two equal groups with 3 in each half. We write that $\frac{1}{2}$ of 6 is 3.



$\frac{1}{2}$ of 6 is 3

We can divide this group into thirds by dividing the 6 members into three equal groups. We write that $\frac{1}{3}$ of 6 is 2.



$\frac{1}{3}$ of 6 is 2

Example 2 (a) What number is $\frac{1}{2}$ of 450?

(b) What number is $\frac{1}{3}$ of 450?

(c) How much money is $\frac{1}{5}$ of \$4.50?

Solution (a) To find $\frac{1}{2}$ of 450 we divide 450 into two equal parts and find the amount in one of the parts. We find that $\frac{1}{2}$ of 450 is **225.**

$$\begin{array}{r} 225 \\ 2\overline{)450} \end{array}$$

$\frac{1}{2}$ of 450 is 225

(b) To find $\frac{1}{3}$ of 450 we divide 450 into three equal parts. Since each part is 150, we find that $\frac{1}{3}$ of 450 is **150.**

$$\begin{array}{r} 150 \\ 3\overline{)450} \end{array}$$

$\frac{1}{3}$ of 450 is 150

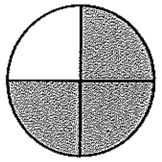
(c) To find $\frac{1}{5}$ of \$4.50 we divide \$4.50 by 5. We find that $\frac{1}{5}$ of \$4.50 is **\$0.90.**

$$\begin{array}{r} \$0.90 \\ 5\overline{)\$4.50} \end{array}$$

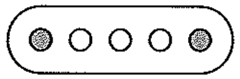
$$\tfrac{1}{5} \text{ of } \$4.50 \text{ is } \$0.90$$

Practice Use both words and digits to write the fraction that is shaded:

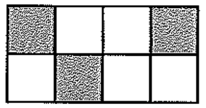
a.



b.



c.



d. What number is $\frac{1}{2}$ of 72?

e. What number is $\frac{1}{2}$ of 1000?

f. What number is $\frac{1}{3}$ of 180?

g. How much money is $\frac{1}{3}$ of \$3.60?

Problem set 6

1. *(6)* What number is $\frac{1}{2}$ of 540?

2. *(6)* What number is $\frac{1}{3}$ of 540?

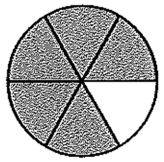
3. *(1)* In four days of sight-seeing the Richmonds drove 346 miles, 417 miles, 289 miles, and 360 miles, respectively. How many miles did they drive in all?

4. *(1)* Brad paid \$20 for a book that cost \$12.08. How much money should he get back?

5. *(2)* How many days are in 52 weeks?

6. *(2)* How many \$20 bills would it take to make \$1000?

7. (6) Use words and digits to write the fraction that is shaded.



8. (1)
$$\begin{array}{r} 3604 \\ 5186 \\ +\ 7145 \\ \hline \end{array}$$

9. (1)
$$\begin{array}{r} \$30.01 \\ -\ \$15.76 \\ \hline \end{array}$$

10. (2)
$$\begin{array}{r} 376 \\ \times\ \ 87 \\ \hline \end{array}$$

11. (2)
$$\begin{array}{r} 470 \\ \times\ 203 \\ \hline \end{array}$$

12. (1) $20 − $11.98

13. (5) 596 − (400 − 129)

14. (5) 32 ÷ (8 × 4)

15. (2) $8\overline{)4016}$

16. (2) $15\overline{)6009}$

17. (2) $36\overline{)9000}$

18. (4)
$$\begin{array}{r} W \\ \times\ \ 8 \\ \hline 480 \end{array}$$

19. (3)
$$\begin{array}{r} X \\ -\ 64 \\ \hline 46 \end{array}$$

20. (4) $\frac{49}{N} = 7$

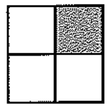
21. (4) $\frac{M}{7} = 15$

22. (2) $6.35 × 12

23. (3) $365 + P = 653$

24. (4) $9 \times \square = 720$

25. (6) This square was divided in half. Then each half was divided in half. What fraction of the square is shaded?



26. (3) 36¢ + 25¢ + m = 99¢

27. (1) Use the numbers 2, 4, and 6 to make two addition facts and two subtraction facts.

28. (2) Write two multiplication facts and two division facts using the numbers 2, 4, and 8.

29. (2) Multiply to find the answer to this addition problem.

38 + 38 + 38 + 38 + 38 + 38 + 38 + 38 + 38 + 38

30. (6) Make up a fractional part question about money like Example 2 (c). Then find the answer.

LESSON 7

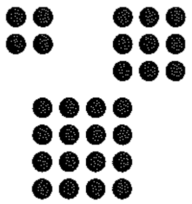
Linear Measure

Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

Mental Math: Count up and down by $\frac{1}{2}$'s between $\frac{1}{2}$ and 10. Count up and down by 2's between 2 and 40.

a. 800 − 300 **b.** 3000 − 2000 **c.** 450 − 100
d. 2500 − 300 **e.** 480 − 80 **f.** 750 − 250

Problem Solving: Sharon made three square patterns using 4 coins, 9 coins, and 16 coins. If she continues the patterns, how many coins will she need for each of the next two square patterns?



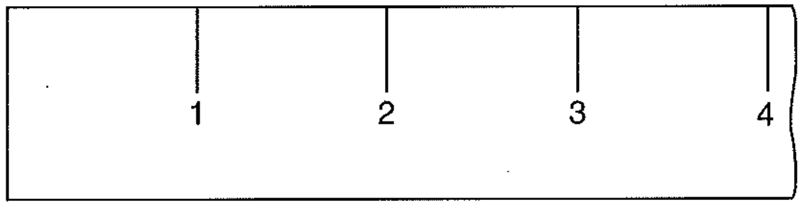
As civilized people we have agreed upon certain units of measure. In the United States we have two systems of units that we use to measure length. One system is the **U.S. Customary System.** Some of the units in this system are inches (in.), feet (ft), yards (yd), and miles (mi). The other system is the **metric system.** Some of the units in the metric system are millimeters (mm), centimeters (cm), meters (m), and kilometers (km).

Activity: Inch Ruler

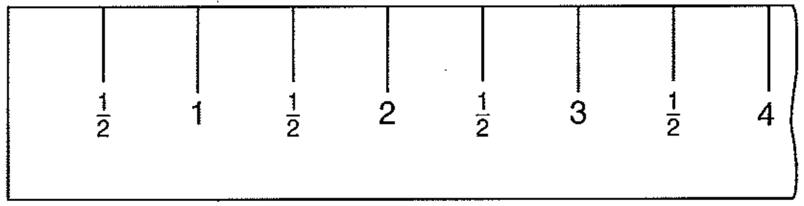
Materials needed by each student:

- Inch ruler
- Narrow strip of tagboard about 6 in. long by 1 in. wide
- Pencil

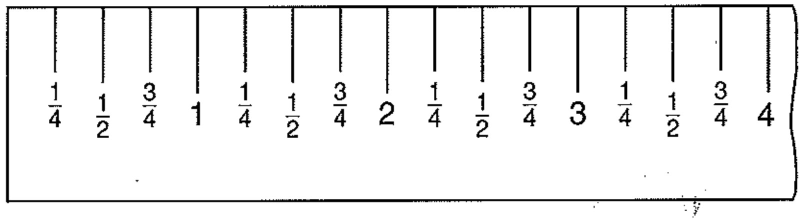
We will use estimation to make an inch ruler out of tagboard marked to fourths of an inch. Use your pencil and ruler to draw inch marks on the strip of tagboard. Number the inch marks. When you are done, the tagboard strip should look like this.



Now set your ruler aside. We will use estimation to make the rest of the marks on the tagboard strip. Estimate the halfway point between inch marks and make the half inch marks a little bit shorter than the inch marks. Write "$\frac{1}{2}$" below each of these marks.



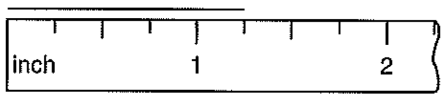
Every half inch is marked on your ruler. Now we will show every quarter inch by making a shorter mark halfway between each mark on the tagboard ruler. Label the marks as we show here.



You may use this ruler to help you with the measurements in the problem sets. Save this tagboard ruler. We will be making more marks on it in a few days.

A metric ruler is divided into centimeters. Each centimeter is divided into ten millimeters. So one centimeter equals 10 millimeters, and two centimeters equals 20 millimeters.

Example 1 How long is the line segment?



Solution The line is one whole inch plus a fraction. The fraction is one fourth. The length of the line is **$1\frac{1}{4}$ in.**

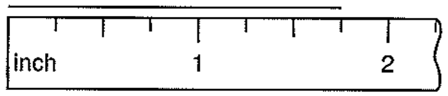
Note: In this book, the abbreviation for "inches" ends with a period. The abbreviations for other units do not end with periods.

Example 2 How long is the line segment?

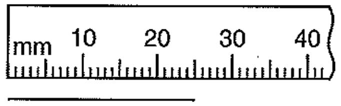
Solution We simply read the scale to see that the line is **2 cm** long. The segment is also **20 mm** long.

Practice How long is each line segment?

a.



b.



Problem set 7

1. (2) To earn money for gifts, Debbie sold decorated pine cones. If she sold 100 cones at $0.25 each, how much money did she earn?

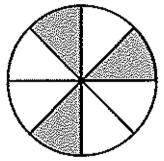
2. There are 365 days in a normal year. April 1 is the 91st day. How many days are left in the year after April 1?
(1)

3. The Smiths are planning to complete a 1890-mile trip in 3 days. If they drive 596 miles the first day and 612 miles the second day, how far must they travel the third day? (*Hint*: This is a two-step problem. First find how far the Smiths traveled the first two days.)
(5)

4. What number is $\frac{1}{2}$ of 234?
(6)

5. How much money is $\frac{1}{3}$ of \$2.34?
(6)

6. Use words and digits to write the fraction of the circle that is shaded.
(6)



7. *(1)*
$$\begin{array}{r} 3654 \\ 2893 \\ +\ 5614 \\ \hline \end{array}$$

8. *(1)*
$$\begin{array}{r} \$41.01 \\ -\ \$15.76 \\ \hline \end{array}$$

9. *(2)*
$$\begin{array}{r} 28¢ \\ \times\ 74 \\ \hline \end{array}$$

10. *(2)*
$$\begin{array}{r} 906 \\ \times\ \ 47 \\ \hline \end{array}$$

11. $6\overline{)5000}$
(2)

12. 800 ÷ 16
(2)

13. $60\overline{)3174}$
(2)

14. $3 + 6 + 5 + w + 4 = 30$
(3)

15. 300 − 30 + 3
(5)

16. 300 − (30 + 3)
(5)

17. *(2)*
$$\begin{array}{r} \$4.32 \\ \times\ \ \ 20 \\ \hline \end{array}$$

18. *(2)*
$$\begin{array}{r} 48¢ \\ \times\ 24 \\ \hline \end{array}$$

19. \$8.75 ÷ 25
(2)

Find the missing numbers:

20. $W \div 6 = 7$
(4)

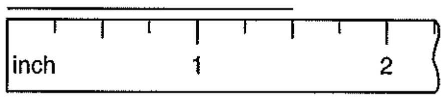
21. $6n = 96$
(4)

22. $58 + r = 213$
(3)

23. Rearrange the numbers in this subtraction fact to make another subtraction fact and two addition facts.
(1)

$$60 - 24 = 36$$

24. How long is the line segment?
(7)



25. Find the length, in centimeters and in millimeters, of the line segment.
(7)

26. Use the numbers 9, 10, and 90 to make two multiplication facts and two division facts.
(2)

27. Describe how to find a missing dividend in a division problem.
(3)

28. $w - 12 = 8$
(3)

29. $12 - x = 8$
(3)

30. A meter stick is 100 centimeters long. One hundred centimeters is how many millimeters?
(7)

LESSON
8
Perimeter

Facts Practice: 64 Addition Facts (Test A in Test Masters)

Mental Math: Count up by $\frac{1}{4}$'s from $\frac{1}{4}$ to 10.

a. 400 + 2400 **b.** 750 + 36 **c.** 8400 + 520
d. 980 − 60 **e.** 4400 − 2000 **f.** 480 − 120

Problem Solving: As you sit straight in your desk, you can describe the locations of people and objects in your classroom compared to your position. Perhaps a friend is in front and to the left. Perhaps a door is directly to your right. Describe the location of your teacher's desk, the pencil sharpener, and a person or object of your choice.

The distance around a shape is its **perimeter.** The perimeter of a square is the distance around it. The perimeter of a room is the distance around the room.

Activity: Perimeter

Walk the perimeter of the classroom. Start at a point along a wall of the classroom. Staying close to the walls, walk around the room until you return to your starting point. Count your steps as you travel around the room. How many of your steps is the perimeter of the room? When you are finished, discuss these questions:

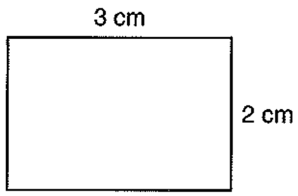
a. Did everyone count the same number of steps?

b. Does the perimeter depend upon who is measuring it?

c. Which of these is the best physical example of perimeter?

1. The tile or carpet that covers the floor.
2. The molding along the base of the wall.

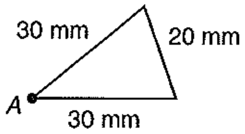
Here we have a rectangle that is 3 cm long and 2 cm wide.



If we were to trace the perimeter of the rectangle, our pencil would travel 3 cm, then 2 cm, then 3 cm, then 2 cm to get all the way around the rectangle. We add these lengths to find the perimeter of the rectangle.

3 cm + 2 cm + 3 cm + 2 cm = **10 cm**

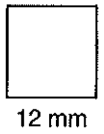
Example What is the perimeter of the triangle?



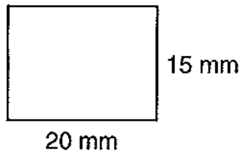
Solution The perimeter of a shape is the distance around it. If we use a pencil to trace a triangle of the given dimensions from point *A*, the point of the pencil would travel 30 mm, then 20 mm, then 30 mm. Adding these distances, we find that the perimeter is **80 mm.**

Practice What is the perimeter of each shape?

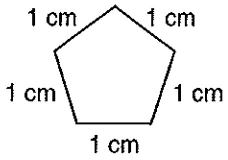
a. Square



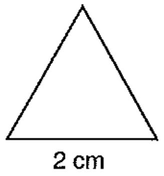
b. Rectangle



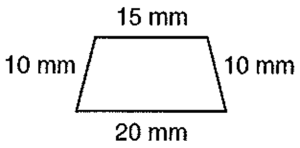
c. Pentagon



d. Equilateral triangle



e. Trapezoid



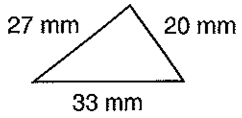
Problem set 8

1. (2) In an auditorium there are 25 rows of chairs with 18 chairs in each row. How many chairs are in the auditorium?

2. (1) All the king's horses numbered 765. All the king's men numbered 1750. Find how many fewer horses than men the king had by subtracting 765 from 1750.

3. (2) Robin Hood divided 140 of his merry men into 5 equal groups. How many were in each group?

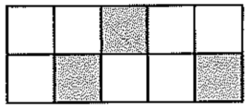
4. (8) What is the perimeter of the triangle?



5. (6) How much money is $\frac{1}{2}$ of \$6.54?

6. (6) What number is $\frac{1}{3}$ of 654?

7. (6) What fraction of the rectangle is shaded?



8. (2) $4\overline{)\$9.00}$ **9.** (2) $10\overline{)373}$ **10.** (2) $12\overline{)1500}$ **11.** (2) $39\overline{)800}$

12. (5) $400 \div 20 \div 4$ **13.** (5) $400 \div (20 \div 4)$

14. (2) Use the numbers 240, 20, and 12 to make two multiplication facts and two division facts.

15. (1) Rearrange the numbers in this addition fact to make another addition fact and two subtraction facts.

$$60 + 80 = 140$$

16. (8) What is the perimeter of a square tile with sides 12 inches long?

17. (1,2) Find the sum of 6 and 4 and find the product of 6 and 4.

18. $\$5 - M = \$1.48$
(3)

19. $10 \times 20 \times 30$
(5)

20. $825 \div 8$
(2)

Find the missing numbers:

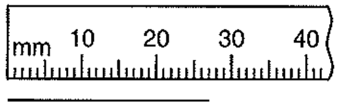
21. $w - 63 = 36$
(3)

22. $150 + 165 + a = 397$
(3)

23. $12w = 120$
(4)

24. If the divisor is 8 and the quotient is 24, then what is the dividend?
(4)

25. How many millimeters long is the line segment?
(7)



26. Use your ruler to help you draw a line segment that is $2\frac{3}{4}$ in. long.
(7)

27. $w - 27 = 18$
(3)

28. $27 - x = 18$
(3)

29. Multiply to find the answer to this addition problem:
(2)

$$35 + 35 + 35 + 35$$

30. Describe how to find the perimeter of a rectangle.
(8)

LESSON 9

The Number Line: Ordering and Comparing

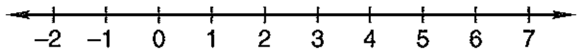
Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

Mental Math: Count up and down by 25's between 25 and 1000.

a. 48 + 120 **b.** 76 + 10 + 3 **c.** 7400 + 320
d. 860 − 50 **e.** 960 − 600 **f.** 365 − 200

Problem Solving: The digits 2, 4, and 6 can be arranged to form six different three-digit numbers. List the six numbers in order from least to greatest.

A **number line** is a way to show numbers in order.

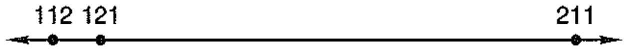


The arrowheads show that the line continues without end and that the numbers continue without end. The small marks crossing the horizontal line are called **tick marks.** Number lines may be labeled with various types of numbers. The numbers we say when we count (1, 2, 3, 4, and so on) are called **counting numbers. Whole numbers** are all of the counting numbers plus the number zero. To the left of zero on this number line are **negative numbers,** which will be described in later lessons. As we move to the right on this number line, the numbers are greater and greater in value. As we move to the left, the numbers are less and less in value.

Example 1 Arrange the numbers in order from least to greatest:

121 112 211

Solution On a number line these three numbers appear in order from least (on the left) to greatest (on the right).



For our answer we write

112 121 211

When we **compare** two numbers we decide whether the numbers are equal or if one number is greater and the other is less. We may show a comparison with symbols. If the numbers are equal, the comparison sign we use is the **equals sign** (=).

$$1 + 1 = 2$$

If the numbers are not equal, we use the **greater than/less than sign** (>). The greater than/less than sign may point to the right or to the left (> or <). When the symbol is properly placed between two numbers, the small point of the symbol points to the smaller number.

Example 2 Compare: $5012 \bigcirc 5102$

Solution In place of the circle we should write =, >, or < to make the statement true. Since 5012 is less than 5102, we point the small end to the 5012.

$$\mathbf{5012 < 5102}$$

Example 3 Compare: $16 \div 8 \div 2 \bigcirc 16 \div (8 \div 2)$

Solution Before we compare the two expressions, we find the number each expression equals.

$$\underbrace{16 \div 8 \div 2}_{1} \bigcirc \underbrace{16 \div (8 \div 2)}_{4}$$

Since 1 is less than 4, the point of the comparison symbol points to the left.

$$\mathbf{16 \div 8 \div 2 < 16 \div (8 \div 2)}$$

Example 4 Use digits and other symbols to write this comparison:

One fourth is less than one half.

Solution We write the numbers in the order stated.

$$\mathbf{\frac{1}{4} < \frac{1}{2}}$$

Practice

a. Arrange these amounts of money in order from least to greatest:

12¢ $12 $1.20

b. Compare: $16 - 8 - 2$ ○ $16 - (8 - 2)$

c. Compare: $8 \div 4 \times 2$ ○ $8 \div (4 \times 2)$

d. 2×3 ○ $2 + 3$

e. $1 \times 1 \times 1$ ○ $1 + 1 + 1$

f. Use digits and other symbols to write this comparison:

One half is greater than one fourth.

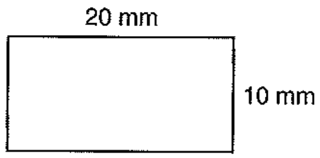
Problem set 9

1. (2) Tamara arranged 144 books into 8 equal stacks. How many books were in each stack?

2. (1) Find how many years there were from 1492 to 1603 by subtracting 1492 from 1603.

3. (2) Martin is carrying groceries in from the car. If he can carry 2 bags at a time, how many trips will it take him to carry in 9 bags?

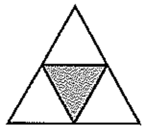
4. (8) What is the perimeter of the rectangle?



5. (6) How much money is $\frac{1}{2}$ of $5.80?

6. (6) How many cents is $\frac{1}{4}$ of a dollar?

7. (6) Use words and digits to name the fraction of the triangle that is shaded.



8. (9) Compare: 5012 ○ 5120

9. (9) Arrange these numbers in order from least to greatest:

$1, 0, \frac{1}{2}$

10. *(9)* Compare: 100 − 50 − 25 ○ 100 − (50 − 25)

11. *(1)*
$$\begin{array}{r} 478 \\ 3692 \\ +\ \ 45 \\ \hline \end{array}$$

12. *(1)*
$$\begin{array}{r} \$50.00 \\ -\ \$31.76 \\ \hline \end{array}$$

13. *(2)*
$$\begin{array}{r} \$4.20 \\ \times\ \ \ \ 60 \\ \hline \end{array}$$

14. *(2)*
$$\begin{array}{r} 78 \\ \times\ 36 \\ \hline \end{array}$$

15. *(2)* $9\overline{)7227}$

16. *(2)* $25\overline{)7600}$

17. *(2)* $20\overline{)8014}$

18. *(2)* 7136 ÷ 100

19. *(2)* 736 ÷ 736

Find the missing numbers:

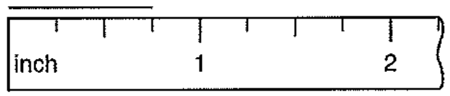
20. *(3)* $165 + a = 300$

21. *(3)* $b - 68 = 86$

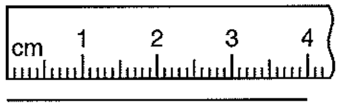
22. *(4)* $9c = 144$

23. *(4)* $d \div 15 = 7$

24. *(7)* How long is the line segment?



25. *(7)* How many millimeters long is the line segment?



26. *(9)* Use digits and symbols to write this comparison:

One half is greater than one third.

27. *(2)* Arrange the numbers 9, 11, and 99 to form two multiplication facts and two division facts.

28. *(9)* Compare: 25 + 0 ○ 25 × 0

29. *(3)* $100 = 20 + 30 + 40 + x$

30. *(9)* Describe how to properly position a greater than/less than sign (>) to correctly show the comparison in problem 8.

LESSON
10

Sequences • Scales

Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

Mental Math: Count by $\frac{1}{4}$'s from $\frac{1}{4}$ to 10.

a. 43 + 20 + 5
b. 670 + 200
c. 254 + 20 + 5
d. 100 − 50
e. 300 − 50
f. 3600 − 400

Problem Solving: Use the digits 5, 6, 7, and 8 to complete this addition problem. There are two possible arrangements.

$$\begin{array}{r} \_\,\_ \\ +\ \ 9 \\ \hline \_\,\_ \end{array}$$

Sequences

A **sequence** is an ordered list of numbers that follows a certain rule. Here are two different sequences.

(a) 5, 10, 15, 20, 25, ...

(b) 5, 10, 20, 40, 80, ...

Sequence (a) is an **addition sequence** because the same number is added to each term of the sequence to get the next term. In this case, 5 is added to each term. Sequence (b) is a **multiplication sequence** because each term of the sequence is multiplied by the same number to get the next term. In (b) each term is multiplied by 2. When we are asked to find missing numbers in a sequence, we inspect the numbers to discover the rule for the sequence. Then we use the rule to find other numbers in the sequence.

Example 1 What is the next number in this sequence?

1, 3, 9, 27, ______, ...

Solution Inspecting the numbers, we find that each term in the sequence can be found by multiplying the term before it by 3. Multiplying 27 by 3, we find that the next term in the sequence is **81.**

The numbers ..., 0, 2, 4, 6, 8, ... form a special sequence called **even numbers.** We say the even numbers when we "count by twos." Notice that zero is an even number. A whole number with a last digit of 0, 2, 4, 6, or 8 is an even number. The whole numbers which are not even numbers are **odd numbers.** The odd numbers are ... 1, 3, 5, 7, 9, ... An even number of objects can be divided into two equal groups. An odd number of objects cannot be divided into two equal groups.

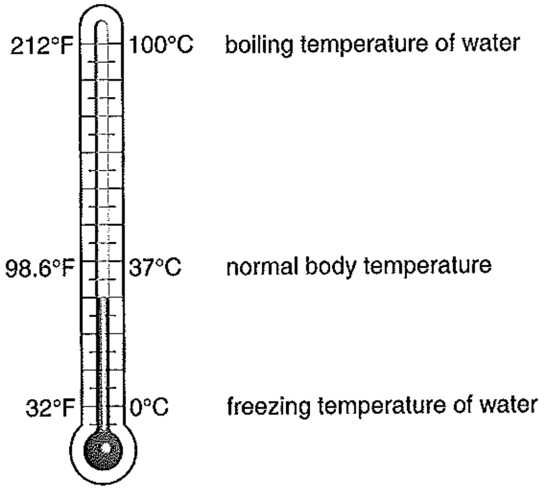
Example 2 Think of a whole number. Double that number. Is the answer even or odd?

Solution The answer is **even.** Doubling any whole number—odd or even—results in an even number.

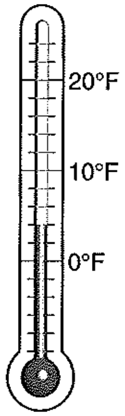
Scales

Numerical information is often presented to us in the form of a **scale** or **graph.** A scale is a display of numbers with an indicator to show where a certain measure falls on the scale. **The trick to reading a scale is to discover the value of the marks on the scale.** Marks on a scale may show every unit or only every two, five, ten, or another number of units. We study the scale to find the value of the units before we try to read the indicated number.

Two commonly used scales on thermometers are the Fahrenheit scale and the Celsius scale. The temperature at which water freezes under standard conditions is 32 degrees Fahrenheit (abbreviated 32°F) and zero degrees Celsius (0°C). The boiling temperature of water is 212°F and 100°C. Normal body temperature is 98.6°F and 37°C. A cool room may be 68°F and 20°C.



Example 3 What temperature is shown on the thermometer?



Solution As we look at the scale on this Fahrenheit thermometer, we see that the tick marks on the scale divide the distance from 0°F to 10°F into five equal sections. So the number of degrees from one tick mark to the next tick mark must be 2°F. Since the fluid in the thermometer is two marks above the zero mark, the temperature shown is **4°F.**

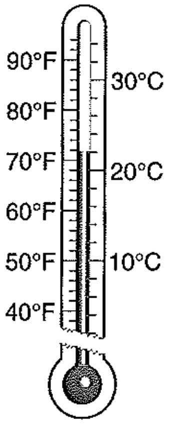
Practice Find the next three numbers in each sequence:

a. 18, 27, 36, 45, _____, _____, _____, ...

b. 1, 2, 4, 8, _____, _____, _____, ...

c. Think of a whole number. Double that number. Then add 1 to the answer. Is the final number even or odd?

d. Find the temperature indicated on this thermometer to the nearest degree Fahrenheit and to the nearest degree Celsius.



Problem set 10

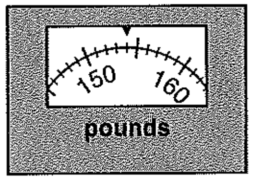
1. (10) Find the next three numbers in this sequence:

16, 24, 32, _____, _____, _____, ...

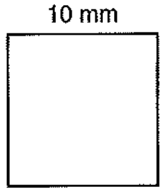
2. (1) Find how many years there were from 1620 to 1776 by subtracting 1620 from 1776.

3. (10) Is the number 1492 even or odd? How can you tell?

4. (10) What weight is indicated on this scale?



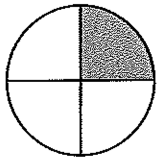
5. (8) What is the perimeter of the square?



6. (6) How much money is $\frac{1}{2}$ of \$6.50?

7. (9) Compare: $4 \times 3 + 2 \bigcirc 4 \times (3 + 2)$

8. (6) Use words and digits to write the fraction of the circle that is **not** shaded.



9. (1,2) What is the product of 100 and 100 and the sum of 100 and 100?

10. (2) $\begin{array}{r} 365 \\ \times\ 100 \\ \hline \end{array}$

11. (2) $\begin{array}{r} 146 \\ \times\ 240 \\ \hline \end{array}$

12. (2) $\begin{array}{r} 78¢ \\ \times\ 48 \\ \hline \end{array}$

13. (2) $\begin{array}{r} 907 \\ \times\ 36 \\ \hline \end{array}$

14. (2) $\frac{4260}{10}$

15. (2) $\frac{4260}{20}$

16. (2) $\frac{4260}{15}$

17. (3) $56 + 28 + 37 + n = 200$

18. (1) 28,347 − 9,637

19. (3) $\$8 + w = \$11.49$

20. (1) \$10 − \$0.75

21. (2) \$0.56 × 60

22. (2) \$6.20 ÷ 4

23. (3) $a - 67 = 49$

24. (3) $67 - b = 49$

25. (4) $8c = 120$

26. (4) $\frac{d}{8} = 24$

27. Here are three ways to write "12 divided by 4."
(2)

$$4\overline{)12} \qquad 12 \div 4 \qquad \frac{12}{4}$$

Show three ways to write "20 divided by 5."

28. What number is one third of 36?
(6)

29. Arrange the numbers 346, 463, and 809 to form two addition equations and two subtraction equations.
(1)

30. At what temperature on the Fahrenheit scale does water freeze?
(10)

LESSON
11

"Some and Some More" and "Some Went Away" Stories

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count up and down by $\frac{1}{2}$'s between $\frac{1}{2}$ and 12.

a. 3 × 40 **b.** 3 × 400 **c.** \$4.50 + \$1.25
d. 451 + 240 **e.** 4500 − 400 **f.** \$5.00 − \$1.50
g. Start with 10; add 2; divide by 2; add 2; divide by 2; subtract 2.

Problem Solving: Sandra has 2¢ stamps, 3¢ stamps, 10¢ stamps, and 29¢ stamps. What is the fewest number of stamps she can use to mail a package that requires \$1.15 postage?

Millions of people go to the theaters to watch stories. Millions buy books to read stories. Stories can entertain us, inspire us, and instruct us. When we analyze stories we talk about the characters, the settings, and the plots. The plot is often about a problem that develops for the main characters and how that problem is resolved.

Many of the stories we analyze in mathematics also have plots. Two common mathematical stories are stories with "some and some more" plots and stories with "some went away" plots. Here are some examples.

A "some and some more" story:

Before he went to work, Tom had $24.50. He earned $12.50 more putting up a fence. Then Tom had $37.00. (Plot: Tom had some money and then he earned some more money.)

A "some went away" story:

Tom took $37.00 to the music store. He bought a pair of headphones for $26.17. Then Tom had $10.83. (Plot: Tom had some money, but some went away when he spent it.)

A "some and some more" story has an addition thought pattern. We fit the numbers from the story into the pattern.

Pattern	Problem
Some	$24.50
+ Some More	+ $12.50
Total	$37.00

A "some went away" story has a subtraction thought pattern. We fit the numbers from the story into the pattern.

Pattern	Problem
Some	$37.00
– Some Went Away	– $26.17
What is left	$10.83

Questions arise when one of the numbers in the story is missing. Since each of these stories has three numbers, three different questions could be asked for each story.

To answer a question about a "some and some more" story or about a "some went away" story we follow these four steps.

Step 1. Read the story and recognize the pattern.

Step 2. Sketch the pattern and record the given information.

Step 3. Find the missing number to complete the pattern.

Step 4. Answer the question.

Note: When referring to "some and some more" stories and to "some went away" stories, we will sometimes use the abbreviations SSM and SWA, respectively.

Example 1 Jenny rode her bike on a trip with her bicycling club. After the first day her trip odometer showed that she had traveled 86 miles. After the second day the trip odometer showed that she had traveled a total of 163 miles. How far did Jenny ride the second day?

Solution

Step 1. Jenny rode some miles and then rode some more miles. We **recognize** that this is a "some and some more" story.

Step 2. The trip odometer showed how far she traveled the first day and the total of the first two days. We **record** the information in the pattern. We'll use a letter in place of the missing number.

$$\begin{array}{r} \text{Some} \\ + \text{ Some More} \\ \hline \text{Total} \end{array} \qquad \begin{array}{r} 86 \text{ miles} \\ + \; m \text{ miles} \\ \hline 163 \text{ miles} \end{array}$$

Step 3. We **find the missing number** that completes the pattern. From Lesson 3 we know that we can find the missing addend by subtracting 86 miles from 163 miles. We check the answer.

$$\begin{array}{r} 163 \text{ miles} \\ - \;\; 86 \text{ miles} \\ \hline 77 \text{ miles} \end{array} \longrightarrow \begin{array}{r} 86 \text{ miles} \\ + \; 77 \text{ miles} \\ \hline 163 \text{ miles} \end{array}$$

Step 4. We answer the question. On the second day of the trip Jenny rode **77 miles.**

Example 2 Nancy counted 47 prairie dogs standing in the field. When a hawk flew over the field some of the prairie dogs ducked into their burrows. Then Nancy counted 29 prairie dogs. How many prairie dogs ducked into their burrows when the hawk flew over the field?

Solution **Step 1.** We recognize this as a "some went away" story.

Step 2. We sketch the pattern and record the information.

$$\begin{array}{r} \text{Some} \\ -\ \text{Some Went Away} \\ \hline \text{What is left} \end{array} \qquad \begin{array}{r} 47\ \text{prairie dogs} \\ -\ \ d\ \text{prairie dogs} \\ \hline 29\ \text{prairie dogs} \end{array}$$

Step 3. We find the missing number by subtracting 29 from 47. We check the answer.

$$\begin{array}{r} 47\ \text{prairie dogs} \\ -\ 29\ \text{prairie dogs} \\ \hline 18\ \text{prairie dogs} \end{array} \longrightarrow \begin{array}{r} 47\ \text{prairie dogs} \\ -\ 18\ \text{prairie dogs} \\ \hline 29\ \text{prairie dogs} \end{array}$$

Step 4. We answer the question. When the hawk flew over the field **18 prairie dogs** ducked into their burrows.

Practice Identify each story below as a SSM or a SWA story, and answer the question.

a. When Tim finished page 129 of a 314-page book, how many pages did he still have to read?

b. The football team scored 19 points in the first half of the game and 42 points by the end of the game. How many points did the team score in the second half of the game?

Problem set 11

1. (11) John ran 8 laps and rested. Then he ran some more laps. If John ran 21 laps in all, how many laps did he run after he rested? (Use the SSM pattern.)

2. (1,2) What is the product of 8 and 4? What is the sum of 8 and 4?

3. (5) Here we show the product of 6 and 4 divided by the difference of 8 and 5. What is the quotient?

$$(6 \times 4) \div (8 - 5)$$

4. (11) Marcia went to the store with $20.00 and returned home with $7.75. How much money did Marcia spend at the store? (Use a SWA pattern.)

5. (11) When Jack went to bed at night, the beanstalk was one meter tall. When he woke up in the morning, the beanstalk was one thousand meters tall. How many meters had the beanstalk grown during the night? (Use a SSM pattern.)

6. (1) $0.65 + $0.40

7. (3) $87 + w = 155$

8. (3) $1000 - x = 386$

9. (3) $y - 1000 = 386$

10. (3) $42 + 596 + m = 700$

11. (9) Compare: $1000 - (100 - 10)$ ○ $1000 - 100 - 10$

12. (2) $8\overline{)1000}$

13. (2) $10\overline{)987}$

14. (4) $\begin{array}{r} 35 \\ 12\overline{)w} \end{array}$

15. (2) 600×300

16. (4) $365w = 365$

17. (10) What are the next three numbers in the sequence?

2, 6, 10, _____, _____, _____, ...

18. (5) $2 \times 3 \times 4 \times 5$

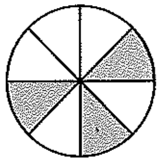
19. (6) What number is $\frac{1}{2}$ of 360?

20. (6) What number is $\frac{1}{4}$ of 360?

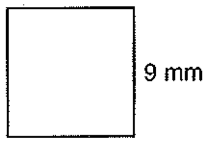
21. (2) What is the product of eight and one hundred twenty-five?

22. How long is the line segment?
(7)

23. What fraction of the circle is not shaded?
(6)



24. What is the perimeter of the square?
(8)



25. What is the sum of the first five odd numbers greater than zero?
(10)

26. Here are three ways to write "24 divided by 4."
(2)

$$4\overline{)24} \qquad 24 \div 4 \qquad \frac{24}{4}$$

Show three ways to write "30 divided by 6."

27. Seventeen of the 30 students in a class are girls. So the girls are $\frac{17}{30}$ of the students in the class. The boys are what fraction of the students in the class?
(6)

28. At what temperature on the Celsius scale does water freeze?
(10)

29. Use the numbers 24, 6, and 4 to write two multiplication facts and two division facts.
(2)

30. In the third paragraph of this lesson there is a "some and some more" story. Rewrite the story as a problem by removing one of the numbers from the story and asking a question instead.
(11)

LESSON 12

Place Value Through Trillions' • Multiple-Step Problems

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count by $\frac{1}{4}$'s from $\frac{1}{4}$ to 12.

a. 6 × 40 **b.** 6 × 400 **c.** \$12.50 + \$5.00

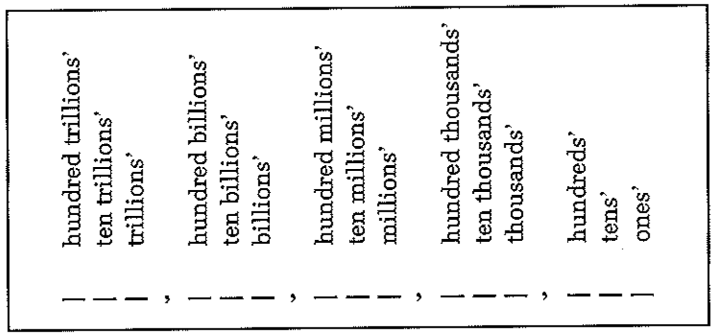
d. 451 + 24 **e.** 7500 − 5000 **f.** \$10.00 − \$2.50

g. Start with 12; divide by 2; subtract 2; divide by 2; subtract 2.

Problem Solving: A number cube (a die) has six surfaces (faces) marked with one through six dots. Altogether, how many dots are on a number cube?

Place value through trillions'

In our number system the value of a digit depends upon its position. The value of each position is called its **place value.**



Example 1 In the number 123,456,789,000 which digit is in the ten-millions' place?

Solution Either by counting places or looking at the chart we find that the digit in the ten-millions' place is **5.**

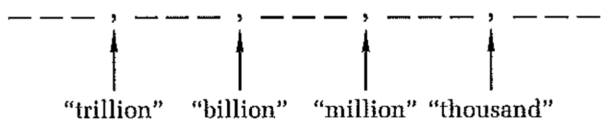
Example 2 In the number 5,764,283 what is the place value of the digit 4?

Solution By counting places or looking at the chart we can see that the place value of 4 is **thousands'.**

Large numbers are easy to read and write if we use commas to group the digits. To place commas, we begin at the right and move to the left, writing a comma after each three digits.

Putting commas in 1234567890 we get 1,234,567,890.

Commas help us read large numbers by marking the end of the trillions, billions, millions, and thousands. We need only to read the three-digit number in front of each comma, then say "trillion," "billion," "million," or "thousand" when we reach the comma.



Example 3 Use words to write the number 1024305.

Solution First we insert commas.

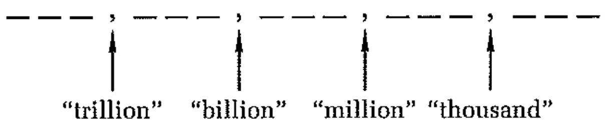
1,024,305

We write **one million, twenty-four thousand, three hundred five.**

Note: We write commas after the words "trillion," "billion," "million," and "thousand." We hyphenate compound numbers from 21 through 99. We do not say or write "and" when naming whole numbers.

Example 4 Use digits to write the number one trillion, two hundred fifty billion.

Solution When writing large numbers it may help to sketch the pattern before writing the digits.



We write a 1 to the left of the trillions' comma and 250 in the three places to the left of the billions' comma. The remaining places are filled with zeros.

1,250,000,000,000

Multiple-step problems

The **operations of arithmetic** are addition, subtraction, multiplication, and division. In this table we list the terms for the answers we get when we perform these operations.

Sum	The answer when we add
Difference	The answer when we subtract
Product	The answer when we multiply
Quotient	The answer when we divide

We will use these terms in problems that have several steps.

Example 5 What is the difference between the product of 6 and 4 and the sum of 6 and 4?

Solution We see the words "difference," "product," and "sum" in this question. We first look for phrases like "the product of 6 and 4." We will rewrite the question, emphasizing these phrases.

> What is the difference between the **product of 6 and 4** and the **sum of 6 and 4?**

For each phrase we find one number. "The product of 6 and 4" is 24, and "the sum of 6 and 4" is 10. So we can replace the two phrases with the numbers 24 and 10.

> What is the difference between 24 and 10?

We find this answer by subtracting 10 from 24. The difference between 24 and 10 is **14.**

Practice*

a. Which digit is in the millions' place in 123,456,789?

b. What is the place value of the 1 in 12,453,000,000?

c. Use words to write 21,350,608.

d. Use digits to write four billion, five hundred twenty million.

e. When the product of 6 and 4 is divided by the difference of 6 and 4, what is the quotient?

Problem set 12

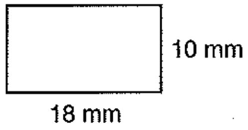
1. *(12)* What is the difference between the product of 1, 2, and 3 and the sum of 1, 2, and 3?

2. *(12)* The earth is about ninety-three million miles from the sun. Use digits to write that number.

3. *(11)* Gilbert and Sharon cooked 342 pancakes for the pancake breakfast. If Gilbert cooked 167 pancakes, how many pancakes did Sharon cook? (Use a SSM pattern.)

4. *(5)* Robin bought two arrows for $1.75 each. If he paid the good merchant with a five-dollar bill, how much did he receive in change?

5. *(8)* What is the perimeter of the rectangle?



6. *(4)* $6m = 60$

7. *(6)* (a) What number is $\frac{1}{2}$ of 100?

   (b) What number is $\frac{1}{4}$ of 100?

8. *(9)* Compare: $300 \times 1 \bigcirc 300 \div 1$

9. *(5)* $(3 \times 3) - (3 + 3)$

10. (10) What are the next three numbers in this sequence?

1, 2, 4, 8, ______, ______, ______, ...

11. (3) $1 + m + 456 = 480$

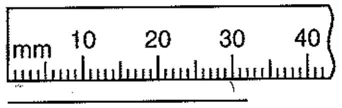
12. (3) $1010 - n = 101$

13. (2) $1234 \div 10$

14. (2) $1234 \div 12$

15. (10) What is the sum of the first five even numbers greater than zero?

16. (7) How many millimeters long is the line segment?



17. (12) In the number 123,456,789,000 which digit is in the ten-billions' place?

18. (12) In the number 5,764,283,000 what is the place value of the digit 4?

19. (12) Which digit is in the hundred-thousands' place in 987,654,321?

20. (5) $1 \times 10 \times 100 \times 1000$

21. (2) 3.75×3

22. (4) $22y = 0$

23. (3) $100 + 200 + 300 + 400 + w = 2000$

24. (2) 24×26

25. (4) $\begin{array}{r} 25 \\ m\overline{)625} \end{array}$

26. (4) If the divisor is 4 and the quotient is 8, then what is the dividend?

27. Show three ways to write "27 divided by 3."
(2)

28. Seven of the ten marbles in a bag are red. So $\frac{7}{10}$ of the marbles are red. What fraction of the marbles are not red?
(6)

29. Use digits to write four trillion.
(12)

30. Make up a question similar to Example 5 in this lesson using different numbers. Then find the answer.
(12)

LESSON 13

"Larger-Smaller-Difference" Stories • "Later-Earlier-Difference" Stories

Facts Practice: 64 Addition Facts (Test A in Test Masters)

Mental Math: Count up and down by 25's between 25 and 1000. Count up and down by 2's between 2 and 40.

a. 5×300
b. 5×3000
c. $7.50 + $1.75
d. $3600 + 230$
e. $4500 - 500$
f. $20.00 − $5.00

Problem Solving: Tom studied the treasure map. He started at the tree and walked north 5 steps. He turned right and walked 7 steps. He turned right again and walked 9 steps. Then he turned left and walked 3 steps. Finally, he turned left and walked 4 steps and stopped. In what direction was he facing? How many steps away was the tree?

"Larger-smaller-difference" stories

"Larger-smaller-difference" stories compare two numbers. We can find how much greater or how much less one number is than another number by subtracting. "Larger-smaller-difference" stories have a subtraction pattern.

$$\begin{array}{r} \text{Larger} \\ -\ \text{Smaller} \\ \hline \text{Difference} \end{array}$$

We will sometimes use the abbreviation L-S-D to refer to "larger-smaller-difference" stories.

Example 1 There were 324 girls and 289 boys in the school. How many fewer boys than girls were there in the school?

Solution **Step 1.** We are asked to compare the number of boys to the number of girls. We recognize this as a "larger-smaller-difference" story.

Step 2. We draw the L-S-D pattern and record the numbers.

PATTERN	PROBLEM
Larger	324 girls
− Smaller	− 289 boys
Difference	d fewer boys

Step 3. We find the missing number by subtracting.

$$\begin{array}{r} 324 \text{ girls} \\ - \ 289 \text{ boys} \\ \hline 35 \text{ fewer boys} \end{array}$$

Step 4. We answer the question. There were **35 fewer boys** than girls in the school. We can also state that there were 35 more girls than boys in the school.

"Later-earlier-difference" stories

We use a "larger-smaller-difference" pattern to compare sizes. We use a "later-earlier-difference" pattern to compare times.

$$\begin{array}{l} \text{Later} \\ - \ \text{Earlier} \\ \hline \text{Difference} \end{array}$$

We will sometimes abbreviate this pattern as L-E-D.

Example 2 How many years were there from 1492 to 1620?

Solution **Step 1.** We recognize the question has a "later-earlier-difference" pattern.

Step 2. We sketch the pattern and record the years. The year 1620 is later than the year 1492.

PATTERN	PROBLEM
Later	1620
− Earlier	− 1492
Difference	d

Step 3. We find the missing number by subtracting.

$$\begin{array}{r} 1620 \\ -\ 1492 \\ \hline 128 \end{array}$$

Step 4. We answer the question. There were **128 years** from 1492 to 1620.

Example 3 Abraham Lincoln was born in 1809 and died in 1865. How many years did he live?

Solution **Step 1.** We recognize that this is a "later-earlier-difference" story.

Step 2. We sketch the pattern and record the numbers.

PATTERN	PROBLEM
Later	1865 (death)
− Earlier	− 1809 (birth)
Difference	L (years lived)

Step 3. We find the missing number by subtracting.

$$\begin{array}{r} 1865 \\ -\ 1809 \\ \hline 56 \end{array}$$

Step 4. We answer the question. Abraham Lincoln lived **56 years.**

Practice **a.** The population of Castor is 26,290. The population of Weston is 18,962. How many more people live in Castor than live in Weston?

b. How many years were there from 1066 to 1215?

Problem set 13

1. (12) When the sum of 8 and 5 is subtracted from the product of 8 and 5, what is the difference?

2. (12) The moon is about two hundred fifty thousand miles from the earth. Use digits to write that number.

3. (12) Use words to write 521,000,000,000.

4. (12) Use digits to write five million, two hundred thousand.

5. (2) Robin Hood roamed Sherwood Forest with sevenscore merry men. A score is twenty. So sevenscore is seven twenties. Find how many merry men roamed with Robin.

6. (11) The beanstalk was 1000 meters tall. The giant had climbed down 487 meters before Jack could chop down the beanstalk. How far did the giant fall? (Use a SWA pattern.)

7. (13) At Big River Middle School there are 503 girls and 478 boys. How many more girls than boys attend Big River Middle School? (Use a L-S-D pattern.)

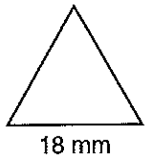
8. (5) 99 + 100 + 101

9. (5) 9 × 10 × 11

10. (12) Which digit is in the thousands' place in 54,321?

11. (12) What is the place value of the 1 in 1,234,567,890?

12. (8) The three sides of an equilateral triangle are equal in length. What is the perimeter of this equilateral triangle?



13. (2) 5432 ÷ 100

14. (2) $\frac{60,000}{30}$

15. (2) 1000 ÷ 7

16. (2) \$4.56 ÷ 3

17. *(9)* Compare: $3 + 2 + 1 + 0 \bigcirc 3 \times 2 \times 1 \times 0$

18. *(10)* The sequence in this problem has a rule that is different from the rules for an addition sequence or a multiplication sequence. What is the next number in the sequence?

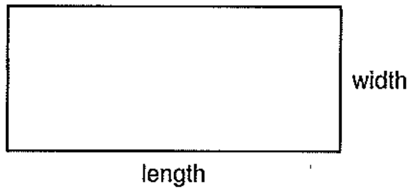
1, 4, 3, 6, 5, 8, _____, ...

19. *(6)* What is $\frac{1}{2}$ of 5280?

20. *(4)* $365 \div w = 365$

21. *(5)* $(5 + 6 + 7) \div 3$

22. *(7)* Use your ruler to find the length of this rectangle in inches.



23. *(8)* Describe two ways to find the perimeter of a square, one way by adding and the other way by multiplying.

24. *(2)* Multiply to find the answer to this addition problem:

$$125 + 125 + 125 + 125 + 125 + 125$$

25. *(10)* At what temperature on the Fahrenheit scale does water boil?

26. *(2)* Show three ways to write "21 divided by 7."

Find each missing number:

27. *(4)* $8a = 816$

28. *(4)* $\frac{b}{4} = 12$

29. *(4)* $\frac{12}{c} = 4$

30. *(3)* $d - 16 = 61$

LESSON
14

The Number Line: Negative Numbers

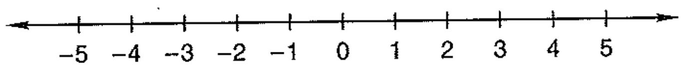
Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 6.

a. 8×400 b. 6×3000 c. \$7.50 + \$7.50
d. $360 + 230$ e. $1250 - 1000$ f. \$10.00 − \$7.50
g. Start with 10; add 2; divide by 3; multiply by 4; subtract 5.

Problem Solving: Andy, Bob, and Carol stood side by side for a picture. Then they changed their order for another picture. Then they changed their order again. List all the possible side-by-side arrangements. Three students may demonstrate the arrangements.

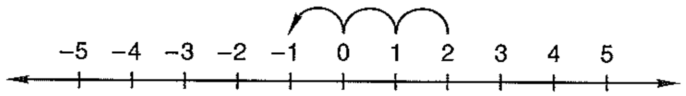
We have seen that a number line can be used to arrange numbers in order.



On the number line above, the points to the right of zero represent **positive** numbers. The points to the left of zero represent **negative** numbers. Zero is neither positive nor negative.

The number −5 is read "negative five." Notice that the points marked 5 and −5 are the same distance from zero but are on opposite sides of zero. We say that 5 and −5 are **opposites.** The numbers −2 and 2 are opposites. The tick marks on this number line show the location of **integers.** Integers include all of the counting numbers and their opposites and zero.

If you subtract a larger number from a smaller number (like $2 - 3$), the answer will be a negative number. One way to find the answer to such questions is to use the number line. We start at 2 and count back (left) three integers. Maybe you can figure out a faster way to find the answer.



2 − 3 = −1

Example 1 Subtract 5 from 2.

Solution **Order matters in subtraction.** Start at 2 and count to the left 5 integers. You should end up at **−3.** Try this problem with a calculator. Enter 2 − 5 =. What number is displayed after the = is pressed?

Example 2 Arrange these four numbers in order from least to greatest:

1, −2, 0, −1

Solution A number line shows numbers in order. By arranging these numbers in the order they appear on a number line, we arrange them in order from least to greatest.

−2, −1, 0, 1

Example 3 What number is 7 less than 3?

Solution The phrase "7 less than 3" means to start with 3 and subtract 7.

3 − 7

We count to the left 7 integers from 3. The answer is **−4.**

Practice

a. Use words to write this number: −8.

b. What number is the opposite of 3?

c. Arrange these numbers in order from least to greatest:

0, −1, 2, −3

d. What number is 5 less than 0?

e. What number is 10 less than 5?

f. $5 - 8$ **g.** $1 - 5$

h. All five of these numbers are integers: true or false?

$$-3, 0, 2, -10, 50$$

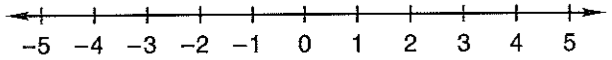
Problem set 14

1. (12) What is the quotient when the sum of 15 and 12 is divided by the difference of 15 and 12?

2. (12) What is the place value of the 7 in 987,654,321,000?

3. (12) Light travels at a speed of about one hundred eighty-six thousand miles per second. Use digits to write that number.

4. (14) What number is three integers to the left of 2 on the number line?



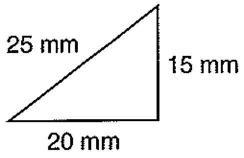
5. (9) What number is halfway between 1 and 5 on the number line?

6. (14) What number is halfway between −4 and 0 on the number line?

7. (11) Seventy-two of the 140 Merry Men remained in Sherwood Forest while the rest rode out with Robin. How many of the Merry Men rode out of the forest with Robin? (Use a SWA pattern.)

8. (9) Compare: $1 + 2 + 3 + 4 \bigcirc 1 \times 2 \times 3 \times 4$

9. (8) What is the perimeter of the right triangle?



10. (10) What are the next two numbers in this sequence?

16, 8, 4, _____, _____, ...

11. (13) There are 365 days in a common year. How much less than 500 is 365? (Use a L-S-D pattern.)

12. (14) What number is 8 less than 6?

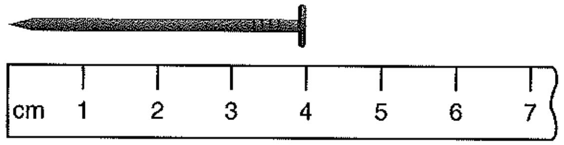
13. (2) 1020 ÷ 100 **14.** (2) $\frac{36{,}180}{12}$ **15.** (2) $18\overline{)564}$

16. (5) 1234 + 567 + 89 **17.** (3) $n - 310 = 186$

18. (5) 10 × 11 × 12 **19.** (3) $\$3.05 - m = \$2.98$

20. (7) (a) How many centimeters long is the nail?

(b) How many millimeters long is the nail?



21. (5) 100 × 100 × 100

22. (12) What digit is in the ten-thousands' place in 123,456,789?

23. (7) If you know the length of an object in centimeters, how can you figure out the length of the object in millimeters without remeasuring?

24. (2) Use the numbers 19, 21, and 399 to write two multiplication facts and two division facts.

25. (9) Compare: 12 ÷ 6 × 2 ○ 12 ÷ (6 × 2)

26. (2) Show three ways to write "60 divided by 6."

27. (12) The human brain has about nine trillion nerve cells. Use digits to write that number.

28. (6) One third of the 12 eggs in the carton were cracked. How many eggs were cracked?

29. (14) What number is the opposite of 10?

30. (14) Arrange these numbers in order from least to greatest.

$$1, 0, -1, \tfrac{1}{2}$$

LESSON 15

"Equal Groups" Stories

Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

Mental Math: Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 10.

a. 7 × 4000 **b.** 8 × 300 **c.** \$12.50 + \$12.50

d. 80 + 12 **e.** 6250 − 150 **f.** \$20.00 − \$2.50

g. Start with a dozen; subtract 3; divide by 3; subtract 3; multiply by 3.

Problem Solving: Copy this subtraction problem and fill in the missing digits.

$$\begin{array}{r} 4\_7 \\ -\ \_9\_ \\ \hline 21 \end{array}$$

We have studied some mathematical story problems. "Some and some more" stories have an addition pattern. "Some went away" stories and "larger-smaller-difference" stories have subtraction patterns. Another type of mathematical story is the "equal groups" story.

> *In the auditorium there were 15 rows of chairs with 20 chairs in each row. Altogether, there were 300 chairs in the auditorium.*

The chairs were arranged in groups (rows) of 20 chairs in each group. There were 15 groups. Here is how we draw the pattern.

PATTERN	PROBLEM
Number in each group	20 chairs in each row
× Number of groups	× 15 rows
Number in all groups	300 chairs in all rows

Any one of the numbers may be missing in an "equal groups" problem. Since an "equal groups" story has a multiplication pattern, we multiply to find the total, and we divide to find either the number of groups or the number in each group. We will sometimes use the abbreviation EG to stand for "equal groups."

Example At Russell Middle School there were 232 seventh-grade students in 8 classrooms. If there were the same number of students in each classroom, how many students would be in each seventh-grade classroom at Russell Middle School?

Solution **Step 1.** A number of students is divided into equal groups (classrooms). We recognize this as an "equal groups" story. The words "in each" often appear in "equal groups" stories, signifying the top number of the pattern.

Step 2. We draw the pattern and record the numbers, writing a letter in place of the missing number.

PATTERN	PROBLEM
Number in each group	n in each classroom
× Number of groups	× 8 classrooms
Number in all groups	232 in all classrooms

Step 3. We find the missing number. We find the missing factor by dividing. We check our work.

$$\begin{array}{r} 29 \\ 8\overline{)232} \end{array} \qquad \begin{array}{r} 29 \\ \times\ \ 8 \\ \hline 232 \end{array}$$

Step 4. We answer the question. If there were the same number of students in each classroom, there would be **29 students** in each seventh-grade classroom.

Practice

a. Marcie collected \$4.50 selling lemonade at 25¢ for each cup. How many cups of lemonade did Marcie sell? (*Hint*: Record \$4.50 as 450¢.)

b. In the store parking lot there were 18 parking spaces in each row and there were 12 rows of parking spaces. Altogether, how many parking spaces were in the parking lot?

Problem set 15

1. (15) The second paragraph of this lesson contains an "equal groups" story. Rewrite the story as a problem by removing one of the numbers and writing a question.

2. (13) On the Fahrenheit scale of temperature, water freezes at 32°F and boils at 212°F. How many degrees difference is there between the freezing and boiling points? (Use a L-S-D pattern.)

3. (15) There are about three hundred twenty little O's of cereal in an ounce. About how many O's are there in a one-pound box? (1 pound = 16 ounces) (Use an EG pattern.)

4. (11) There are 31 days in August. How many days are left in August after August 3? (Use a SSM pattern.)

5. (9,14) Compare: $3 - 1 \bigcirc 1 - 3$

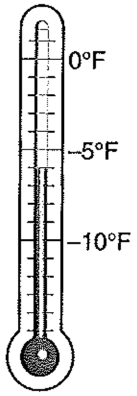
6. (14) Subtract 5 from 2. Use words to write the answer.

7. (14) What number is 8 less than 5?

8. (10) What are the next three numbers in this sequence?

6, 4, 2, 0, ____, ____, ____, ...

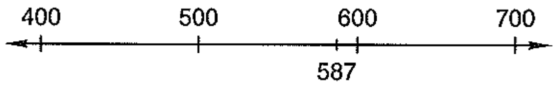
9. (10) What is the temperature reading on this thermometer? Write your answer twice, once with digits and an abbreviation and once with words.



10. (1) \$10 – 10¢

11. (6) How much money is $\frac{1}{2}$ of \$3.50?

12. (9) To which hundred is 587 closest?



13. (5) $9 + 87 + 654 + 3210$

14. (3) $w + \$65 = \$1000$

15. (2) $\dfrac{4320}{9}$

16. (2) $36\overline{)493}$

17. (5) $(8 + 9 + 16) \div 3$

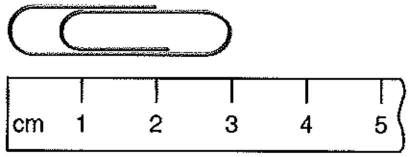
18. (4) $63w = 63$

19. (4) $\dfrac{76}{m} = 1$

20. (2) 574×76

21. (3) $3 + n + 12 + 27 = 50$

22. (7) There are 10 millimeters in 1 centimeter. How many millimeters long is the paper clip?



23. (4) $1200 \div w = 300$

24. (12) What is the place value of 5 in 12,345,678?

25. (12) Which digit occupies the ten-billions' place in 123,456,789,000?

26. Use the numbers 19, 21, and 40 to write two addition
(1) facts and two subtraction facts.

27. Arrange these numbers in order from least to greatest:
(14)

0, –1, 2, –3

28. Susan sold seven of her seventeen seashells down by
(6) the seashore. What fraction of her seashells did she sell?

29. Susan sold seashells for 75¢ each. How much money
(15) did Susan receive selling seven seashells?

30. Which number is neither positive nor negative?
(14)

LESSON 16

Rounding Whole Numbers • Estimating • Bar Graphs

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

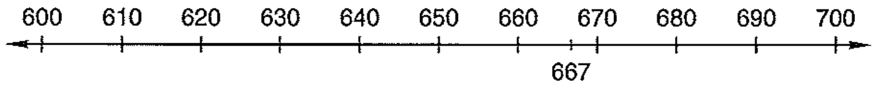
Mental Math: Count by 3's from 3 to 60.

a. 3×30 plus 3×2
b. 4×20 plus 4×3
c. $150 + 20$
d. $75 + 9$
e. $800 - 50$
f. $8000 - 500$
g. Start with 1; add 2; multiply by 3; subtract 4; divide by 5.

Problem Solving: Fran has 8 coins that total exactly $1.00. If at least one of the coins is a dime, what are Fran's 8 coins?

Rounding whole numbers

When we **round** a number, we are finding another number, usually ending in zero, that is close to the number we are rounding. The number line can help us visualize rounding.



If we are to round 667 to the nearest ten, we can see that 667 is closer to 670 than it is to 660. If we are to round 667 to the nearest hundred, we can see that 667 is closer to 700 than 600.

Example 1 Round 6789 to the nearest thousand.

Solution The number we are rounding is between 6000 and 7000. It is closer to **7000.**

Example 2 Round 550 to the nearest hundred.

Solution The number we are to round is halfway between 500 and 600. When the number we are rounding is halfway between two round numbers, we round **up.** So 550 rounds to **600.**

Estimating Rounding can help us estimate the answer to a problem. Estimating is a quick, easy way to get "close" to the answer. Estimating can help us decide if an answer is reasonable. To estimate, we round the numbers before we add, subtract, multiply, or divide.

Example 3 Estimate the sum of 467 and 312.

Solution Estimating is a skill we can learn to do "in our head." First we round each number. Since both numbers are in the hundreds we will round each number to the nearest hundred.

467 rounds to 500

312 rounds to 300

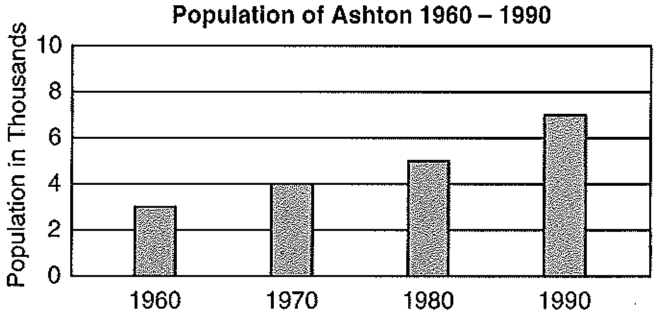
To estimate the sum we add the rounded numbers.

$$\begin{array}{r} 500 \\ +\ 300 \\ \hline 800 \end{array}$$

The estimated sum of 467 and 312 is **800.**

Bar graphs Bar graphs display numerical information with shaded rectangles (bars) of various lengths. Bar graphs are often used to show comparisons.

Example 4 According to this graph, about how many more people lived in Ashton in 1990 than in 1970?



Solution In 1990 the population was about 7000. In 1970 the population was about 4000. The question is a "larger-smaller-difference" question. We subtract and find that about **3000** more people lived in Ashton in 1990 than in 1970.

Practice* Round each of these numbers to the nearest ten:

a. 57 **b.** 63 **c.** 45

Round each of these numbers to the nearest hundred:

d. 282 **e.** 350 **f.** 426

Round each of these numbers to the nearest thousand:

g. 4387 **h.** 7500 **i.** 6750

Use rounded numbers to estimate each answer:

j. $397 + 206$ **k.** $703 - 598$

l. 29×31 **m.** $29\overline{)591}$

Use the information in the graph in Example 4 to answer these questions:

n. About how many fewer people lived in Ashton in 1980 than in 1990?

o. The graph shows an upward trend in the population of Ashton. Based upon the trend, what would be a reasonable projection for the population of Ashton in the year 2000?

Problem set 16

1. (12) What is the difference between the product of 20 and 5 and the sum of 20 and 5?

2. (13) Columbus landed in the Americas in 1492. The Pilgrims landed in 1620. How many years after Columbus did the Pilgrims land in America? (Use a L-E-D pattern.)

3. (15) Robin Hood separated his 140 merry men into 5 equal groups. One group he sent north, one south, one east, and one west. The remaining group stayed in camp. How many merry men stayed in camp? (Use an EG pattern.)

4. (12) Which digit is in the hundred-thousands' place in 159,342,876?

5. (12) Use words to write the number 5,010,000,000.

6. (9) What number is halfway between 5 and 11 on the number line?

7. (16) Round 56,789 to the nearest thousand.

8. (16) Round 550 to the nearest hundred.

9. (16) Estimate the product of 295 and 406 by rounding each number to the nearest hundred before multiplying.

10. (5) 45 + 5643 + 287

11. (1) 40,312 − 14,908

12. (2) $\frac{7308}{12}$

13. (2) $100\overline{)5367}$

14. (5) (5 + 11) ÷ 2

15. (6) How much money is $\frac{1}{2}$ of $5?

16. (6) How much money is $\frac{1}{4}$ of $5?

17. (2) $0.25 × 10

18. (5) 325 × (324 − 323)

19. (9) Compare: 1 + (2 + 3) ○ (1 + 2) + 3

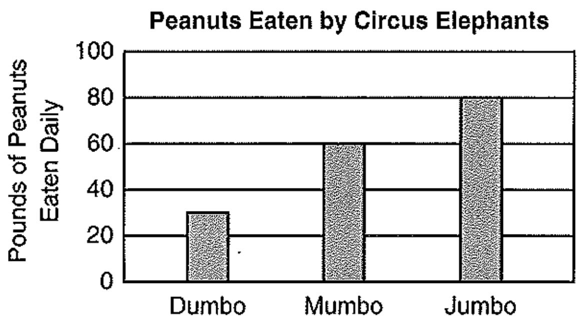
20. (14) What number is five less than 1?

21. (15) Your heart beats about 72 times per minute. At that rate, how many times will it beat in one hour? (Use an EG pattern.)

22. (10) What number comes next in this sequence?

100, 80, 60, 40, _____, ...

Use the graph to answer questions 23, 24, 25, and 26.



23. (16) How many more pounds of peanuts does Jumbo eat all day than Dumbo?

24. (16) Altogether, how many pounds do the three elephants eat each day?

25. (16) How many pounds would Mumbo eat in one week?

26. (16) Write a "larger-smaller-difference" problem using the information in this graph.

Find the missing number:

27. (4) $6w = 66$

28. (3) $m - 60 = 37$

29. (3) $60 - n = 37$

30. (16) Chico, Fuji, and Rolo each day eat 6, 8, and 9 bananas, respectively. Sketch a bar graph to illustrate this information.

LESSON
17

Number Line: Fractions and Mixed Numbers

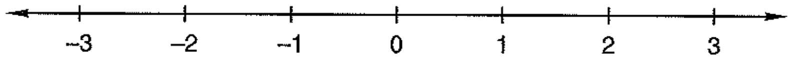
Facts Practice: 100 Multiplication Facts (Test E in Test Masters)

Mental Math: Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 12.

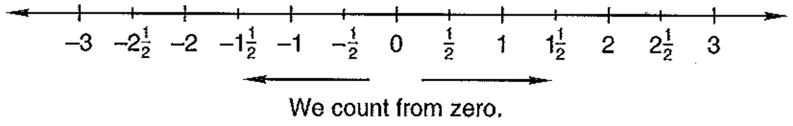
a. 5×30 plus 5×4 **b.** 4×60 plus 4×4
c. $180 + 12$ **d.** $64 + 9$
e. $3000 - 1000 - 100$ **f.** \$10.00 − \$7.50
g. Start with 5; multiply by 4; add 1; divide by 3; subtract 2.

Problem Solving: If you pick up a number cube with two fingers by holding your fingers against opposite faces, your fingers will cover a total of how many dots? (Use a number cube to find out.)

On this number line the tick marks show the location of the integers.

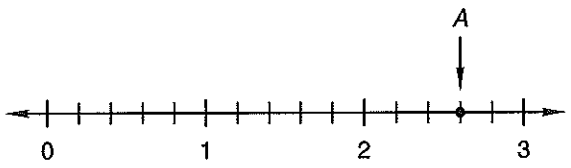


There are points on the number line between the integers that can be named with fractions or mixed numbers. Halfway between 0 and 1 is $\frac{1}{2}$. Halfway between 1 and 2 is $1\frac{1}{2}$. Halfway between –1 and –2 is $-1\frac{1}{2}$.



The distance between consecutive integers on a number line may be divided into halves, thirds, fourths, fifths, or any other number of equal divisions. To determine which fraction or mixed number is represented by a point on the number line, we follow the steps described in the next example.

Example 1 What mixed number is marked as point *A* on this number line?

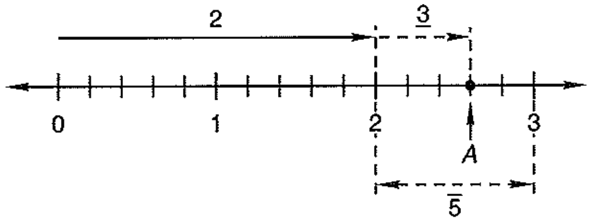


Solution Point *A* represents a mixed number. A mixed number is a whole number plus a fraction.

Step 1. As we move from zero to point *A*, we pass points for the whole numbers 1 and 2. We do not pass 3. So the whole number part of the answer is 2.

Step 2. From the tick mark for 2 we count three small segments to point *A*. So 3 is the numerator of the fraction.

Step 3. From the tick mark for 2 to the tick mark for 3 are five small segments. So 5 is the denominator of the fraction.



Point *A* represents the mixed number **$2\frac{3}{5}$**.

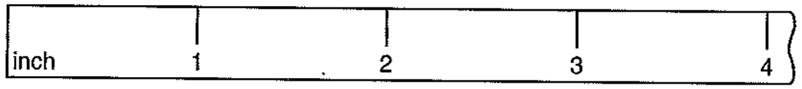
Activity: Inch Ruler to Sixteenths

Materials needed:

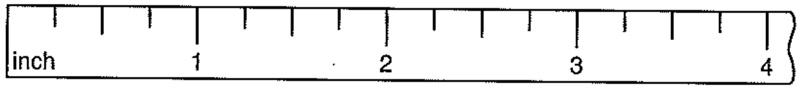
- Inch ruler made in Lesson 7

In Lesson 7 we made an inch ruler divided into fourths. In this activity we will divide the ruler into eighths and sixteenths. First we will review what we did in Lesson 7.

We used a ruler to make one-inch divisions on a strip of tag board.

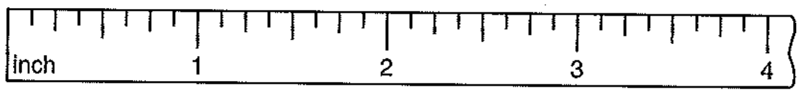


Then we estimated the halfway point between inch marks and drew new marks. The new marks were half-inch divisions. Then we estimated the halfway point between the half-inch marks and made quarter-inch divisions.

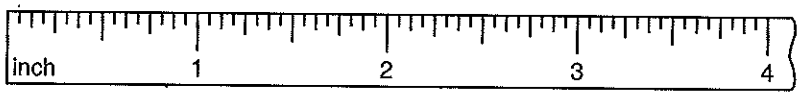


We made the half-inch marks a little shorter than the inch marks and the quarter-inch marks a little shorter than the half-inch marks.

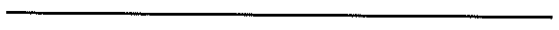
Now divide your ruler into eighths of an inch by estimating the halfway point between the quarter-inch marks. Make these eighth-inch marks a little shorter than the quarter-inch marks.



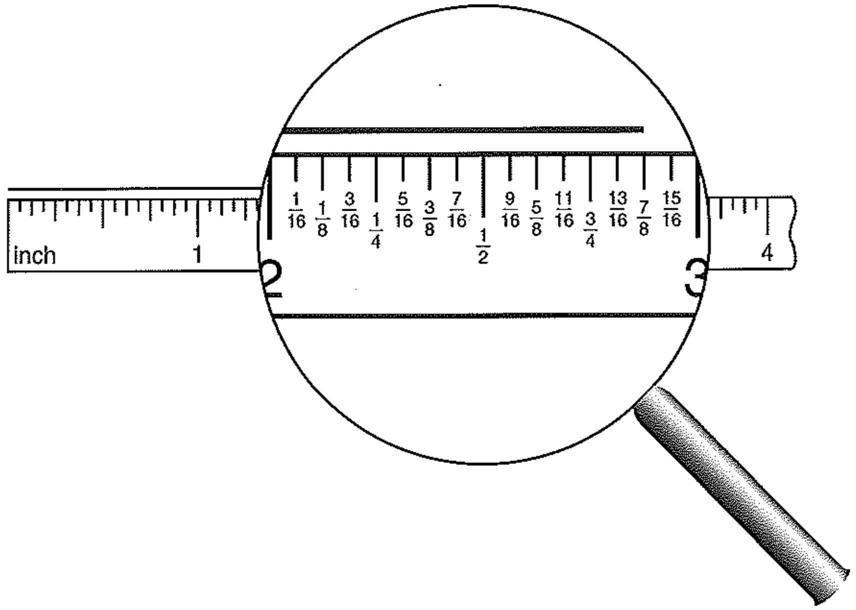
Finally, divide your ruler into sixteenths by estimating the halfway point between the eighth-inch marks. Make these marks the shortest marks on the ruler.



Example 2 Use your ruler to find the length of this line segment to the nearest sixteenth of an inch.



Solution The ruler has been divided into sixteenths. We align the zero mark (or end of the ruler) with one end of the line segment. Then we find the mark on the ruler closest to the other end of the line segment and read this mark. We will enlarge a portion of a ruler to show how each mark is read.



We find that the line segment is about **$2\frac{7}{8}$ inches long.** This is the nearest sixteenth because the end of the segment aligns more closely to the $\frac{7}{8}$ mark $\left(\text{which equals } \frac{14}{16}\right)$ than it does to the $\frac{13}{16}$ mark or to the $\frac{15}{16}$ mark.

Practice **a.** Continue this sequence to $1\frac{1}{2}$:

$$\frac{1}{16}, \frac{1}{8}, \frac{3}{16}, \frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{7}{16}, \frac{1}{2}, \ldots$$

b. What number is halfway between -2 and -3?

c. What number is halfway between 2 and 5?

Use your ruler to find the length of each of these line segments to the nearest sixteenth of an inch.

d. ________

e. ________________________

f. __________________________________

Problem set 17

1. *(12)* What is the sum of twelve thousand, five hundred and ten thousand, six hundred ten?

2. *(13)* In 1903 the Wright brothers made the first powered airplane flight. In 1969 Americans first landed on the moon. How many years was it from the first powered airplane flight to the first moon landing? (Use the L-E-D pattern.)

3. *(15)* Captain Hook often ran from the sound of ticking clocks. If he could run 6 yards in one second, how far could he run in 12 seconds? (Use an "equal groups" pattern.)

4. *(15)* Jack found two dozen golden eggs. If the value of each egg was $1000, what was the value of all the eggs Jack found? (Use an "equal groups" pattern.)

5. *(16)* Estimate the sum of 5280 and 1760 by rounding each number to the nearest thousand before adding.

6. *(2)* $\frac{480}{3}$

7. *(5)* $\frac{6 - 6}{3}$

8. *(1)* The letters a, b, and c represent three different numbers. The sum of a and b is c.

$$a + b = c$$

Rearrange the letters to make another addition fact and two subtraction facts.

9. *(2)* Rewrite $2 \div 3$ with a division bar but do not divide.

10. *(8)* A square has sides 10 cm long. What is its perimeter?

11. *(17)* Use your ruler to find the length of this line segment to the nearest sixteenth of an inch.

12. *(3)* $\$3 - y = \$1.75$

13. *(5)* $365 + 4576 + 50{,}287$

14. (4) $12n = 0$

15. (9) Compare: 19×21 ○ 20×20

16. (2) $\frac{5280}{44}$

17. (10) What number is missing in this sequence?

5, 10, ______, 20, 25, ...

18. (12) Which digit is in the hundred-millions' place in 987,654,321?

19. (2) $250{,}000 \div 100$

20. (2) $3.75 × 10

21. (3) $16 + 14 = 14 + w$

22. (15) The magician pulled 38 rabbits out of a hat. Half of the rabbits were white. How many were not white? (Use an EG pattern.)

23. (5) $100 - (50 - 25)$

24. (3) $m - 20 = 30$

25. (10) What is the sum of the first six positive odd numbers?

26. (6) Describe how to find $\frac{1}{4}$ of 52.

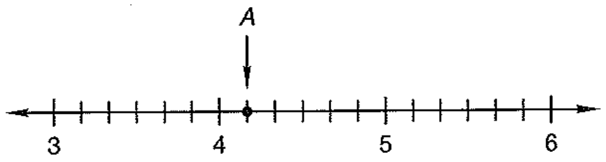
27. (6) A quarter is $\frac{1}{4}$ of a dollar.

(a) How many quarters are in one dollar?

(b) How many quarters are in three dollars?

28. (17) On an inch ruler, which mark is halfway between the $\frac{1}{4}$-inch mark and the $\frac{1}{2}$-inch mark?

29. (17) Point *A* represents what mixed number on this line?



30. (17) A segment that is $\frac{1}{2}$ of an inch long is how many sixteenths of an inch long?

LESSON 18

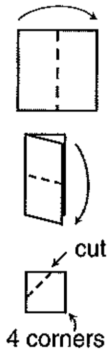
Average • Line Graphs

Facts Practice: 100 Addition Facts (Test B in Test Masters)

Mental Math: Count by 3's from 3 to 60.

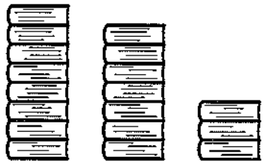
a. 4 × 23 equals 4 × 20 plus 4 × 3. Find 4 × 23.
b. 4 × 32 **c.** 3 × 42 **d.** 3 × 24
e. Start with half a dozen; add 2; multiply by 3; divide by 4; subtract 5.

Problem Solving: Jeana folded a square paper in half so that the left edge aligned with the right edge. Then she folded the paper again so that the top edge aligned with the bottom edge. (The four corners of the paper aligned at the lower right.) Then she used scissors and cut off the upper left corners. What will the paper look like when it is unfolded?

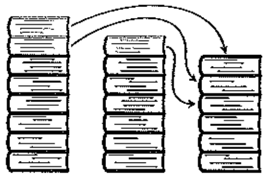


Average

Here we show three stacks of books. In the three stacks there are 8 books, 7 books, and 3 books, respectively. Altogether, there are 18 books, but the number of books in each stack is not equal.



If we move some of the books from the taller stacks to the shortest stack we can make the three stacks the same height. Then there will be 6 books in each stack.



By making the stacks equal we have found the **average** number of books in the three stacks. One way to find an average is to make equal groups.

Example 1 In four classrooms there were 28 students, 27 students, 26 students, and 31 students, respectively. What was the average number of students in each classroom?

Solution The average number of students in each classroom is how many students there would be in each room if we made the numbers equal. So we will take all of the students and make four equal groups. To find all of the students, we add the numbers in each classroom.

$$\begin{array}{rl} & 28 \text{ students} \\ & 27 \text{ students} \\ & 26 \text{ students} \\ + & 31 \text{ students} \\ \hline & 112 \text{ students in all} \end{array}$$

We make four equal groups by dividing the total number of students by four.

$$\begin{array}{r} 2\,8 \text{ students} \\ 4\overline{)11^{3}2 \text{ students}} \end{array}$$

If the groups were equal there would be 28 students in each classroom. The average number of students in each classroom was **28.**

Notice that this average problem is an "equal groups" problem. First we found the number of students in all the groups (classrooms). Then we found the number of students that would be in each group if the groups were equal.

Example 2 What is the average of 3, 7, and 8?

Solution The question does not tell us if the numbers 3, 7, and 8 refer to books or students or coins or quiz scores. We can still find the average by making equal groups. Since there are three numbers, there will be three groups. First we find the total. Then we divide the total by three.

$$3 + 7 + 8 = 18 \text{ (Find the total.)}$$

Then we divide the total into three equal groups.

$$18 \div 3 = 6$$

The average of 3, 7, and 8 is **6.**

Example 3 What number is halfway between 27 and 81?

Solution The number halfway between two numbers is also the average of the two numbers. For example, the average of 7 and 9 is 8, and 8 is halfway between 7 and 9. So the average of 27 and 81 will be the number halfway between 27 and 81. We add 27 and 81 and divide by 2.

$$\text{Average of 27 and 81} = \frac{27 + 81}{2} = \frac{108}{2} = 54$$

The number halfway between 27 and 81 is **54.**

Line graphs **Line graphs** display numerical information as points on a line. Line graphs are often used to show how a measurement is changing.

Example 4 This line graph shows Margie's height in inches from her 8th birthday to her 14th birthday. During which year did Margie grow the most?

Margie's height

Solution From Margie's 8th birthday to her 9th birthday she grew about two inches. She also grew about two inches from her 9th to her 10th birthday. From her 10th to her 11th birthday Margie grew about five inches. Notice that this is the steepest part of the growth line. So the year Margie grew the most was **the year she was ten.**

Your teacher may like for you to keep a line graph of your math test scores from week to week. "Activity Master 1" in the *Math 76 Test Masters* may be used for this purpose.

Practice*

a. There were 26 books on the first shelf, 36 books on the second shelf, and 43 books on the third shelf. Martin rearranged the books so that there were the same number of books on each shelf. After Martin rearranged the books, how many were on the first shelf?

b. What is the average of 96, 44, 68, and 100?

c. What number is halfway between 28 and 82?

d. What number is halfway between 86 and 102?

e. Find the average of 3, 6, 9, 12, and 15.

Use the information in the graph in Example 4 to answer these questions:

f. How many inches did Margie grow from her 8th to her 12th birthday?

g. During which year did Margie grow the least?

h. Predicting from the information in the graph, does it seem likely that Margie will grow to be 68 inches tall?

Problem set 18

1. (11) Jumbo ate two thousand, sixty-eight peanuts in the morning and three thousand, nine hundred forty in the afternoon. How many peanuts did Jumbo eat in all? What kind of a pattern did you use?

2. (11) Jimmy counted his permanent teeth. He had eleven on top and twelve on the bottom. An adult has 32 permanent teeth. How many more of Jimmy's teeth need to grow in? What kind of pattern did you use?

3. (15) Olive bought one dozen cans of spinach as a birthday present for her boyfriend. The spinach cost 53¢ per can. How much did Olive spend on spinach? What kind of pattern did you use?

4. (16) Estimate the difference of 5035 and 1987 by rounding to the nearest thousand before subtracting.

5. (18) Find the average of 9, 7, and 8.

6. (18) What number is halfway between 59 and 81?

7. (14) What number is 6 less than 2?

8. (4) $\frac{234}{n} = 6$

9. (2) $10{,}010 \div 10$

10. (2) $34{,}180 \div 17$

11. (5) \$3.64 + \$94.28 + 87¢

12. (1) $41{,}375 - 13{,}576$

13. (3) $w - 84 = 48$

14. (5) $4 \times 3 \times 2 \times 1 \times 0$

15. (2) 125×16

16. (2) \$0.35 × 100

17. (8) Draw a rectangle 5 cm long and 3 cm wide. What is its perimeter?

18. (10) What is the sum of the first six positive even numbers?

19. (10) What number is missing in this sequence?

1, 2, 4, ______, 16, 32, 64, ...

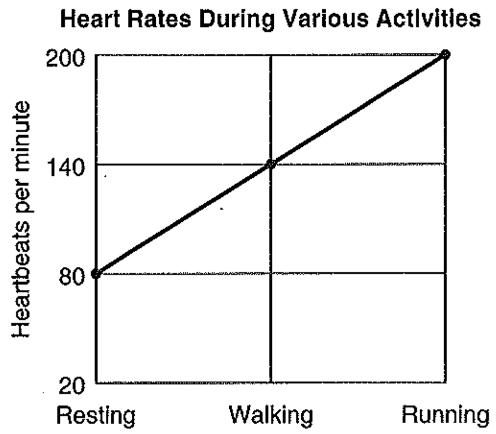
20. (9) Compare: 500×1 ○ $500 \div 1$

21. (3) $(1 + 2) \times 3 = (1 \times 2) + m$

22. (6) What number is $\frac{1}{2}$ of 1110?

23. (12) What is the place value of the 7 in 987,654,321?

Use the graph to answer questions 24, 25, and 26.



24. *(18)* Running increases a resting person's heartbeat by about how many beats per minute?

25. *(18)* About how many times would a person's heart beat during a 10-minute run?

26. *(18)* Write a larger-smaller-difference problem using the information in the line graph and answer the problem.

27. *(18)* In three classrooms there are 24, 27, and 33 students, respectively. How many students would be in each classroom if some students were moved from one classroom to the other classrooms so that the number of students in the three classrooms were equal?

28. *(6)* A dime is $\frac{1}{10}$ of a dollar.

(a) How many dimes are in a dollar?

(b) How many dimes are in three dollars?

29. *(17)* Use your ruler to draw a rectangle that is $2\frac{1}{4}$ inches long and $1\frac{3}{4}$ inches wide.

30. *(14)* What number is the opposite of 12?

LESSON
19

Factors • Divisibility

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 12.

a. 3 × 64 **b.** 3 × 46 **c.** 120 + 18
d. 34 + 40 + 9 **e.** 34 + 50 − 1 **f.** \$20.00 − \$12.50
g. Start with 100; divide by 2; subtract 1; divide by 7; add 3.

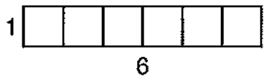
Problem Solving: Alex wanted to make a 40¢ phone call at a pay phone. He had a nickel, a dime, and a quarter. He could put in the nickel, then the dime, then the quarter. Or he could put in the quarter, then the dime, then the nickel. What are the other possible orders of coin drops for his phone call?

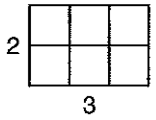
Factors

A whole number **factor** is a whole number that divides another whole number. For example, there are four whole numbers that divide 6. These four numbers are 1, 2, 3, and 6. All these numbers are factors of 6.

$$\begin{array}{r}6\\1\overline{)6}\end{array}\qquad\begin{array}{r}3\\2\overline{)6}\end{array}\qquad\begin{array}{r}2\\3\overline{)6}\end{array}\qquad\begin{array}{r}1\\6\overline{)6}\end{array}$$

We can illustrate the factors of 6 by arranging 6 tiles to form rectangles. With 6 tiles we can make a 1-by-6 rectangle. With 6 tiles we can also make a 2-by-3 rectangle.





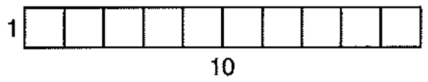
The number of tiles along the sides of these two rectangles (1, 6, 2, 3) are the four factors of 6.

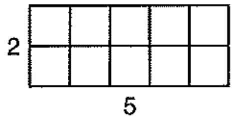
Example 1 What are the factors of 10?

Solution The factors of 10 are all the numbers that divide 10 with no remainder. They are **1, 2, 5,** and **10.**

$$\begin{array}{r} 10 \\ \mathbf{1}\overline{)10} \end{array} \qquad \begin{array}{r} 5 \\ \mathbf{2}\overline{)10} \end{array} \qquad \begin{array}{r} 2 \\ \mathbf{5}\overline{)10} \end{array} \qquad \begin{array}{r} 1 \\ \mathbf{10}\overline{)10} \end{array}$$

We can illustrate the factors of 10 with two rectangular arrays of tiles.



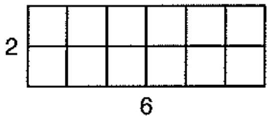


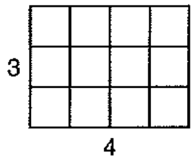
The number of tiles along the sides of the two rectangles (1, 10, 2, 5) are the factors of 10.

Example 2 How many different whole numbers are factors of 12?

Solution Twelve can be divided by 1, 2, 3, 4, 6, and 12. The question asked "How many?" Counting, we find that 12 has **6** different factors.

Twelve tiles can be arranged in three rectangular arrays demonstrating that the six factors of 12 are 1, 12, 2, 6, 3, and 4.





Divisibility There are ways of discovering whether some numbers are factors of other numbers without actually dividing. For instance, even numbers can be divided by 2. Therefore, 2 is a factor of every even counting number. Since even numbers are "able" to be divided by 2, we say that even numbers are "divisible" by 2. The tests for **divisibility** can help us find the factors of a number. Here we list the tests for divisibility of 2, 3, 5, 9, and 10.

Last-Digit Tests

Inspect the last digit of the number. A number is able to be divided by ...

2 if the last digit is even
5 if the last digit is 0 or 5
10 if the last digit is 0

Sum-of-Digits Tests

Add the digits of the number and inspect the total. A number is able to be divided by...

3 if the sum of the digits can be divided by 3
9 if the sum of the digits can be divided by 9

Example 3 Which of these numbers is divisible by 2?

365 1179 1556

Solution To decide if a number can be divided by 2, we inspect the last digit of the number. If the last digit is an even number, then the number can be divided by 2. The last digit of these three numbers are 5, 9, and 6. Since 5 and 9 are not even numbers, 365 and 1179 are not divisible by 2. Since 6 is an even number, 1556 is divisible by 2. It is not necessary to perform the division to answer the question. By inspecting the last digits we see that the number that is divisible by 2 is **1556.**

Example 4 Which of these numbers is divisible by 3?

365 1179 1556

Solution To decide if a number can be divided by 3, we add the digits of the number and inspect the sum. If the sum of the digits is divisible by 3, then the number is also divisible by 3.

The digits of 365 are 3, 6, and 5. We add these digits and get 14.

$$3 + 6 + 5 = 14$$

We try to divide 14 by 3 and find that there is a remainder of 2. Since 14 is not divisible by 3, we know that 365 is not divisible by 3 either.

The digits of 1179 are 1, 1, 7, and 9. The sum of these digits is 18.

$$1 + 1 + 7 + 9 = 18$$

We divide 18 by 3 and there is no remainder. We see that 18 is divisible by 3, so 1179 is also divisible by 3.

The sum of the digits of 1556 is 17.

$$1 + 5 + 5 + 6 = 17$$

Since 17 is not divisible by 3, the number 1556 is not divisible by 3.

By using the divisibility test for 3, we find that the number that is divisible by 3 is **1179.**

Example 5 Which of the numbers 2, 3, 5, 9, and 10 are factors of 135?

Solution First we will use the last-digit tests. The last digit of 135 is 5, so 135 is divisible by 5 but not by 2 or by 10. Next we use the sum-of-digit tests. The sum of the digits in 135 is 9 ($1 + 3 + 5 = 9$). Since 9 can be divided by both 3 and 9, we know that 135 can also be divided by 3 and 9. So **3, 5,** and **9** are factors of 135.

Practice Most of the time we just say **factors** instead of saying **whole number factors.** List the factors of the following numbers:

a. 14

b. 15

c. 16

d. 17

How many different factors do each of these numbers have?

e. 18 **f.** 19

g. 20 **h.** 21

Use the tests for divisibility to decide which of the numbers 2, 3, 5, 9, and 10 are factors of the following numbers:

i. 120 **j.** 102

Problem set 19

1. (4) If two hundred fifty-two is the dividend and six is the quotient, then what is the divisor?

2. (15) Lincoln began his speech, "Fourscore and seven years ago ...." A score is twenty. How many years is fourscore and seven?

3. (14) Overnight the temperature dropped from 4°C to –3°C. This was a drop of how many degrees?

4. (15) If 203 turnips are to be shared equally among seven dwarfs, how many should each receive? What kind of pattern did you use?

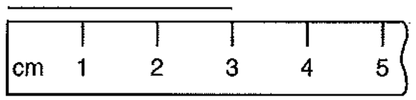
5. (18) What is the average of 1, 2, 4, and 9?

6. (10) What is the next number in the sequence?

1, 4, 9, 16, 25, _____, ...

7. (8) A regular hexagon has six sides of equal length. If each side of a hexagon is 25 mm, what is the perimeter?

8. (7) One centimeter equals ten millimeters. How many millimeters long is the line segment?



9. What are the whole number factors of 20?
(19)

10. How many different whole numbers are factors of 15?
(19)

11. Which of these numbers is divisible by 9?
(19)
A. 365 B. 1179 C. 1556

12. $250{,}000 \div 100$
(2)

13. $1234 \div 60$
(2)

14. $\dfrac{6 + 18 + 9}{3}$
(5)

15. $\$42 + \$375$
(1)

16. $\$3.45 \times 10$
(2)

17. $\$10.00 - w = \$1.93$
(3)

18. The letters a, b, and c represent three different numbers. The product of a and b is c.
(2)

$$ab = c$$

Rearrange the letters to make another multiplication fact and two division facts.

19. $\dfrac{w}{3} = 4$
(4)

20. Compare: $123 \div 1 \bigcirc 123 - 1$
(9)

21. Which digit is in the ten-millions' place in 135,792,468,000?
(12)

22. Round 123,456,789 to the nearest million.
(16)

23. How much money is $\frac{1}{2}$ of \$11.00?
(6)

24. If a square has a perimeter of 40 inches, how long is each side of the square?
(8)

25. $(51 + 49) \times (51 - 49)$
(5)

26. Which of these numbers is divisible by both 2 and 3?
(19)
A. 4671 B. 3858 C. 6494

27. By which whole numbers is 24 divisible?
(19)

28. The dictionaries were piled in three stacks. There were 6 dictionaries in one stack and 12 dictionaries in each of the other two stacks. How many dictionaries would be in each stack if some dictionaries were moved from the taller stacks to the shortest stack so that there were the same number of dictionaries in each stack?
(18)

29. Draw a square with sides that are $1\frac{3}{8}$ inches long.
(17)

30. Describe a method for deciding if a number is divisible by 3.
(19)

LESSON 20

Greatest Common Factor (GCF)

Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

Mental Math: Count up and down by 3's between 3 and 60.

a. 6 × 23 **b.** 6 × 32 **c.** 640 + 1200
d. 63 + 20 + 9 **e.** 63 + 30 − 1 **f.** $100.00 − $75.00
g. Start with 10; multiply by 10; subtract 1; divide by 9; add 1.

Problem Solving: Use the digits 5, 6, 7, and 8 to complete this subtraction problem. There are two possible arrangements.

$$\begin{array}{r} \_\,\_ \\ -\ 9 \\ \hline \_\,\_ \end{array}$$

The factors of 8 are

1, 2, 4, and 8

The factors of 12 are

1, 2, 3, 4, 6, and 12

We see that 8 and 12 have some of the same factors. They have three factors in common. Their three common factors are 1, 2, and 4. Their **greatest common factor**—the largest factor which they both have—is 4. Greatest common factor is often abbreviated **GCF.** The letters GCF stand for **G**reatest **C**ommon **F**actor.

Example 1 Find the greatest common factor of 12 and 18.

Solution The factors of 12 are

1, 2, 3, 4, 6, and 12

The factors of 18 are

1, 2, 3, 6, 9, and 18

We see that 12 and 18 share four common factors. The greatest of these is **6.**

Example 2 Find the GCF of 6, 9, and 15.

Solution The factors of 6 are

1, 2, 3, and 6.

The factors of 9 are

1, 3, and 9.

The factors of 15 are

1, 3, 5, and 15.

The GCF of 6, 9, and 15 is **3.**

Note: The search for the greatest common factor of two or more numbers is a search for the **largest** number which divides them. In this problem we can quickly determine by inspecting the numbers that 3 is the largest number that divides 6, 9, and 15. A complete listing of the factors may be helpful but is not required.

Practice* Find the greatest common factor (GCF) of the following:

a. 10 and 15
b. 18 and 27

c. 18 and 24
d. 12, 18, and 24

e. 15 and 25
f. 20, 30, and 40

g. 12 and 15
h. 20, 40, and 60

Problem set 20

1. (12) What is the difference between the product of 12 and 8 and the sum of 12 and 8?

2. (12) Saturn's average distance from the sun is one billion, four hundred twenty-seven million kilometers. Write that number.

3. (12) Which digit in 497,325,186 is in the ten-millions' place?

4. *(16)* Ernie actually had $427,872, but when Bert asked him how much money he had, Ernie rounded the amount to the nearest thousand dollars. How much did he say he had?

5. *(14)* The morning temperature was –3°C. By afternoon it had warmed to 8°C. How many degrees had the temperature risen?

6. *(18)* What is the average of 31, 52, and 40?

7. *(20)* Find the greatest common factor of 12 and 20.

8. *(20)* Find the GCF of 9, 15, and 21.

9. *(6)* How much money is $\frac{1}{4}$ of $3.24?

10. *(2)* $5432 \div 10$

11. *(5)* $\frac{28 + 42}{14}$

12. *(1)* $56{,}042 + 49{,}985$

13. *(3)* $14{,}009 - w = 9670$

14. *(3)* $w - 76 = 528$

15. *(5)* $5 \times 4 \times 3 \times 2 \times 1$

16. *(2)* $6.47 × 10

17. *(2)* $37{,}080 \div 12$

18. *(10)* Which number is missing in this sequence?

_____, 10, 16, 22, 28, ...

19. *(4)* $6w = 90$

20. *(9)* Compare: $50 - 1 \bigcirc 49 + 1$

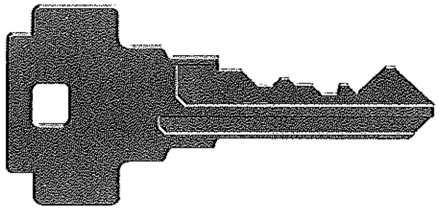
21. *(3)* $q - 365 = 365$

22. *(3)* $365 - p = 365$

23. *(10)* The first positive odd number is 1. What is the tenth positive odd number?

24. *(8)* The perimeter of a square is 100 cm. What is the length of each side?

25. Use your ruler to find the length of the key to the nearest sixteenth of an inch.
(17)



26. A "bit" is $\frac{1}{8}$ of a dollar.
(6)

(a) How many bits are in a dollar?

(b) How many bits are in three dollars?

27. In four boxes there are 12, 24, 36, and 48 ping pong balls, respectively. If the ping pong balls are rearranged so that there are the same number of ping pong balls in each of the four boxes, then how many ping pong balls would be in each box?
(18)

28. Which of these numbers is divisible by both 9 and 5?
(19)

A. 567 B. 875 C. 675

29. List the whole number factors of 24.
(19)

30. Ten billion is how much less than one trillion?
(12)

LESSON
21

Comparing Fractions with Pictures

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

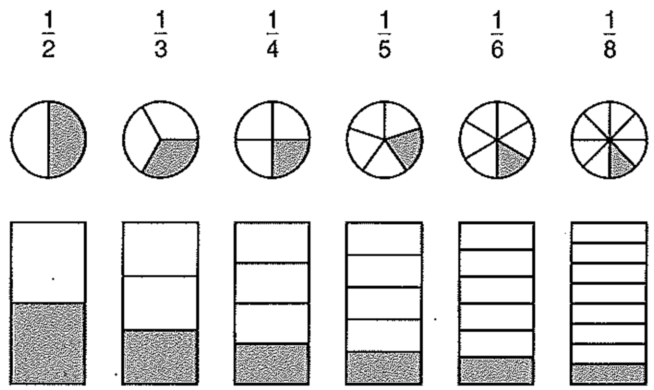
Mental Math: Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 12.

a. 4 × 42 **b.** 3 × 76 **c.** 64 + 19 *(19 is 20 − 1)*
d. 450 + 37 **e.** $10.00 − $6.50 **f.** $\frac{1}{2}$ of 24
g. Start with 25, × 2, − 1, ÷ 7, + 1, ÷ 2*

Problem Solving: Alexis has 6 coins that total exactly one dollar. Name three sets of coins that she **could** have.

*As a shorthand, we will use commas to separate operations to be performed sequentially from left to right. In this case, 25 × 2 = 50, then 50 − 1 = 49, then 49 ÷ 7 = 7, then 7 + 1 = 8, then 8 ÷ 2 = 4. The answer is 4.

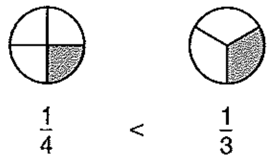
Sketching pictures that represent fractions can help us compare fractions. Recall that common fractions are written with two numbers. The denominator (the bottom number) tells the number of equal parts in the whole. The numerator (the top number) tells how many of the equal parts we are counting. We will usually sketch and shade circles or rectangles to represent fractions. Discussing sketching strategies can help us improve our sketches. Here are sample illustrations for six fractions. Notice that the segments we draw to divide a circle pass through the center of the circle or meet (intersect) at the center of the circle.



Example 1 Draw and shade circles to illustrate this comparison:

$$\frac{1}{4} < \frac{1}{3}$$

Solution We begin by drawing two circles the same size. We will divide one circle into four equal parts and the other circle into three equal parts. To divide a circle into four equal parts, we may draw a "+" in the circle. The segments intersect at the center of the circle. To divide a circle into three equal parts, we may draw an open "Y" in the circle with the three segments intersecting at the center of the circle. Then we shade one part of each circle.



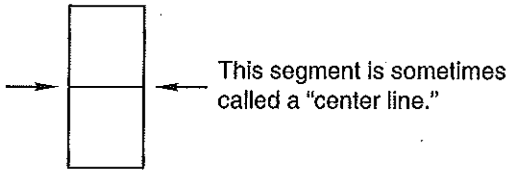
We see that less of the circle is shaded when $\frac{1}{4}$ is shaded than when $\frac{1}{3}$ is shaded. This illustrates that $\frac{1}{4}$ is less than $\frac{1}{3}$.

Example 2 Draw and shade rectangles to illustrate this comparison:

$$\text{Compare: } \frac{3}{4} \bigcirc \frac{3}{5}$$

Solution We begin by drawing two congruent rectangles (two rectangles that are the same size and shape). We will divide one rectangle into four equal sections and the other rectangle into five equal sections. We describe a possible strategy for each sketch.

To divide the rectangle into fourths, we may first divide the rectangle in half. (To sketch fractions with denominators that are even numbers, we may begin by drawing a segment that divides the figure in half.)

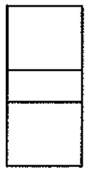


Then we divide each half in half to make four equal sections.

This rectangle is divided into fourths.

Notice that we draw three segments to make four parts. Now we will divide the other rectangle into five parts by drawing four segments.

Since five is not an even number, we will not begin by dividing the rectangle in half. Instead we will draw two segments across the rectangle, one just above and the other just below what would be the center line of the rectangle.

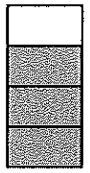


Draw segments a little above and a little below where the center line would be drawn.

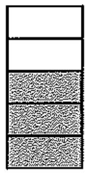
Then draw segments that divide the upper section and the lower section into two equal parts.

This rectangle is divided into fifths.

Once the rectangles are divided, we shade them and compare the fractions by comparing the shaded sketches.



$\frac{3}{4}$



$\frac{3}{5}$

We see that the $\frac{3}{4}$-shaded rectangle is slightly more shaded than the $\frac{3}{5}$-shaded rectangle. This illustrates that $\frac{3}{4}$ is greater than $\frac{3}{5}$.

$$\frac{3}{4} > \frac{3}{5}$$

Practice Draw and shade circles to illustrate these comparisons:

a. Compare: $\frac{1}{2}$ ○ $\frac{2}{4}$ **b.** Compare: $\frac{2}{3}$ ○ $\frac{3}{5}$

Draw and shade rectangles to illustrate these comparisons:

c. Compare: $\frac{1}{2}$ ○ $\frac{3}{6}$ **d.** Compare: $\frac{2}{5}$ ○ $\frac{1}{4}$

> **Teacher Note:** Fraction manipulatives will be used by students beginning in Lesson 23. Masters to produce these manipulatives are included in the *Math 76 Test Masters.* Please refer to Lessons 23, 25, and 28 for a list of materials that will be required.

Problem set 21

1. *(12)* What is the product of the sum of 8 and 5 and the difference of 8 and 5?

2. *(11)* Delaware became the first state in 1787. Hawaii became the fiftieth state admitted to the Union in 1959. How many years were there between these two events?

3. *(15)* Tom figured that the bowling balls on the rack weighed a total of 240 pounds. How many 16-pound bowling balls weigh a total of 240 pounds?

4. *(6)* An apple pie was cut into four equal slices. One slice was quickly eaten. What fraction of the pie was left?

5. *(6)* There are 17 girls in a class of 30 students. What fraction of the class is made up of girls?

6. Use digits to write the fraction three hundredths.
(6)

7. How much money is $\frac{1}{2}$ of \$2.34?
(6)

8. What is the place value of the 7 in 987,654,321?
(12)

9. What number comes next in this multiplication sequence?
(10)

1, 4, 16, 64, _____, ...

10. Compare: $64 \times 1 \bigcirc 64 + 1$
(9)

11. $50 - 1 = 49 + n$
(3)

12. Estimate the sum of 396, 197, and 203 by rounding to the nearest hundred before adding.
(16)

13. What is the greatest common factor (GCF) of 12 and 16?
(20)

14. $100\overline{)4030}$
(2)

15. $48{,}840 \div 24$
(2)

16. $\frac{678}{6}$
(2)

17. $\$4.75 \times 10$
(2)

18. $\$10 - w = 87¢$
(3)

19. $463 + 27 + m = 500$
(3)

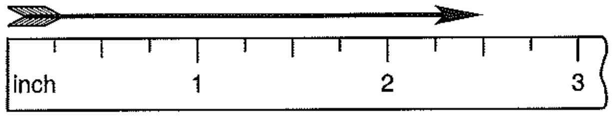
20. What number is 10 less than 3?
(14)

21. What is the average of 12, 16, and 23?
(18)

22. List the whole numbers that are factors of 28.
(19)

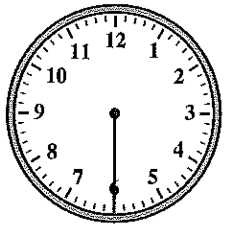
23. A regular octagon has eight equal-length sides. What is the perimeter of a regular octagon with sides 18 cm long?
(8)

24. How long is the arrow?
(7)



25. (12 × 12) − (11 × 13)
(5)

26. To divide a circle into thirds, John first imagined the face of a clock. From the center of the "clock" he drew one segment down to the six. Then starting from the center John drew two other segments. To which two numbers on the "clock" did John draw the two segments when he divided the circle into thirds?
(21)



27. Draw and shade rectangles to illustrate this comparison:
(21)

$$\frac{2}{3} < \frac{3}{4}$$

28. A "bit" is $\frac{1}{8}$ of a dollar.
(6)

(a) How many bits are in a dollar?

(b) How many bits are in a half dollar?

29. What numbers are factors of both 20 and 30?
(19)

30. Describe a method for dividing a circle into eight equal parts that involves drawing a plus sign and a times sign. Illustrate the explanation.
(21)

LESSON 22

"Equal Groups" Stories with Fractions

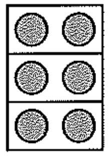
Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

Mental Math: Count by 2's from 2 to 40. Count by 4's from 4 to 40. Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 12.

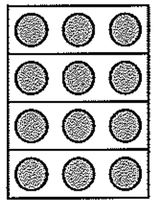
a. 4×54 **b.** 3×56 **c.** $36 + 29$ (*29 is 30 − 1*)
d. $359 - 42$ **e.** \$10.00 − \$3.50 **f.** $\frac{1}{2}$ of 48
g. Start with 100, − 1, ÷ 9, + 1, ÷ 2, − 1, × 5

Problem Solving: Two number cubes are rolled. The total number of dots on the two top faces is 6. What is the total of the dots on the bottom faces of the two number cubes?

Here we show a collection of six objects. The collection is divided into three equal groups. We see that there are two objects in $\frac{1}{3}$ of the collection. We also see that there are four objects in $\frac{2}{3}$ of the collection.

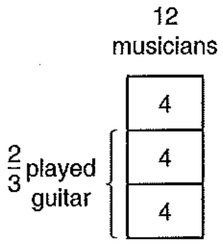


This collection of 12 objects has been divided into four equal groups. There are three objects in $\frac{1}{4}$ of the collection. There are nine objects in $\frac{3}{4}$ of the collection.



Example 1 Two thirds of the 12 musicians played guitars. How many of the musicians played guitars?

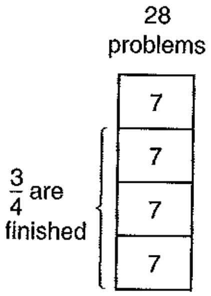
Solution This is a two-step problem. First we divide the 12 musicians into three equal groups (thirds). There are four musicians in each group. Then we count the number of musicians in two of the three groups.



Since there are four musicians in each third, the number of musicians in two thirds is eight. We find that **eight** of the 12 musicians played guitars.

Example 2 Cory has finished $\frac{3}{4}$ of the 28 problems on the assignment. How many problems has Cory finished?

Solution First we will divide the 28 problems into four equal groups (fourths). Then we will find the number of problems in three of the four groups. Since 28 ÷ 4 is 7, there are 7 problems in each group (in each fourth).

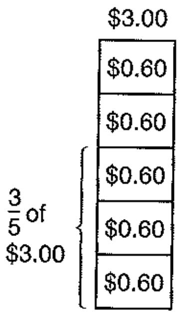


In one group there are 7 problems; in two groups there are 14 problems; and in three groups there are 21 problems. We see that Cory has finished **21 problems.**

Example 3 How much money is $\frac{3}{5}$ of \$3.00?

Solution First we will divide \$3.00 into 5 equal groups. Then we will find the amount of money in 3 of the 5 groups. We will divide \$3.00 by 5 to find the amount of money in each group.

$$\begin{array}{r}\$0.60\\5\overline{)\$3.00}\end{array}\text{ in each group.}$$



Now we multiply \$0.60 by 3 to find the amount of money in 3 groups.

$$\begin{array}{r}\$0.60\\\times\quad 3\\\hline\$1.80\end{array}$$

We find that $\frac{3}{5}$ of \$3.00 is **\$1.80.**

Practice* Answer the following questions. Draw a diagram to illustrate each problem.

a. Three fourths of the 12 musicians could play the piano. How many of the musicians could play the piano?

b. How much money is $\frac{2}{3}$ of \$4.50?

c. What number is $\frac{4}{5}$ of 60?

d. What number is $\frac{3}{10}$ of 80?

e. Five sixths of 24 is what number?

Problem set 22

1. *(12)* When the sum of 15 and 12 is subtracted from the product of 15 and 12, what is the difference?

2. *(6)* There were 13 original states. There are now 50 states. What fraction of the states are the original states?

3. *(11,15)* A marathon race is 26 miles plus 385 yards. A mile is 1760 yards. Altogether, how many yards long is a marathon? (First use an EG pattern to find the number of yards in 26 miles. Then use a SSM pattern to include the 385 yards.)

4. *(22)* If $\frac{2}{3}$ of the 12 jelly beans are eaten, how many are eaten? Draw a diagram to illustrate the problem.

5. *(22)* What number is $\frac{3}{4}$ of 16? Draw a diagram to illustrate the problem.

6. *(22)* How much money is $\frac{3}{10}$ of \$3.50? Draw a diagram to illustrate the problem.

7. *(14)* The temperature rose from −3°F to 4°F. How many degrees did the temperature rise?

8. *(3)* $w - 15 = 8$

9. *(5)* 36¢ + \$4.78 + \$34.09

10. *(4)* $w \div 67 = 345$

11. *(2)* $\$12.45 \div 3$

12. *(2)* $35\overline{)1000}$

13. *(5)* $\frac{7 + 9 + 14}{3}$

14. *(16)* Find the product of 36 and 124, and then round the answer to the nearest hundred.

15. *(12)* Which digit is in the ten-millions' place in 375,426,198,000?

16. *(20)* Find the greatest common factor of 12 and 15.

17. *(19)* List the factors of 30.

18. *(19)* The number 100 is divisible by which of these numbers: 2, 3, 5, 9, 10?

19. *(21)* Compare: $\frac{1}{3} \bigcirc \frac{1}{2}$

20. *(4)* $\frac{64}{m} = 64$

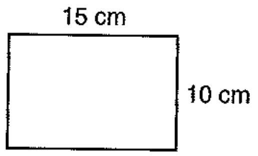
21. *(10)* Here are the first five numbers of a sequence. What would be the **seventh** number in the sequence?

1, 4, 7, 10, 13, ..., ________, ...

22. *(5)* $(3 + 3) - (3 \times 3)$

23. *(18)* Find the number halfway between 27 and 43.

24. *(8)* What is the perimeter of the rectangle?



25. *(7)* Use your ruler to find the length of this line segment.

26. (21) An apple pie was cut into six equal slices. A cherry pie, baked in the same size pie pan, was cut into five equal slices. Which was larger, a slice of apple pie or a slice of cherry pie?

27. (21) Compare these fractions. Draw and shade rectangles to illustrate the comparison.

$$\frac{2}{4} \bigcirc \frac{3}{5}$$

28. (22) A quarter of a year is $\frac{1}{4}$ of a year. There are 12 months in a year. How many months are in a quarter of a year?

29. (6) A "bit" is one eighth of a dollar.

(a) How many bits are in a dollar?

(b) How many bits are in a quarter of a dollar?

30. (1) The letters c, p, and t represent three different numbers. When p is subtracted from c, the answer is t.

$$c - p = t$$

Use these letters to write another subtraction fact and two addition facts.

LESSON 23

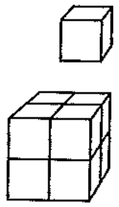
Fraction Manipulatives, Part 1

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count up and down by 3's between 3 and 60. Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 12.

a. 5×62 **b.** 5×36 **c.** $87 + 9$ (*9 is 10 − 1*)
d. $1200 + 350$ **e.** \$20.00 − \$15.50 **f.** $\frac{1}{2}$ of 84
g. $10 \times 3, + 2, \div 4, + 1, \div 3, \times 4, \div 6$

Problem Solving: Sarah used 8 sugar cubes to make a larger cube as shown. The cube she made was two cubes high, two cubes wide, and two cubes deep. How many cubes will she need to make a cube that has three cubes along each edge?



In this lesson you will make your own set of fraction manipulatives to use as you answer the questions in this lesson and in future problem sets.

Activity: Fraction Manipulatives $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$

Materials needed:

- Each student needs a copy of "Activity Master 2," which includes patterns for halves, fourths, and eighths (available in the *Math 76 Test Masters*).
- Scissors
- Envelopes or locking plastic bags in which to store fraction pieces
- Colored pencils or markers if the fraction manipulatives are to be color-coded

Note: Color-coding the fraction manipulatives makes sorting easier. If you wish to color-code the manipulatives, agree upon a different color for each fraction circle. Students may lightly color the front and back of each circle before cutting. Following the activity, each student should store the fraction manipulatives in an envelope or plastic bag for use in later lessons.

Preparation for activities:

- Distribute materials. Have students color-code the manipulatives if desired. Then have students separate the fraction manipulatives by cutting out the fraction circles and cutting apart the fraction slices along the lines. After the activities, store the manipulatives for later use.

Use your fraction manipulatives to help you with these activities and questions.

a. What percent of a circle is $\frac{1}{2}$ of a circle?

b. What fraction is half of $\frac{1}{2}$?

c. What fraction is half of $\frac{1}{4}$?

d. Fit three $\frac{1}{4}$ pieces together to form $\frac{3}{4}$ of a circle. Three fourths of a circle is what percent of a circle?

e. Fit four $\frac{1}{8}$ pieces together to form $\frac{4}{8}$ of a circle. Four eighths of a circle is what percent of a circle?

f. If you add $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$, the sum is $\frac{3}{8}$. If you add $\frac{3}{8} + \frac{2}{8}$, what is the sum?

Using your halves, fourths, and eighths manipulatives, a whole circle can be formed using two through eight pieces. Write a number sentence for each blank in the chart.

NUMBER OF PIECES USED	NUMBER SENTENCE WITH A SUM OF 1
2	$\frac{1}{2} + \frac{1}{2} = 1$
3	$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$
4	$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$ or **g.**
5	$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = 1$ or **h.**
6	**i.**
7	**j.**
8	**k.**

l. Show that both $\frac{4}{8}$ and $\frac{2}{4}$ make $\frac{1}{2}$ of a circle. (We say that $\frac{4}{8}$ and $\frac{2}{4}$ both "reduce" to $\frac{1}{2}$.)

m. The fraction $\frac{2}{8}$ is equivalent to which single fraction piece?

n. The fraction $\frac{6}{8}$ is equivalent to how many $\frac{1}{4}$'s?

Work in groups of two or three students for the remaining exercises.

o. Show that the improper fraction $\frac{5}{4}$ is equal to the mixed number $1\frac{1}{4}$ by combining four of the $\frac{1}{4}$ pieces to make a whole circle.

p. To what mixed number is the improper fraction $\frac{7}{4}$ equal?

After the activities have been completed, each student should gather and store his or her own fraction manipulatives in a bag for later use.

Problem set 23

1. How many millimeters long is a ruler that is 30 cm long? *(7)*

2. Dan has finished $\frac{2}{3}$ of the 27 problems on an assignment. How many problems has Dan finished? *(22)*

3. William Tell shot at the apple from 100 paces. If each pace was 36 inches, how many inches away was the apple? *(15)*

4. There are 31 days in December. After December 25, what fraction of the month remains? *(6)*

5. What number is $\frac{3}{5}$ of 25? *(22)*

6. How much money is $\frac{7}{10}$ of \$36.00? *(22)*

You may use your fraction manipulatives to help you answer questions 7 through 9.

7. What is the sum of $\frac{3}{8}$ and $\frac{4}{8}$? *(23)*

8. The improper fraction $\frac{9}{8}$ equals what mixed number? *(23)*

9. (23) Two eighths of a circle is what percent of a circle?

10. (3)
$$\begin{array}{r} \$10.20 \\ -\quad m \\ \hline \$3.46 \end{array}$$

11. (2)
$$\begin{array}{r} \$3.75 \\ \times\quad 16 \\ \hline \end{array}$$

12. (2) $\dfrac{\$3.75}{25}$

13. (12) What is the place value of the 6 in 36,174,591?

14. (22) Describe a way to find $\frac{2}{3}$ of a number.

15. (4) $\$0.35n = \$35.00$

16. (21) Compare: $\frac{3}{4}$ ○ 1

17. (8) The length of a rectangle is 20 inches. The width is half the length. What is the perimeter of the rectangle?

18. (10) What is the sixth number in this sequence?

2, 4, 8, 16, ...

19. (16) Estimate the sum of 3174 and 4790 to the nearest thousand.

20. (9) Compare: $12 \div 6 - 2$ ○ $12 \div (6 - 2)$

21. (20) What is the greatest common factor (GCF) of 24 and 32?

22. (10) What is the sum of the first seven positive odd numbers?

You may use your fraction manipulatives to help you answer questions 23, 24, and 25.

23. (23) (a). How many $\frac{1}{4}$'s are in 1?

(b) How many $\frac{1}{4}$'s are in $\frac{1}{2}$?

24. (23) One eighth of a circle is what percent of a circle?

25. (23) Write a fraction with a denominator of 8 that is equal to $\frac{1}{2}$.

LESSON 24

Adding and Subtracting Fractions That Have Common Denominators

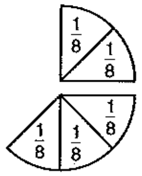
Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

Mental Math: Count by 4's from 4 to 80.
Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 12.

a. 6×24
b. 4×75
c. $47 + 39$
d. $1500 - 250$
e. \$20.00 − \$14.50
f. $\frac{1}{2}$ of 68
g. $6 \times 7, - 2, \div 5, \times 2, - 1, \div 3$

Problem Solving: Xavier, Yolanda, and Zollie finished first, second, and third in the race, though not necessarily in that order. (a) List all the possible orders of finish. (b) If Xavier was not first, how many possible orders of finish were there?

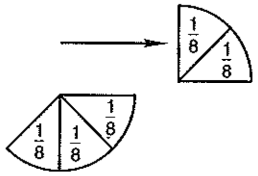
Using our fraction manipulatives, we see that when we add $\frac{2}{8}$ to $\frac{3}{8}$ the sum is $\frac{5}{8}$.



$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

Three eighths plus two eighths equals five eighths.

Likewise, if we subtract $\frac{2}{8}$ from $\frac{5}{8}$, then $\frac{3}{8}$ are left.



$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

Five eighths minus two eighths equals three eighths.

Notice that we add the numerators when we add fractions and we subtract the numerators when we subtract fractions. The denominator of the fraction is not changed by adding or subtracting.

Example 1 $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

Solution The denominators are the same. We add the numerators.

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \mathbf{\frac{3}{4}}$$

Example 2 $\frac{1}{2} + \frac{1}{2}$

Solution One half and one half is two halves, which is one whole.

$$\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$$

$$\frac{2}{2} = \mathbf{1}$$

Example 3 $\frac{7}{8} - \frac{2}{8}$

Solution The denominators are the same. We subtract two eighths from seven eighths.

$$\frac{7}{8} - \frac{2}{8} = \mathbf{\frac{5}{8}}$$

Example 4 $\frac{2}{4} - \frac{1}{4}$

Solution The denominators are the same. We subtract $\frac{1}{4}$ from $\frac{2}{4}$ and get $\frac{1}{4}$.

$$\frac{2}{4} - \frac{1}{4} = \mathbf{\frac{1}{4}}$$

Example 5 $\frac{1}{2} - \frac{1}{2}$

Solution If we start with $\frac{1}{2}$ and subtract $\frac{1}{2}$, then what is left is zero.

$$\frac{1}{2} - \frac{1}{2} = \frac{0}{2}$$

$$\frac{0}{2} = \mathbf{0}$$

Practice Add or subtract as shown:

a. $\frac{3}{8} + \frac{4}{8}$

b. $\frac{3}{4} + \frac{1}{4}$

c. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

d. $\frac{4}{8} - \frac{1}{8}$

e. $\frac{3}{4} - \frac{2}{4}$

f. $\frac{1}{4} - \frac{1}{4}$

Problem set 24

1. (11,15) Martin worked in the yard for five hours and was paid $6.00 per hour. Then he washed the car for $5. Altogether, how much money did Martin earn? What pattern did you use to find Martin's yard-work earnings? Then what pattern did you use to find his total earnings?

2. (22) In one bite, Cookie ate $\frac{3}{4}$ of a dozen chocolate chip cookies. How many cookies did Cookie eat in that bite? Draw a diagram that illustrates the problem.

3. (22) One mile is one thousand, seven hundred sixty yards. How many yards is $\frac{1}{8}$ of a mile?

You may use your fraction manipulatives to help answer problems 4 through 8.

4. (24) $\frac{1}{4} + \frac{2}{4}$

5. (24) $\frac{7}{8} - \frac{4}{8}$

6. (24) $\frac{1}{2} + \frac{1}{2}$

7. (24) $\frac{1}{2} - \frac{1}{2}$

8. (23) What percent of a circle is $\frac{1}{2}$ of a circle plus $\frac{1}{4}$ of a circle?

9. (19) Which of these numbers is divisible by both 2 and 5?
A. 1760 B. 365 C. 1492

10. (18) What number is halfway between 123 and 321?

11. (8) Paul wanted to fence in a square pasture for Babe, his blue ox. Each side was to be 25 miles long. How many miles of fence did Paul need?

12. (16) Round 32,987,145 to the nearest million.

13. (10) What number is missing in this sequence?

1, 7, ______, 19, 25, ...

14. (2) $9\overline{)1000}$

15. (2) 22,422 ÷ 32

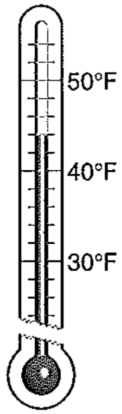
16. (4) $w \div 8 = 20$

17. (2) $350.00 ÷ 100

18. (4) $7x = 84$

19. (21) Compare: $\frac{1}{2} \bigcirc \frac{1}{4}$

20. (10) What temperature is shown on the thermometer?



21. (5) $(35 \times 35) - (5 \times 5)$

22. (16) Estimate the product of 385 and 214.

23. (20) What is the GCF of 21 and 28?

24. (19) Which of these numbers is divisible by 9?
A. 123 B. 234 C. 345

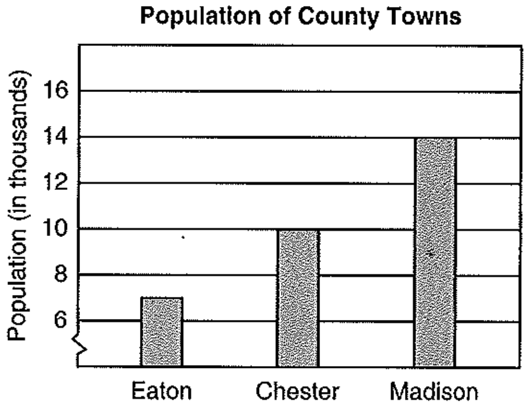
25. (23) Write a fraction equal to 1 that has 4 as the denominator.

Find the missing numbers:

26. $376 + w = 481$
(3)

27. $m - 286 = 592$
(3)

Use the information in this graph to help answer questions 28 through 30.



28. Which town has about twice the population of Eaton?
(16)

29. About how many more people live in Madison than in Chester?
(16)

30. Sketch this graph on your paper and add a fourth town to your graph: Wilson, population 11,000.
(16)

LESSON 25

Fraction Manipulatives, Part 2

Facts Practice: 100 Multiplication Facts (Test E in Test Masters)

Mental Math: Count up and down by 3's between 3 and 60. Count up and down by 6's between 6 and 60.

a. 7 × 52 **b.** 6 × 33 **c.** 63 + 19
d. 256 + 50 **e.** \$10.00 − \$7.25 **f.** $\frac{1}{2}$ of 86
g. 8 × 8, − 1, ÷ 7, × 2, + 2, ÷ 2

Problem Solving: The digits 1 through 9 are used in this subtraction problem. Copy the problem and fill in the missing digits.

$$\begin{array}{r} \_\,\_\,\_ \\ -\ 452 \\ \hline 3\,\_\,\_ \end{array}$$

In this lesson you will make fraction manipulatives for thirds, sixths, and twelfths.

Activity: Fraction Manipulatives $\left(\frac{1}{3}, \frac{1}{6}, \frac{1}{12}\right)$

Materials needed:

- Each student needs a copy of "Activity Master 3" (available in the *Math 76 Test Masters*).
- Scissors
- Envelopes or locking plastic bags in which to store fraction pieces (re-use bag from Lesson 23)
- Colored pencils or markers
- Fraction manipulatives made in Lesson 23

Preparation for activities:

- Distribute materials. If fraction manipulatives will be color-coded, we suggest agreeing upon colors and lightly coloring the front and back of each circle before cutting. After the activities, store the manipulatives for later use.

Use all the fraction manipulatives you have made to help you with these activities and questions.

a. What fraction is half of $\frac{1}{3}$?

b. What fraction is half of $\frac{1}{6}$?

c. How many sixths equal $\frac{1}{2}$?

d. How many twelfths equal $\frac{1}{2}$?

e. Form a whole circle using six of the $\frac{1}{6}$ pieces. Then remove (subtract) $\frac{1}{6}$. What fraction of the circle is left?

f. Demonstrate subtracting $\frac{1}{3}$ from 1 by forming a circle of $\frac{3}{3}$ and then removing $\frac{1}{3}$. What fraction is left?

g. Use four fourths to demonstrate the subtraction $1 - \frac{1}{4}$ and write the answer.

h. Use eight eighths to demonstrate the subtraction $1 - \frac{3}{8}$ and write the answer.

i. What percent of a circle is $\frac{1}{3}$ of a circle?

j. What percent of a circle is $\frac{3}{6}$ of a circle?

k. Make a fraction equivalent to $\frac{4}{6}$ using thirds.

l. Find a single fraction piece that matches $\frac{3}{12}$.

m. What percent of a circle is $\frac{3}{12}$ of a circle?

n. Find a single fraction piece that matches $\frac{4}{12}$.

o. How many twelfths equal $\frac{2}{3}$?

p. How many twelfths equal $\frac{3}{4}$?

q. The improper fraction $\frac{4}{3}$ equals what mixed number?

r. Convert $\frac{11}{6}$ to a mixed number.

Store your manipulatives for later use.

Problem set 25

1. (12) What is the product of the sum of 55 and 45 and the difference of 55 and 45?

2. (22) Potatoes are three fourths water. If a sack of potatoes weighs 20 pounds, how many pounds of water are in the potatoes?

3. (11) Frankie found three hundred six fleas on his dog. He caught two hundred forty-nine of them. How many fleas got away?

4. (17) What number is halfway between 8 and 9?

5. Which of these numbers is divisible by both 2 and 3?
(19)
A. 122 B. 123 C. 132

6. Round 1,234,567 to the nearest ten thousand.
(16)

7. If ten pounds of apples cost \$4.90, then what is the price per pound?
(15)

8. What is the denominator of $\frac{23}{24}$?
(6)

9. What number is $\frac{3}{5}$ of 65?
(22)

10. How much money is $\frac{2}{3}$ of \$15?
(22)

You may use your fraction manipulatives to help answer problems 11 through 18.

11. $\frac{1}{6} + \frac{2}{6} + \frac{3}{6}$
(24)

12. $\frac{11}{12} - \frac{5}{12}$
(24)

13. $\frac{6}{6} - \frac{5}{6}$
(25)

14. $\frac{2}{8} + \frac{5}{8}$
(24)

15. (a) What percent of a circle is $\frac{1}{12}$ of a circle?
(25)
(b) What percent of a circle is $\frac{2}{12}$ of a circle?

16. (a) How many $\frac{1}{12}$'s are in 1?
(25)
(b) How many $\frac{1}{12}$'s are in $\frac{1}{2}$?

17. What fraction is half of $\frac{1}{4}$?
(23)

18. What fraction of a circle is 50% of a circle?
(23)

19. $52\overline{)2100}$
(2)

20. $\frac{432}{18}$
(2)

21. If a 36-inch-long string is made into the shape of a square, how long will each side be?
(8)

22. Convert $\frac{7}{6}$ to a mixed number.
(25)

23. $(55 + 45) \div (55 - 45)$
(5)

24. Which of these numbers is divisible by both 2 and 5?
(19)
A. 502 B. 205 C. 250

25. Describe a method for deciding if a number is divisible by 9.
(19)

LESSON 26

Writing Division Answers as Mixed Numbers • Multiples

Facts Practice: 90 Division Facts (Test F in Test Masters)

Mental Math: Count by $\frac{1}{8}$'s from $\frac{1}{8}$ to 2.

a. 6×43 **b.** 3×75 **c.** $57 + 29$ (*29 is 30 − 1*)
d. $2650 - 150$ **e.** \$10.00 − \$6.25 **f.** $\frac{1}{2}$ of 30
g. 10 × 2, + 1, ÷ 3, + 2, ÷ 3, × 4, ÷ 3

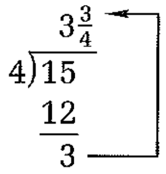
Problem Solving: Alexis has 6 coins that total exactly \$1.00. Name the one coin she **must** have.

Writing division answers as mixed numbers

We have been writing division answers with remainders. However, not all questions involving division can be appropriately answered using remainders. Some story problems have answers that are mixed numbers, as we see in the following example:

Example 1 A 15-inch length of ribbon was cut into four equal lengths. How long was each piece of ribbon?

Solution We divide 15 by 4 and write the answer as a mixed number.



Notice that the remainder is the numerator of the fraction and the divisor is the denominator of the fraction. We find that the length of each piece of ribbon is **$3\frac{3}{4}$ inches.**

Example 2 A whole circle is 100% of a circle. One third of a circle is what percent of a circle?

Solution If we divide 100% by 3, we will find the percent equivalent of $\frac{1}{3}$.

$$\begin{array}{r} 33\frac{1}{3}\% \\ 3\overline{)100\%} \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

We write the answer **$33\frac{1}{3}$%.** Notice that our answer matches our fraction manipulative piece for $\frac{1}{3}$.

Example 3 Write $\frac{25}{6}$ as a mixed number.

Solution The fraction line in $\frac{25}{6}$ is also a division sign. We divide 25 by 6 and write the remainder as the numerator of the fraction.

$$\begin{array}{r} 4\frac{1}{6} \\ 6\overline{)25\phantom{\frac{1}{6}}} \\ \underline{24}\phantom{\frac{1}{6}} \\ 1\phantom{\frac{1}{6}} \end{array}$$

We find that the improper fraction $\frac{25}{6}$ equals the mixed number **$4\frac{1}{6}$.**

Multiples

We find **multiples** of a number by multiplying the number by 1, 2, 3, 4, 5, 6, and so on.

The first six multiples of 2 are 2, 4, 6, 8, 10, and 12.
The first six multiples of 3 are 3, 6, 9, 12, 15, and 18.
The first six multiples of 4 are 4, 8, 12, 16, 20, and 24.
The first six multiples of 5 are 5, 10, 15, 20, 25, and 30.

Example 4 What are the first four multiples of 8?

Solution Multiplying 8 by 1, 2, 3, and 4 gives the first four multiples: **8, 16, 24,** and **32.**

Example 5 What number is the eighth multiple of 7?

Solution The eighth multiple of 7 is 8 times 7, which is **56.**

Practice*

a. A 28-inch long ribbon was cut into eight equal lengths. How long was each piece of ribbon?

b. A whole circle is 100% of a circle. What percent of a circle is $\frac{1}{7}$ of a circle?

c. Divide 467 by 10 and write the quotient as a mixed number.

d. What are the first four multiples of 12?

e. What are the first six multiples of 8?

f. What number is both the third multiple of 8 and the second multiple of 12?

Write each of these improper fractions as a mixed number:

g. $\frac{35}{6}$ **h.** $\frac{49}{10}$ **i.** $\frac{65}{12}$

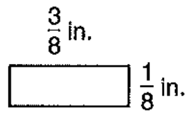
Problem set 26

1. (25) What is the difference between the sum of $\frac{1}{2}$ and $\frac{1}{2}$ and the sum of $\frac{1}{3}$ and $\frac{1}{3}$?

2. (18) In three tries Carlos punted the football 35 yards, 30 yards, and 37 yards, respectively. How can Carlos find the average distance of his punts?

3. (12) The earth's average distance from the sun is one hundred forty-nine million, six hundred thousand kilometers. Use digits to write that number.

4. (8) What is the perimeter of the rectangle?



5. (26) A 30-inch length of ribbon was cut into 4 equal lengths. How long was each piece of ribbon?

6. (25) Two thirds of the class finished the test on time. What fraction of the class did not finish the test on time?

7. (22) Compare: $\frac{1}{2}$ of 12 ○ $\frac{1}{3}$ of 12

8. (23) What fraction is half of the fraction that is half of $\frac{1}{2}$?

9. (26) A whole circle is 100% of a circle. What percent of a circle is $\frac{1}{9}$ of a circle?

10. (25) (a) How many $\frac{1}{6}$'s are in 1?

(b) How many $\frac{1}{6}$'s are in $\frac{1}{2}$?

11. (26) What fraction of a circle is $33\frac{1}{3}\%$ of a circle?

12. (26) Divide 365 by 7 and write the answer as a mixed number.

13. (24) $\frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

14. (24) $\frac{6}{6} - \frac{5}{6}$

15. (25) $\frac{5}{8} + m = 1$

16. (24) $\frac{5}{12} - \frac{5}{12}$

17. (10) What number comes next in this sequence?

81, 64, 49, 36, ______, ...

18. (15) Cheryl bought 10 pens for 25¢ each. How much did she pay for all ten pens?

19. (20) What is the greatest common factor (GCF) of 24 and 30?

20. (5) $30 \times 40 \div 60$

21. (22) What number is $\frac{1}{100}$ of 100?

22. *(16)* Estimate the sum of 3142, 6328, and 4743 to the nearest thousand.

23. *(22)* Two thirds of the students liked hamburgers. If 60 students were asked, how many liked hamburgers? Draw a diagram that illustrates the problem.

24. *(4)* $\frac{144}{n} = 12$

25. *(17)* Use your ruler to find the length of this line segment.

26. *(21)* To divide a circle into thirds, Jan imagined the circle was the face of a clock. She drew one segment from the center of the "clock" up to where the "12" would be on a clock. Then she drew two more segments from the center of the circle. To which two numbers on the face of a clock did Jan draw the segments to divide the circle into thirds?

27. *(26)* Write $\frac{15}{4}$ as a mixed number.

28. *(21)* Draw and shade rectangles to illustrate and complete this comparison:

$$\frac{3}{4} \bigcirc \frac{4}{5}$$

29. *(26)* What are the first four multiples of 25?

30. *(19)* Which of these numbers is divisible by both 9 and 10?

A. 910 B. 8910 C. 78,910

LESSON 27

Using Manipulatives to Reduce Fractions • Adding and Subtracting Mixed Numbers

Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

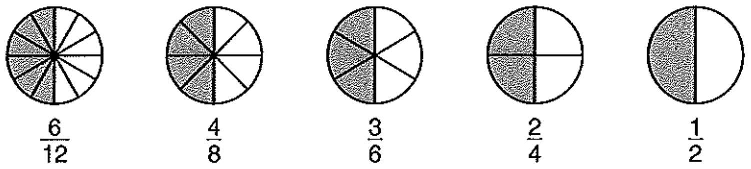
Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 2.

a. 7×34
b. 4×56
c. $74 + 19$
d. $475 + 125$
e. \$5.00 − \$1.75
f. $\frac{1}{2}$ of 32
g. $7 \times 5, + 1, \div 6, \times 3, \div 2, + 1, \div 5$

Problem Solving: Tad picked up a number cube. His thumb and forefinger covered opposite faces. He counted the dots on the other four faces. How many dots did he count?

Using manipulatives to reduce fractions

Use your fraction manipulatives to form these fraction models.

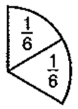


We see that each of these models illustrates half of a circle. The model that uses the fewest number of pieces is $\frac{1}{2}$. We say that each of the other fractions **reduces** to $\frac{1}{2}$.

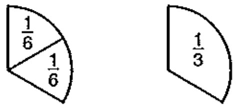
We can use our fraction manipulatives to reduce a given fraction by making an equivalent model that uses fewer pieces.

Example 1 Use your fraction manipulatives to reduce $\frac{2}{6}$.

Solution First we use our manipulatives to form $\frac{2}{6}$.



Then we search for a fraction piece that is equivalent to the $\frac{2}{6}$ model. We find $\frac{1}{3}$.

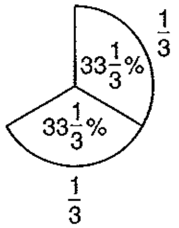


The models illustrate that $\frac{2}{6}$ reduces to $\frac{1}{3}$.

Adding and subtracting mixed numbers

When adding mixed numbers, we add the whole number parts together and the fractions parts together. When subtracting mixed numbers, we subtract fraction parts from fractions and whole number parts from whole numbers.

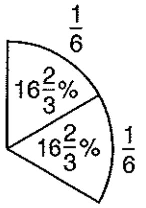
Example 2 Two thirds of a circle is what percent of a circle?



Solution One third is equivalent to $33\frac{1}{3}\%$. So two thirds is equivalent to $33\frac{1}{3}\% + 33\frac{1}{3}\%$. We add.

$$\begin{array}{r} 33\frac{1}{3}\% \\ + \; 33\frac{1}{3}\% \\ \hline \mathbf{66\frac{2}{3}\%} \end{array}$$

Example 3 Two sixths of a circle is what percent of a circle?



Solution We add $16\frac{2}{3}\%$ and $16\frac{2}{3}\%$.

$$\begin{array}{r} 16\frac{2}{3}\% \\ +\ 16\frac{2}{3}\% \\ \hline 32\frac{4}{3}\% \end{array}$$

We notice that the fraction part of the answer, $\frac{4}{3}$, is an improper fraction that equals $1\frac{1}{3}$.

So $32\frac{4}{3}\%$ equals $32 + 1\frac{1}{3}\%$, which is $\mathbf{33\frac{1}{3}\%}$. This makes sense because $\frac{2}{6}$ of a circle equals $\frac{1}{3}$, which is equivalent to $33\frac{1}{3}\%$.

Example 4 Tom lives $2\frac{3}{4}$ miles from school. Tom rode his bike from home to school and back to home. How far did he ride?

Solution This is a "some and some more" story.

$$\begin{array}{r} 2\frac{3}{4}\text{ mi} \\ +\ 2\frac{3}{4}\text{ mi} \\ \hline 4\frac{6}{4}\text{ mi} \end{array}$$

We notice that the fraction part of the answer, $\frac{6}{4}$, reduces to $1\frac{1}{2}$. We find that Tom rode his bike $\mathbf{5\frac{1}{2}}$ **miles.**

Example 5 $5\frac{3}{8} - 1\frac{1}{8}$

Solution We subtract $\frac{1}{8}$ from $\frac{3}{8}$, and we subtract 1 from 5. The difference is $4\frac{2}{8}$.

$$5\frac{3}{8} - 1\frac{1}{8} = 4\frac{2}{8}$$

We reduce the fraction $\frac{2}{8}$ to $\frac{1}{4}$. We write the answer $\mathbf{4\frac{1}{4}}$.

Practice Use your fraction manipulatives to reduce these fractions:

a. $\frac{2}{8}$

b. $\frac{6}{8}$

Add. Reduce the answer when possible.

c. $12\frac{1}{2}\% + 12\frac{1}{2}\%$

d. $16\frac{2}{3}\% + 66\frac{2}{3}\%$

e. $3\frac{3}{4} + 2\frac{3}{4}$

f. $1\frac{1}{8} + 2\frac{7}{8}$

g. $3 + 2\frac{2}{3}$

h. $\frac{3}{4} + 4$

Problem set 27

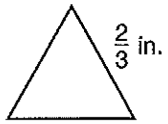
1. (27) Jan rode her bike to the park and back. If the trip was $3\frac{3}{4}$ miles each way, how far did she ride in all?

2. (15) The young elephant was 36 months old. How many years old was the elephant?

3. (6) Gwen bought $2\frac{1}{2}$ dozen cupcakes for the party. That was enough for how many children to have one cupcake each?

4. (15) There are 100 centimeters in a meter. There are 1000 meters in a kilometer. How many centimeters are in a kilometer?

5. (8) What is the perimeter of the equilateral triangle?



6. (23) Compare: $\frac{1}{2}$ plus $\frac{1}{2}$ ○ $\frac{1}{2}$ of $\frac{1}{2}$

7. (27) $5\frac{7}{8} + 7\frac{5}{8}$

8. (27) One eighth of a circle is $12\frac{1}{2}\%$ of a circle. What percent of a circle is $\frac{3}{8}$ of a circle?

9. *(25)* Write a fraction equal to 1 that has a denominator of 12.

10. *(20)* What is the greatest common factor of 15 and 25?

11. *(10)* What is the **seventh** number in this sequence?

8, 16, 24, 32, 40, ...

12. *(26)* Write $\frac{14}{5}$ as a mixed number.

13. *(27)* Add and simplify: $\frac{2}{5} + \frac{4}{5}$

14. *(25)* $\frac{2}{3} + n = 1$

15. *(19)* What is the largest factor of both 12 and 18?

16. *(25)* $1 - \frac{3}{4}$

17. *(27)* $3\frac{3}{4} + 3$

18. *(24)* $2\frac{1}{2} - 2\frac{1}{2}$

19. *(14)* What number is 25 less than 100?

20. *(5)* $(123 + 123 + 123) - (123 + 123)$

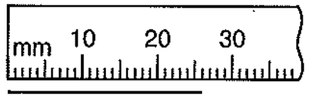
21. *(16)* Estimate the difference of 5063 and 3987 to the nearest thousand.

22. *(27)* Use your fraction manipulatives to reduce $\frac{3}{12}$.

23. *(18)* Find the average of 85, 85, 90, and 100.

24. *(22)* How much money is $\frac{3}{5}$ of $30?

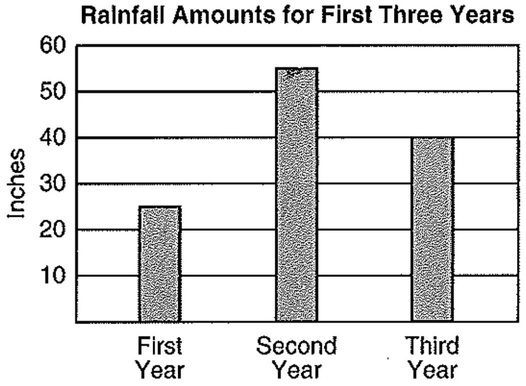
25. *(7)* How many millimeters long is the line segment?



26. *(17)* Arrange these numbers in order from least to greatest:

$\frac{1}{2}$, 0, −1, 1

Sandra began measuring the rainfall when she moved to her new home. The graph below shows the annual rainfall during her first three years in her home. Use the information in this graph to answer questions 27 through 30.



27. About how many more inches of rain fell during the second year than during the first year?
(18)

28. What was the approximate average annual rainfall during the first three years?
(18)

29. The first year's rainfall was about how many inches below the average annual rainfall of the first three years?
(18)

30. Write a "some and some more" problem that refers to this graph and then answer the problem.
(11)

LESSON
28

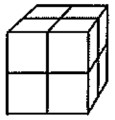
Fraction Manipulatives, Part 3

Facts Practice: 90 Division Facts (Test F in Test Masters)

Mental Math: Count up and down by 3's between 3 and 60. Count up and down by 6's between 6 and 60.

a. 8×42
b. 3×85
c. $36 + 49$
d. $1750 - 500$
e. \$10.00 − \$8.25
f. $\frac{1}{2}$ of 36
g. $8 \times 4, + 1, \div 3, + 1, \times 2, + 1, \div 5$

Problem Solving: Grant glued eight wooden blocks together to make a cube. Then he painted all six faces of the cube. Later the cube broke apart into eight blocks. Describe which sides of the small blocks had been painted.



In this lesson you will make fraction manipulatives for fifths and tenths.

Activity: Fraction Manipulatives $\left(\frac{1}{5}, \frac{1}{10}\right)$

Materials needed:

- Each student needs a copy of "Activity Master 4" (available in the *Math 76 Test Masters*).
- Scissors
- Envelopes or locking plastic bags in which to store fraction pieces (re-use bag from Lesson 23)
- Colored pencils or markers
- Fraction manipulatives made in Lessons 23 and 25

Preparation for activities:

- Distribute materials. Color-code circles, if desired, before cutting. Save fraction manipulatives for later use.

Use all the fraction manipulatives you have made to help you with these activities and questions.

a. What fraction is half of $\frac{1}{5}$?

b. How many tenths equal $\frac{1}{2}$?

c. Two fifths of a circle is what percent of a circle?

d. What percent of a circle is $\frac{7}{10}$ of a circle?

e. Demonstrate the subtraction $1 - \frac{2}{5}$ and write the answer.

f. Demonstrate the subtraction $1 - \frac{3}{10}$ and write the answer.

Use all of your fraction manipulatives to find the fractional answer.

g. $\frac{1}{3}$ of $\frac{1}{2}$ (Start with $\frac{1}{2}$. Which fraction piece covers $\frac{1}{3}$ of it?)

h. $\frac{1}{2}$ of $\frac{1}{4}$ (Start with $\frac{1}{4}$. Which fraction piece covers $\frac{1}{2}$ of it?)

i. $\frac{1}{3}$ of $\frac{1}{4}$

j. $\frac{1}{2}$ of $\frac{1}{3}$

k. $\frac{1}{2}$ of $\frac{2}{3}$ (Start with $\frac{2}{3}$. Then find $\frac{1}{2}$ of it.)

l. $\frac{1}{2}$ of $\frac{4}{5}$

m. $\frac{1}{3}$ of $\frac{3}{4}$

n. $\frac{2}{3}$ of $\frac{3}{4}$

o. $\frac{1}{3}$ of $\frac{9}{10}$

Write each improper fraction as a mixed number:

p. $\frac{17}{10}$ **q.** $\frac{17}{5}$ **r.** $\frac{17}{12}$

Use your manipulatives to reduce each fraction:

s. $\frac{5}{10}$ **t.** $\frac{2}{10}$ **u.** $\frac{4}{6}$

Write each improper fraction as a mixed number with the fraction part of the mixed number reduced:

v. $\frac{12}{10}$ **w.** $\frac{15}{10}$ **x.** $\frac{14}{10}$

Problem set 28

1. (24) What is the sum of $\frac{1}{3}$ and $\frac{2}{3}$ and $\frac{3}{3}$?

2. (22) Two fifths of Robin Hood's one hundred forty men rode with Little John to the castle. How many men went with Little John? Draw a diagram to illustrate the problem.

3. (15) Seven hundred sixty-eight peanuts are to be shared equally by the thirty-two children at the party. How many should each child receive? What type of pattern do we use?

4. (13) Columbus discovered the Americas in 1492. The Declaration of Independence was signed in 1776. How many years were there from Columbus's discovery of the Americas to the signing of the Declaration of Independence? What type of pattern do we use?

5. (26) Convert $\frac{23}{3}$ to a mixed number.

6. (27) $1\frac{2}{3} + 1\frac{2}{3}$

7. (27) $3 + 4\frac{2}{3}$

8. (27) $3\frac{5}{6} - 1\frac{4}{6}$

9. (27) Use your fraction manipulatives to reduce $\frac{4}{8}$.

10. (22) How much money is $\frac{2}{3}$ of $24.00?

11. (28) What percent of a circle is $\frac{3}{10}$ of a circle?

12. (23) Twenty-five percent of a circle is what fraction of a circle?

13. (14) What number is 240 less than 250?

14. (25) $\frac{1}{4} + m = 1$

15. (3) $423 - w = 297$

16. (9) Compare: $20 \times 20 \bigcirc 21 \times 19$

17. *(18)* On the last four papers Christie had 22 right, 20 right, 23 right, and 23 right, respectively. She averaged how many right on each paper?

18. *(8)* A 36-inch-long string is formed into the shape of an equilateral triangle. How long is each side of the triangle?

19. *(20)* What is the greatest common factor (GCF) of 24, 36, and 60?

20. *(1)* 10,010 − 9909

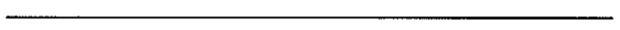
21. *(5)* (100 × 100) − (100 × 99)

22. *(26)* If $\frac{1}{10}$ of the class was absent, what percent of the class was absent?

23. *(26)* Divide 5097 by 10 and write the answer as a mixed number.

24. *(22)* Three fourths of two dozen eggs is how many eggs? Draw a diagram to illustrate the problem.

25. *(17)* Use your ruler to find the length of this line segment to the nearest sixteenth of an inch.



26. *(26)* List the first five multiples of 6 and the first five multiples of 8. Circle any numbers that are multiples of both 6 and 8.

27. *(28)* Which fraction manipulative covers $\frac{1}{2}$ of $\frac{1}{5}$?

INVESTIGATION 1

Frequency Tables • Histograms • Surveys

Frequency tables

Mr. Lawson made a frequency table to record student scores on a math test. As he graded tests he made a tally mark in the row that shows the number of correct answers on the test.

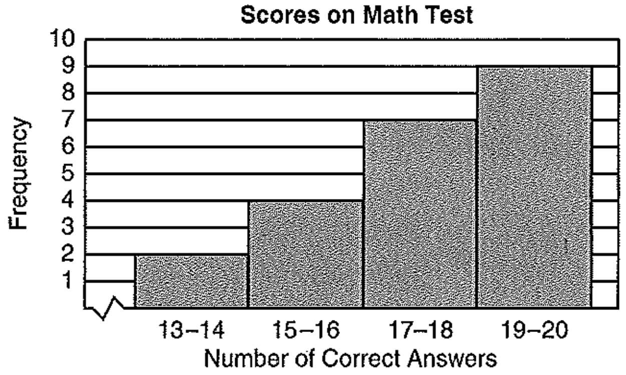
Frequency Table

Number Correct	Tally	Frequency
19–20	𝍸 IIII	9
17–18	𝍸 II	7
15–16	IIII	4
13–14	II	2

When Mr. Lawson finished grading the tests, he counted the number of tally marks in each row, and he recorded the count in the frequency column. For example, the table shows that nine students had either 19 or 20 correct answers on the test. A **frequency table** is a way of pairing selected data, in this case specified test scores, with the number of times the selected data occurred.

Histograms

Using the information in the frequency table, Mr. Lawson created a histogram to display the results of the test.



A **histogram** is a special type of bar graph. This histogram displays the data (test scores) in equal sized intervals (range of scores). There are no spaces between the bars. The break in the horizontal scale (∿) shows that the scale on the graph is broken between 0 and 13. The height of the bar indicates the number of test scores in each interval.

Refer to the histogram to answer questions 1, 2, and 3.

1. Which interval had the lowest frequency of scores?

2. Which interval had the highest frequency of scores?

3. Which interval had twice as many scores as the 13–14 interval?

4. Make a frequency table and histogram for the following set of scores. Use 50–59, 60–69, 70–79, 80–89, and 90–99 for the intervals.

   Test scores: 63, 75, 58, 89, 92, 84, 95, 63, 78, 88, 96, 67, 59, 70, 83, 89, 76, 85, 94, 80

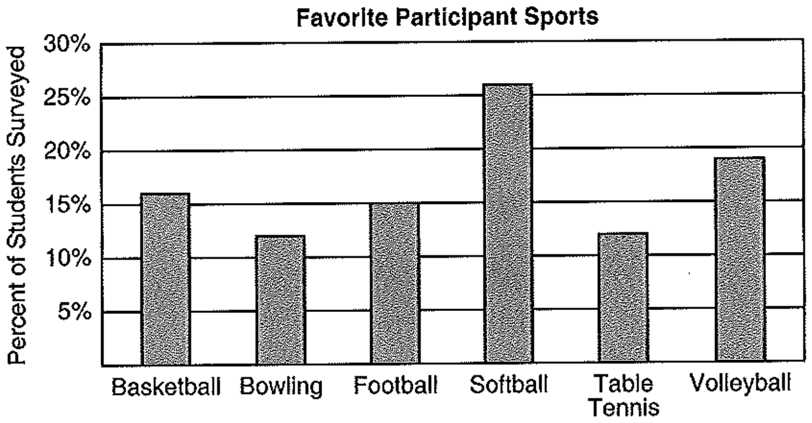
Surveys

A **survey** is a way of collecting data about a population. Rather than study every member of a population, a survey studies a small part of the population, called a **sample.** From the sample, conclusions are formed about the entire population.

Mrs. Patterson's class conducted a survey of 100 male and female students to determine the favorite participant sport of middle school students. Survey participants were given a choice of six different sports and were asked to select the sport they enjoyed participating in the most. The surveyors made a frequency table for the responses.

Sport	Tally	Frequency
Basketball	𝍸𝍸𝍸 I	16
Bowling	𝍸𝍸 II	12
Football	𝍸𝍸𝍸	15
Softball	𝍸𝍸𝍸𝍸𝍸 I	26
Table Tennis	𝍸𝍸 II	12
Volleyball	𝍸𝍸𝍸 IIII	19

From the frequency table Mrs. Patterson's students constructed a bar graph to display the results.



Since 16 out of 100 students selected basketball as their favorite participant sport, basketball was the choice of 16% (which means 16 out of 100) of the students surveyed. Refer to the frequency table and bar graph for this survey to answer questions 5 through 8.

5. Which sport was the favorite sport of about $\frac{1}{4}$ of the students surveyed (1 in 4 students)?

6. Which sport was the favorite sport of the girls who were surveyed?

A. Softball
B. Volleyball
C. Basketball
D. Cannot be determined from information provided.

7. How might changing the sample group change the results of the survey?

8. How might changing the question—the choice of sports—change the results of the survey?

This survey was **closed-option** because survey responses were limited to the six choices offered. An **open-option** survey does not limit the choices. An example of an open-option survey question is, "What is your favorite sport?"

Extensions

a. Consider making a frequency table and histogram based on results from a recent class test. Would raw scores or percentage scores be used? What intervals would be used? What questions can be answered by referring to the histogram?

b. Conduct a class survey of favorite foods. Determine which foods will be included in the survey if the survey question is closed-option. What will be the size of the sample? How will the data gathered by the survey be displayed?

c. Plan a series of surveys that can be conducted throughout the year. Surveys could ask individuals about favorite television shows or books. Opinion surveys could be conducted about current issues. Brainstorm a list of possible topics and design questions to be asked about each topic.

d. Look for surveys in the newspaper or in magazines and bring the publication to class to discuss these questions. What was the population that was surveyed? Did those doing the survey ask open or closed questions? How were the results of the survey displayed? What was the purpose of the survey?

LESSON 29

Multiplying Fractions • Reducing Fractions by Dividing by Common Factors

Facts Practice: 100 Addition Facts (Test B in Test Masters)

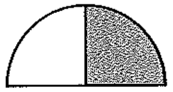
Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 3.

a. 7 × 43 **b.** 4 × 64 **c.** 53 + 39
d. 325 + 50 **e.** \$20.00 − \$17.25 **f.** $\frac{1}{2}$ of 70
g. 4 × 5, − 6, ÷ 7, × 8, + 9, × 2

Problem Solving: Todd flipped a coin three times. It landed up heads, heads, then tails. If he flips the coin three more times, what are the possible outcomes?

Multiplying fractions

We have shaded $\frac{1}{2}$ of $\frac{1}{2}$ of a circle.



We see that $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$.

When we find $\frac{1}{2}$ of $\frac{1}{2}$ we are actually multiplying. The "of" in $\frac{1}{2}$ of $\frac{1}{2}$ means to multiply. The problem becomes

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

When we multiply fractions, we multiply the numerators to find the numerator of the product, and we multiply the denominators to find the denominator of the product.

Example 1 What fraction is $\frac{1}{2}$ of $\frac{3}{5}$?

Solution The word "of" means to multiply. By multiplying $\frac{1}{2}$ and $\frac{3}{5}$ we find $\frac{1}{2}$ of $\frac{3}{5}$.

$$\frac{1}{2} \times \frac{3}{5} = \frac{3 \longleftarrow (1 \times 3 = 3)}{10 \longleftarrow (2 \times 5 = 10)}$$

We find that $\frac{1}{2}$ of $\frac{3}{5}$ is $\frac{3}{10}$. You can illustrate this with your fraction manipulatives.

Example 2 Multiply: $\frac{3}{4} \times \frac{2}{3}$

Solution By performing this multiplication we will find $\frac{3}{4}$ of $\frac{2}{3}$. We multiply the numerators to find the numerator of the product, and we multiply the denominators to find the denominator of the product.

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

The fraction $\frac{6}{12}$ can be reduced to $\frac{1}{2}$, as you can see with your fraction manipulatives.

A whole number may be written as a fraction by writing the whole number as the numerator of the fraction and 1 as the denominator of the fraction. Thus, the whole number 2 may be written as the fraction $\frac{2}{1}$. Writing whole numbers as fractions is helpful when multiplying whole numbers and fractions.

Example 3 Multiply: $4 \times \frac{2}{3}$

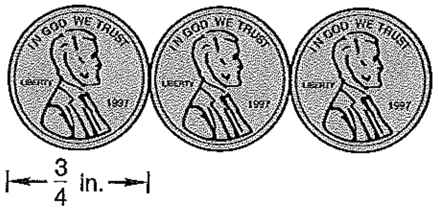
Solution We write 4 as $\frac{4}{1}$ and multiply.

$$\frac{4}{1} \times \frac{2}{3} = \frac{8}{3}$$

Then we convert the improper fraction $\frac{8}{3}$ to a mixed number.

$$\frac{8}{3} = \mathbf{2\frac{2}{3}}$$

Example 4 Three pennies are laid in a row. The diameter of one penny is $\frac{3}{4}$ inch. How long is the row of pennies?



Solution We may find the answer by adding or by multiplying. We will show both ways.

$$\text{Adding: } \frac{3}{4}\text{ in.} + \frac{3}{4}\text{ in.} + \frac{3}{4}\text{ in.} = \frac{9}{4}\text{ in.}$$

Then we convert $\frac{9}{4}$ inches to $2\frac{1}{4}$ inches.

$$\text{Multiplying: } \frac{3}{1} \times \frac{3}{4}\text{ in.} = \frac{9}{4}\text{ in.}$$

$$\frac{9}{4}\text{ in.} = 2\frac{1}{4}\text{ in.}$$

We find that the row of pennies is **$2\frac{1}{4}$ inches** long.

Reducing fractions by dividing by common factors

We can reduce fractions by dividing the numerator and the denominator by a factor of both numbers. To reduce $\frac{6}{12}$, we will divide both the numerator and denominator by 6.

$$\frac{6}{12} \div \frac{6}{6} = \frac{1}{2} \begin{array}{l} \longleftarrow (6 \div 6 = 1) \\ \longleftarrow (12 \div 6 = 2) \end{array}$$

We divide the numerator and the denominator by 6 because 6 is the largest factor (the GCF) of both 6 and 12.

If we had divided by 2 instead of by 6 we would not have completely reduced the fraction.

$$\frac{6}{12} \div \frac{2}{2} = \frac{3}{6}$$

The fraction $\frac{3}{6}$ can be reduced by dividing the numerator and the denominator by 3.

$$\frac{3}{6} \div \frac{3}{3} = \frac{1}{2}$$

It took two steps to reduce $\frac{6}{12}$ to $\frac{1}{2}$. It takes two or more steps to reduce some fractions if we do not divide by the greatest common factor.

Example 5 Reduce: $\frac{8}{12}$

Solution We will show two ways.

Divide numerator and denominator by 2.

$$\frac{8}{12} \div \frac{2}{2} = \frac{4}{6}$$

Divide numerator and denominator by 4.

$$\frac{8}{12} \div \frac{4}{4} = \frac{2}{3}$$

Continue reducing $\frac{4}{6}$ by dividing 4 and 6 by 2.

$$\frac{4}{6} \div \frac{2}{2} = \frac{2}{3}$$

Either way we find that $\frac{8}{12}$ reduces to $\frac{2}{3}$. Since the greatest common factor of 8 and 12 is 4, we reduce $\frac{8}{12}$ in one step by dividing the numerator and denominator by 4.

Example 6 Multiply: $2 \times \frac{5}{12}$

Solution We write 2 as $\frac{2}{1}$ and multiply.

$$\frac{2}{1} \times \frac{5}{12} = \frac{10}{12}$$

We can reduce $\frac{10}{12}$ because both 10 and 12 are divisible by 2.

$$\frac{10}{12} \div \frac{2}{2} = \mathbf{\frac{5}{6}}$$

Practice* Multiply; then reduce if possible.

a. $\frac{1}{2}$ of $\frac{4}{5}$ **b.** $\frac{1}{4}$ of $\frac{2}{3}$ **c.** $\frac{2}{3} \times \frac{3}{4}$

Multiply; then convert each answer from an improper fraction to a whole number or to a mixed number.

d. $\frac{5}{6} \times \frac{6}{5}$ **e.** $5 \times \frac{2}{3}$ **f.** $2 \times \frac{4}{3}$

Reduce each fraction:

g. $\frac{9}{12}$ **h.** $\frac{6}{10}$ **i.** $\frac{18}{24}$

Problem set 29

1. (25) If the product of $\frac{1}{2}$ and $\frac{1}{2}$ is subtracted from the sum of $\frac{1}{2}$ and $\frac{1}{2}$, what is the difference?

2. (15) The African elephant can weigh eight tons. A ton is two thousand pounds. How many pounds can an African elephant weigh?

3. (15) Sixteen jelly beans weigh one ounce. How many jelly beans weigh one pound? (1 pound = 16 ounces)

4. (29) Reduce: $\frac{6}{8}$

5. (29) Reduce: $\frac{16}{24}$

6. (24) $\frac{1}{8} + \frac{3}{8}$

7. (29) $\frac{1}{2} \times \frac{2}{3}$

8. (24) $\frac{7}{12} - \frac{3}{12}$

9. (22) How much money is $\frac{1}{10}$ of \$40.00?

10. (10) Write the next three numbers in the sequence:

1, 4, 7, 10, _____, _____, _____, ...

11. (25) When five months have passed, what fraction of the year remains?

12. (2) \$3.60 × 100

13. (2) 50,000 ÷ 100

14. (26) Convert $\frac{18}{4}$ to a mixed number. Remember to reduce the fraction part of the mixed number.

15. (14) The temperature rose from –8°F to 15°F. This was a rise of how many degrees?

16. (3) $m + 496 + 2684 = 3217$

17. (3) $1000 - n = 857$

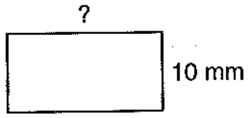
18. (5) $7 \times 11 \times 13$

19. (4) $24x = 480$

20. *(16)* Describe how to estimate the quotient of 4963 ÷ 39.

21. *(29)* Compare: $\frac{2}{3} \times \frac{3}{2} \bigcirc 1$

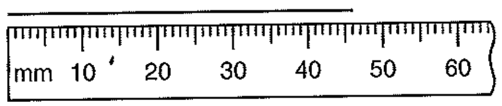
22. *(8)* The perimeter of the rectangle is 60 mm. Its width is 10 mm. What is the length?



23. *(14)* 12 − 40

24. *(29)* $\left(\frac{1}{2} \times \frac{1}{2}\right) - \frac{1}{4}$

25. *(7)* How long is the line segment?



26. *(29)* What fraction is $\frac{2}{3}$ of $\frac{3}{5}$?

27. *(29)* What is the product of $\frac{3}{4}$ and $\frac{4}{3}$?

28. *(29)* What fraction is $\frac{1}{2}$ of $\frac{5}{6}$?

29. *(26)* Convert $\frac{30}{8}$ to a mixed number. Reduce the fraction part of the mixed number.

30. *(26)* What percent of a circle is $\frac{2}{5}$ of a circle?

LESSON 30

Least Common Multiples • Reciprocals

Facts Practice: 100 Multiplication Facts (Test E in Test Masters)

Mental Math: Count up and down by 3's between 3 and 30. Count up and down by 4's between 4 and 40.

a. 9 × 32 b. 5 × 42 c. 45 + 49
d. 436 + 99 e. $20.00 − $12.75 f. $\frac{1}{2}$ of 72
g. 7 × 7, − 1, ÷ 6, × 3, + 1, × 2, − 1

Problem Solving: Use the digits 6, 7, and 8 to complete this multiplication problem.

$$\begin{array}{r} 23\_ \\ \times \quad \_ \\ \hline 166\_ \end{array}$$

Least common multiples

Common multiples are numbers that are multiples of more than one number. Here we show some multiples of 2 and 3. We have emphasized the common multiples.

Multiples of 2: 2, 4, **6,** 8, 10, **12,** 14, 16, **18,** 20, ...

Multiples of 3: 3, **6,** 9, **12,** 15, **18,** 21, ...

We see that 6, 12, and 18 are common multiples of 2 and 3. Since the number 6 is the least of these common multiples, it is called the **least common multiple.** The term "least common multiple" is abbreviated LCM.

Example 1 What is the least common multiple of 3 and 4?

Solution We will list some multiples of each number and emphasize the common multiples.

Multiples of 3: 3, 6, 9, **12,** 15, 18, 21, **24,** ...

Multiples of 4: 4, 8, **12,** 16, 20, **24,** 28, ...

We see that the numbers 12 and 24 are in both lists. Both 12 and 24 are common multiples of 3 and 4. The least of the common multiples, which is the first common multiple, is **12.**

Example 2 What is the LCM of 2 and 4?

Solution We will list some multiples of 2 and 4.

Multiples of 2: 2, **4**, 6, **8**, 10, **12**, ...

Multiples of 4: **4, 8, 12,** 16, 20, ...

The first number that is a common multiple of both 2 and 4 is **4.**

Reciprocals

Reciprocals are two numbers whose product is 1. For example, 2 and $\frac{1}{2}$ are reciprocals because $2 \times \frac{1}{2}$ equals 1.

$$\underbrace{2 \times \frac{1}{2}}_{\text{reciprocals}} = 1$$

We say that 2 is the reciprocal of $\frac{1}{2}$, and $\frac{1}{2}$ is the reciprocal of 2. Sometimes we want to find the reciprocal of a certain number. One way we will practice finding the reciprocal of a number is by solving equations like this.

$$3 \times \square = 1$$

The number that goes in the box is $\frac{1}{3}$ because 3 times $\frac{1}{3}$ is 1. One third is the reciprocal of three.

Reciprocals also answer questions like this one.

How many $\frac{1}{4}$'s are in 1?

The answer 4 is the reciprocal of $\frac{1}{4}$.

Fractions have two terms, the numerator and the denominator. To form the reciprocal of a fraction, we make a new fraction with the terms of the fraction reversed.

$$\frac{3}{4} \rightleftarrows \frac{4}{3}$$

The new fraction, $\frac{4}{3}$, is the reciprocal of $\frac{3}{4}$.

If we multiply $\frac{3}{4}$ and $\frac{4}{3}$, we see that the product, $\frac{12}{12}$, equals 1.

$$\frac{3}{4} \times \frac{4}{3} = 1$$

Example 3 How many $\frac{2}{3}$'s are in 1?

Solution To find the number of $\frac{2}{3}$'s in 1, we need to find the reciprocal of $\frac{2}{3}$. The easiest way to find the reciprocal of $\frac{2}{3}$ is to reverse the positions of the 2 and the 3. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. (We may convert $\frac{3}{2}$ to $1\frac{1}{2}$, but we usually write reciprocals as fractions rather than as mixed numbers.)

Example 4 What number goes into the box to make the equation true?

$$\frac{5}{6} \times \square = 1$$

Solution When $\frac{5}{6}$ is multiplied by its reciprocal, the product is 1. So the answer is the reciprocal of $\frac{5}{6}$, which is $\frac{6}{5}$. When we multiply $\frac{5}{6}$ and $\frac{6}{5}$ we get $\frac{30}{30}$.

$$\frac{5}{6} \times \frac{6}{5} = \frac{30}{30}$$

The fraction $\frac{30}{30}$ equals 1.

Example 5 What is the reciprocal of 5?

Solution Recall that a whole number may be written as a fraction that has a denominator of 1. So 5 can be written $\frac{5}{1}$. (This means five wholes.) Reversing the positions of the 5 and the 1 gives us the reciprocal of 5, which is $\frac{1}{5}$. This makes sense because five $\frac{1}{5}$'s make 1, and $\frac{1}{5}$ of 5 is 1.

Practice* Find the least common multiple of each pair of numbers:

a. 6 and 8 **b.** 3 and 5 **c.** 5 and 10

Write the reciprocal of each number:

d. 6 **e.** $\frac{2}{3}$ **f.** $\frac{8}{5}$ **g.** $\frac{1}{3}$

Find the number that goes into the box that makes each equation true.

h. $\frac{3}{8} \times \square = 1$ **i.** $4 \times \square = 1$

j. $\square \times \frac{1}{6} = 1$

k. $\square \times \frac{7}{8} = 1$

l. How many $\frac{2}{5}$'s are in 1?

m. How many $\frac{5}{12}$'s are in 1?

Problem set 30

1. (26) If the fourth multiple of 3 is subtracted from the third multiple of 4, what is the difference?

2. (22) About $\frac{2}{3}$ of a person's body weight is water. Albert weighs 117 pounds. About how many pounds of Albert's weight is the weight of the water?

3. (15) Cynthia ate 42 pieces of popcorn in the first 15 minutes of a movie. If she kept eating at the same rate, how many pieces of popcorn did she eat in the 2-hour movie?

4. (26) What are the first four multiples of 8?

5. (30) What is the least common multiple of 4 and 6?

6. (30) What is the LCM of 6 and 10?

7. (24) $\frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

8. (28) $1 - \frac{1}{10}$

9. (24) $\frac{11}{12} - \frac{1}{12}$

10. (29) $\frac{3}{4} \times \frac{4}{3}$

11. (29) $5 \times \frac{3}{4}$

12. (29) $\frac{5}{2} \times \frac{5}{3}$

13. (19) The number 24 has how many different whole number factors?

14. (5) \$3 + \$24 + \$6.50

15. (1) \$5 − \$1.50

16. (16) Estimate the product: 596 × 405.

17. (12) Find the difference of one billion and nine hundred eight million, fifty-three thousand.

18. *(29)* Compare: $\frac{2}{3} \times \frac{2}{3} \bigcirc \frac{2}{3} \times 1$

19. *(2)* 500,000 ÷ 100

20. *(2)* $35\overline{)8540}$

21. *(26)* $\frac{100\%}{7}$

22. *(29)* Reduce: $\frac{4}{12}$

23. *(18)* What is the average of 375, 632, and 571?

24. *(8)* A regular hexagon has six sides of equal length. If a regular hexagon is made from a 36-inch-long string, what will be the length of each side?

25. *(30)* What is the product of a number and its reciprocal?

26. *(30)* How many $\frac{2}{5}$'s are in 1?

27. *(30)* What number goes in the box to make the equation true?

$$\frac{3}{8} \times \square = 1$$

28. *(30)* What is the reciprocal of 6?

29. *(26)* Convert the improper fraction $\frac{45}{10}$ to a mixed number with the fraction part of the mixed number reduced.

30. *(29)* Four pennies are placed in a row. The diameter of one penny is $\frac{3}{4}$ inch. What is the length of the row of pennies?

LESSON 31

Areas of Rectangles • Comparing Differences

Facts Practice: 90 Division Facts (Test F in Test Masters)

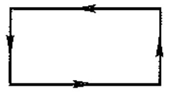
Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 2.

a. 4 × 25 **b.** 6 × 37 **c.** 28 + 29 *(29 is 30 – 1)*
d. \$6.25 + \$2.50 **e.** $\frac{1}{3}$ of 63 **f.** $\frac{600}{10}$
g. 10 × 10, – 20, + 1, ÷ 9, × 2, ÷ 3, × 5, + 2, ÷ 4

Problem Solving: The bus has 40 seats for passengers. Two passengers can sit in each seat. Sixty passengers got on the bus. What is the largest number of seats that could be empty? What is the largest number of seats that could have just one passenger?

Areas of rectangles

We have measured the distance around a shape. The distance around a shape is called its perimeter.

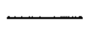


The perimeter of a shape is the distance around it.

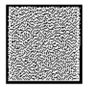
We can also measure how much surface is enclosed by the sides of a shape. When we measure the "inside" of a flat shape we are measuring its **area.**

The area of a shape is the amount of surface enclosed by its sides.

We use different kinds of units to measure perimeter and area. To measure perimeter we use units of length like centimeters. To measure area we use units that have area, like square centimeters.



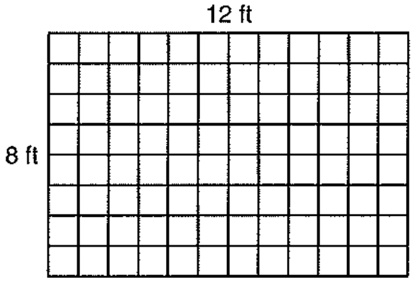
This is one centimeter.



This is one square centimeter.

Units of area such as square centimeters, square feet, and square yards are like floor tiles. The area of a shape is the number of "floor tiles" of a certain size that completely cover the shape. Note that we can use "sq." to abbreviate the word "square."

Example 1 How many floor tiles, one foot on a side, are needed to cover the floor of a room that is 8 feet wide and 12 feet long?

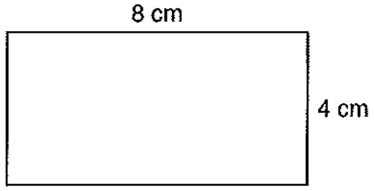


Solution The surface of the floor is covered with tiles. By answering this question we are finding the area of the room in square feet. We could count the tiles, but a faster way to find the number of tiles is to multiply. There are 8 rows of tiles with 12 tiles in each row.

$$\begin{array}{rl} 12 & \text{tiles in each row} \\ \times\ \ 8 & \text{rows} \\ \hline 96 & \text{tiles} \end{array}$$

The number of tiles needed to cover the floor is **96.** This means that the area of the room is 96 sq. ft.

Example 2 What is the area of this rectangle?



Solution The diagram shows the length and width of the rectangle in centimeters. So the unit of area we will use is square centimeters. To calculate the number of square-centimeter tiles needed to cover the rectangle, we multiply the length and width of the rectangle.

$$\text{Length} \times \text{width} = \text{area}$$

$$8 \text{ cm} \times 4 \text{ cm} = \mathbf{32 \text{ sq. cm}}$$

Comparing differences

Recall that when we subtract two numbers the answer is the difference. The difference of 100 and 85 is 15. The difference of 114 and 100 is 14. By comparing the differences, 15 and 14, we can determine that 114 is closer to 100 than 85 is.

Example 3 Which of these numbers is closest to 64?

A. 56 B. 71 C. 58

Solution We can see that 58 is closer than 56 to 64. So we can eliminate 56 from consideration. To determine whether 58 or 71 is closer to 64 we subtract.

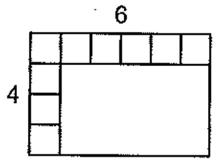
$$\begin{array}{r} 64 \\ -\,58 \\ \hline 6 \end{array} \qquad \begin{array}{r} 71 \\ -\,64 \\ \hline 7 \end{array}$$

We subtract in the order that results in a positive difference. The smaller the difference the closer the number is to 64. Since the difference 6 is less than 7, we find that 58 is closer to 64 than 71 is. So given our three choices the answer is **C. 58.**

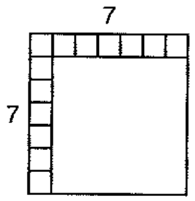
Practice

Find the number of square units needed to cover the area of these shapes.

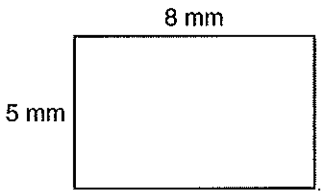
a.



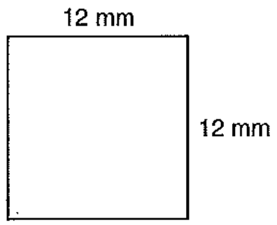
b.



c.



d.



e. Which of these numbers is closest to 500?

A. 449 B. 559 C. 459 D. 549

Problem set 31

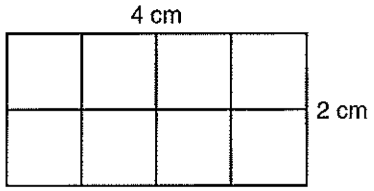
1. *(26)* When the third multiple of 4 is divided by the fourth multiple of 3, what is the quotient?

2. *(30)* How many $\frac{3}{4}$'s are in 1?

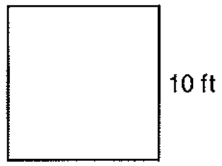
3. *(12)* The distance the earth travels around the sun each year is about five hundred eighty million miles. Use digits to write that number.

4. *(26)* Convert $\frac{10}{3}$ to a mixed number.

5. *(31)* How many square stickers 1 centimeter on a side would be needed to cover this rectangle?



6. *(31)* How many square floor tiles 1 foot on a side would be needed to cover this square?



7. *(31)* What is the area of a rectangle 12 inches long and 8 inches wide?

8. *(10)* What is the next number in the sequence?

1, 4, 9, 16, 25, 36, ____, ...

9. *(22)* What number is $\frac{2}{3}$ of 24?

10. *(3)* $24 + f = 42$

Write each answer in simplest form:

11. *(24)* $\frac{1}{8} + \frac{1}{8}$

12. *(24)* $\frac{5}{6} - \frac{1}{6}$

13. *(29)* $\frac{2}{3} \times \frac{1}{2}$

14. *(29)* $\frac{2}{3} \times 5$

15. *(16)* Estimate the product of 387 and 514.

16. *(2)* $20.00 ÷ 10

17. *(2)* 47¢ × 63

18. *(2)* 4623 ÷ 22

19. *(30)* What is the reciprocal of 2?

20. *(27)* Two thirds of a circle is what percent of a circle?

21. *(31)* Which of these numbers is closest to 100?

A. 90 B. 89 C. 111 D. 109

22. *(12)* Which digit is in the ten-millions' place in 987,654,321?

23. *(10)* The first three positive odd numbers are 1, 3, and 5. What is the sum of the first five positive odd numbers?

24. *(29)* Three of the nine players play outfield. What fraction of the players play outfield? (Reduce.)

25. *(17)* Use your ruler to find the length of this line segment.

26. *(29)* $\frac{3}{10} \times \frac{3}{10}$

27. *(29)* Write a fraction equal to 1 with a denominator of 8.

28. *(22)* Five sixths of the 24 students in the class scored 80% or higher on the test. How many students scored 80% or higher? Draw a diagram to illustrate the problem.

29. (29) Reduce $\frac{30}{100}$.

30. (31) Using a ruler, how could you calculate the floor area of your classroom?

LESSON 32

Expanded Notation • Elapsed Time

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by 25's between 25 and 400.

a. 4×75 **b.** $380 + 1200$ **c.** $54 + 19$

d. \$8.00 − \$1.50 **e.** $\frac{1}{2}$ of 240 **f.** $\frac{600}{100}$

g. $12 \times 3, - 1, \div 5, \times 2, + 1, \div 3, \times 2$

Problem Solving: Cheryl picked up a number cube and held it so that she could see the dots on three faces. The total number of dots on the three faces was 6. What was the number of dots on each of the three faces? What was the total number of dots on the other three faces?

Expanded notation Recall that in our number system the location of a digit in a number has a value called its place value. Consider the value of the 5 in these two numbers.

250 520

In 250 the value of the 5 is 5×10. In 520 the value of the 5 is 5×100.

The value of a digit in a number is the value of the digit times the value of the place the digit occupies. When we write a number in **expanded notation,** we write each non-zero digit times its place value.

Example 1 Write 250 in expanded notation.

Solution The 2 is in the hundreds' place, and the 5 is in the tens' place. In expanded notation we write

$$(2 \times 100) + (5 \times 10)$$

Since there is a zero in the ones' place we could add a third set of parentheses (0 × 1). However, since zero times any number equals zero, it is not necessary to include zeros when writing numbers in expanded notation.

Example 2 Write $(5 \times 1000) + (2 \times 100) + (8 \times 10)$ in standard notation.

Solution Standard notation is our usual way of writing numbers. One way to think about this number is 5000 + 200 + 80. Another way to think about this number is 5 in the thousands' place, 2 in the hundreds' place, and 8 in the tens' place. We may assume a 0 in the ones' place. Either way we think about the number, the standard form is **5280.**

Elapsed time

The 24 hours of the day are divided into two parts: the hours from midnight to noon (a.m.) and the hours from noon to midnight (p.m.). When we calculate the amount of time between two events we are calculating elapsed time—the amount of time that has passed. Elapsed time problems are "later-earlier-difference" problems that we have practiced since Lesson 13. The time elapsed is the difference.

Example 3 The marathon started at 7:15 a.m. Jason finished the race at 10:10 a.m. How long did it take Jason to run the marathon?

Solution This is a "later-earlier-difference" problem. We find Jason's race time (elapsed time) by subtracting the earlier time from the later time.

$$\begin{array}{r} \text{Later} \\ -\ \text{Earlier} \\ \hline \text{Difference} \end{array} \qquad \begin{array}{r} 10{:}10 \text{ a.m.} \\ -\ \ 7{:}15 \text{ a.m.} \\ \hline \end{array}$$

Since we cannot subtract 15 minutes from 10 minutes, we rename one hour as 60 minutes. The 60 minutes and 10 minutes equals 70 minutes. (This means 70 minutes after 9, which is the same as 10:10.)

$$\begin{array}{r} \overset{9}{\cancel{10}}{:}\overset{70}{\cancel{10}} \\ -\ \ 7{:}15 \\ \hline 2{:}55 \end{array}$$

We find that it took Jason **2 hours and 55 minutes** to run the marathon.

Example 4 What time is two and a half hours after 10:43 a.m.?

Solution This is a "later-earlier-difference" problem. The elapsed time, $2\frac{1}{2}$ hours, is the difference. We write $2\frac{1}{2}$ hours as 2:30. The earlier time is 10:43 a.m.

$$\begin{array}{r} \text{Later} \\ -\ \text{Earlier} \\ \hline \text{Difference} \end{array} \qquad \begin{array}{r} \text{Later} \\ -\ 10{:}43 \text{ a.m.} \\ \hline 2{:}30 \end{array}$$

We need to find the later time, so we add $2\frac{1}{2}$ hours to 10:43 a.m. We will describe two methods to do this addition: a mental calculation and a pencil-and-paper calculation. As a mental calculation we could first count two hours after 10:43 a.m. One hour later is 11:43 a.m. Another hour later is 12:43 **p.m.** From 12:43 p.m. we now count 30 minutes (one half hour). We will count 10 minutes at a time, from 12:43 p.m. to 12:53 p.m. to 1:03 p.m. to 1:13 p.m. We find that $2\frac{1}{2}$ hours after 10:43 a.m. is **1:13 p.m.** (Another mental calculation is to add 3 hours, then subtract 30 minutes.)

To perform a pencil-and-paper calculation, we add two hours and 30 minutes to 10:43 a.m.

$$\begin{array}{r} \text{10:43 a.m.} \\ +\ \text{2:30}\phantom{\text{ a.m.}} \\ \hline \text{12:73 p.m.} \end{array}$$

The time of day turns to p.m., but the sum 12:73 p.m. is improper. Seventy-three minutes is more than an hour. We think of 73 minutes as one hour plus 13 minutes. We add one to the number of hours and write 13 as the number of minutes. So $2\frac{1}{2}$ hours after 10:43 a.m. is **1:13 p.m.**

Practice* Write each of these numbers in expanded notation:

a. 205

b. 1760

c. 8050

Write each of these numbers in standard form:

d. (6 × 1000) + (4 × 100) **e.** (7 × 100) + (5 × 1)

f. The marathon started at 7:15 a.m. George finished the race at 11:05 a.m. How long did it take George to run the marathon?

g. What time is $3\frac{1}{2}$ hours after 11:50 p.m.?

Problem set 32

1. (12) When the sum of 24 and 7 is multiplied by the difference of 18 and 6, what is the product?

2. (13) Davy Crockett was born in Tennessee in 1786 and died at the Alamo in 1836. How many years did he live?

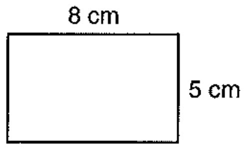
3. (15) A 16-ounce box of a certain cereal costs $2.24. What is the cost per ounce of this cereal?

4. (32) What time is three hours and 30 minutes after 6:50 a.m.?

5. How many $\frac{2}{5}$'s are in 1?
(30)

6. Reduce $\frac{40}{100}$.
(29)

7. How many square centimeters would be needed to cover the area of the rectangle?
(31)



8. How many centimeters is the distance around the same rectangle?
(8)

9. What is the eighth number in this sequence?
(10)

1, 3, 5, 7, ...

10. Write 7500 in expanded notation.
(32)

11. Which of these numbers is closest to 1000?
(31)
A. 990 B. 909 C. 1009 D. 1090

12. In three separate bank accounts Robin has \$623, \$494, and \$380. What is the average amount of money Robin has in each account?
(18)

13. \$0.05 × 100
(2)

14. $8q = 240$
(4)

15. How much money is $\frac{3}{4}$ of \$24?
(22)

Write each answer in simplest form:

16. $\frac{3}{5} + \frac{3}{5}$
(24)

17. $\frac{3}{4} - \frac{1}{4}$
(24)

18. $\frac{3}{4} \times \frac{1}{3}$
(29)

19. $\frac{3}{10} \times \frac{7}{10}$
(29)

20. Three fourths of a circle is what percent of a circle?
(23)

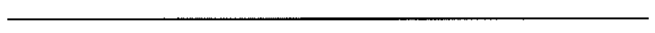
21. $w - 53 = 12$
(3)

22. $1\frac{2}{3} - 1\frac{1}{3}$
(27)

23. *(29)* Reduce $\frac{15}{21}$.

24. *(30)* What is the least common multiple of 4 and 6?

25. *(17)* Use your ruler to find the length of this line segment:

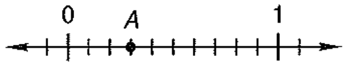


26. *(29)* If 24 of the 30 students finished the assignment in class, then what fraction of the students finished in class?

27. *(32)* Brad and Sharon began the hike at 6:45 a.m. and finished at 11:15 a.m. For how long did they hike?

28. *(32)* Compare: $(3 \times 100) + (5 \times 1)$ ○ 350

29. *(17)* What fraction is represented by Point *A* on this number line?



30. *(15)* Some grocery stores post the price per ounce of different kinds of cereal to help customers compare costs. How can we calculate the cost per ounce of a box of cereal?

LESSON 33

Writing Percents as Fractions, Part 1

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

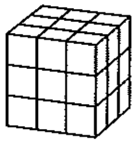
Mental Math: Count by 7's from 7 to 84.

a. (4 × 100) + (4 × 25) **b.** 7 × 29 **c.** 56 + 28

d. \$5.50 + \$1.75 **e.** Double 120 **f.** $\frac{120}{10}$

g. 2 × 3, + 1, × 8, + 4, ÷ 6, × 2, + 1, ÷ 3

Problem Solving: Grant glued 27 small blocks together to make a cube. Then he painted the six faces of the cube. Later the cube broke apart into 27 blocks. How many of the blocks had paint on three faces? on two faces? on one face?



Our fraction manipulatives describe parts of circles as fractions and as percents. The manipulatives show that 50% is equivalent to $\frac{1}{2}$ and that 25% is equivalent to $\frac{1}{4}$. We can find fraction equivalents of other percents by writing a percent as a fraction and then reducing the fraction.

A **percent** actually is a fraction with a denominator of 100. The word "percent" and its abbreviation, %, mean *per hundred*. To write a percent as a fraction, we remove the percent sign and write the number as the numerator and 100 as the denominator.

Example 1 Write 60% as a fraction.

Solution We remove the percent sign and write 60 over 100.

$$60\% = \frac{60}{100}$$

We can reduce $\frac{60}{100}$ in one step by dividing 60 and 100 by 20. If we begin by dividing by a number smaller than 20, it will take more than one step to reduce the fraction.

$$\frac{60}{100} \div \frac{20}{20} = \frac{3}{5}$$

We find that 60% is equivalent to $\frac{3}{5}$.

Example 2 Find the fraction equivalent to 4%.

Solution We remove the percent sign and write 4 over 100.

$$4\% = \frac{4}{100}$$

We reduce the fraction by dividing 4 by 4 and 100 by 4, which is the GCF of 4 and 100.

$$\frac{4}{100} \div \frac{4}{4} = \frac{1}{25}$$

We find that 4% is equivalent to $\frac{1}{25}$.

Practice* Write each percent as a fraction. Reduce when possible.

a. 80% **b.** 5% **c.** 25%

d. 24% **e.** 23% **f.** 10%

> **Teacher Note:** As a class activity, conduct a survey of class members to find out what pets are in their homes. Prepare a frequency table to tally the number of each type of pet. Then create a bar graph to display the results. (Settle questions that arise along the way by consensus.)

Problem set 33

1. (12) When the product of 10 and 15 is divided by the sum of 10 and 15, what is the quotient?

2. (13) The Nile River is 6651 kilometers long. The Mississippi River is 5986 kilometers long. How much longer is the Nile? What type of pattern do we use?

3. (12) Some astronomers think the universe may be fifteen billion years old. Use digits to write that number.

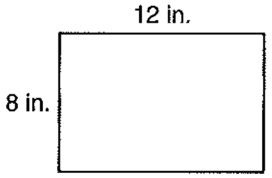
4. (32) Write 3040 in expanded notation.

5. (32) Write $(6 \times 100) + (2 \times 1)$ in standard notation.

6. (29) Write two fractions equal to 1, one with a denominator of 10 and the other with a denominator of 100.

7. (30) By what number should $\frac{5}{3}$ be multiplied for the product to be 1?

8. (8) What is the perimeter of the rectangle?



9. (31) How many square tiles 1 inch on a side would be needed to cover the rectangle?

10. (19) Which of these numbers is divisible by both 2 and 3?

A. 56 B. 75 C. 83 D. 48

11. (16) Estimate the difference of 4968 and 2099.

12. (2) $4.30 × 100 **13.** (3) $Q - 24 = 23$ **14.** (22) $\frac{3}{5}$ of 20

Write each answer in simplest form:

15. (24) $\frac{4}{5} + \frac{4}{5}$ **16.** (24) $\frac{5}{8} - \frac{1}{8}$

17. (29) $\frac{5}{2} \times \frac{3}{2}$ **18.** (29) $\frac{3}{10} \times \frac{3}{100}$

19. (10) What is the tenth number of this sequence?

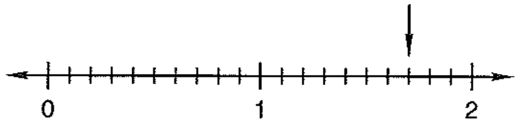
2, 4, 6, 8, ...

20. (2) $402.00 ÷ 25 **21.** (2) 348 × 67 **22.** (30) $\frac{1}{2}w = 1$

23. (7) A meter is about one big step. About how many meters high is a door?

24. (29) Five of the 30 students in the class were absent. What fraction of the class was absent? (Reduce.)

25. To what mixed number on the line is the arrow pointing?
(17)



26. Write 70% as a reduced fraction.
(33)

27. Four fifths of Gina's 20 answers were correct. How many of Gina's answers were correct? Draw a diagram that illustrates the problem.
(22)

28. By looking at the numerator and denominator of a fraction, how can you tell if the fraction is greater than or less than $\frac{1}{2}$?
(28)

29. What time is $6\frac{1}{2}$ hours after 8:45 p.m.?
(32)

30. Arrange these fractions in order from least to greatest:
(17)

$$\frac{1}{8}, \frac{1}{4}, \frac{1}{16}, \frac{1}{2}$$

LESSON 34

Decimal Place Value

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 2.

a. $(4 \times 200) + (4 \times 25)$

b. $1480 - 350$

c. $45 + 18$ (*18 is $20 - 2$*)

d. \$12.00 − \$2.50

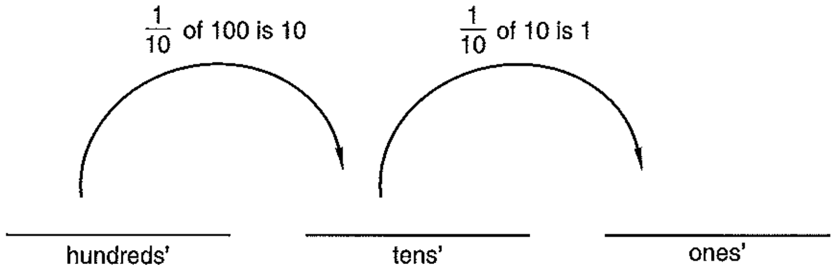
e. Double 250

f. $\frac{1500}{100}$

g. $3 \times 3, \times 9, - 1, \div 2, + 2, \div 7, \times 2$

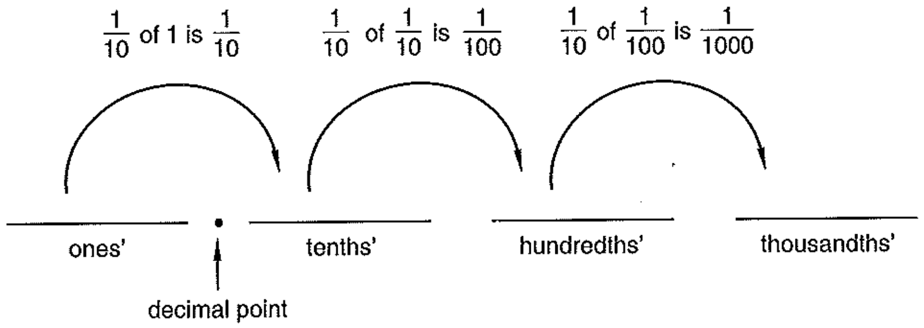
Problem Solving: In the deck there were red cards and black cards. Kathy selected three cards. Two were red and one was black. If she selects three more cards, what possible combinations of three cards could she select?

Since Lesson 12 we have studied place value from the ones' place leftward to the hundred trillions' place, noting as we move to the left that each place is ten times as large as the preceding place. If we move to the right instead of to the left, each place is one tenth of the preceding place.

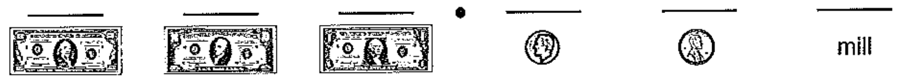


Places to the right of the ones' place also have a value one tenth of the value of the place to their left. These places have a value less than one (but not less than zero). We use a decimal point to mark the separation between the ones' place and places with a value less than one.

Places to the right of a decimal point are often called **decimal places.** Here we show three decimal places.



Thinking about money is a helpful way to remember decimal place values.



A mill is $\frac{1}{1000}$ of a dollar and $\frac{1}{10}$ of a cent. We do not have a coin for a mill. However, purchasers of gasoline are charged mills at the gas pump. A price of $\$1.49\frac{9}{10}$ per gallon is one mill less than \$1.50 and nine mills more than \$1.49.

Example 1 Which digit in 123.45 is in the hundredths' place?

Solution The *-ths* ending of "hundredths" indicates that the hundredths' place is to the right of the decimal point. The first place to the right of the decimal point is the tenths' place. The second place is the hundredths' place. The digit in the hundredths' place is **5.**

Example 2 What is the place value of the 8 in 67.89?

Solution The 8 is in the first place to the right of the decimal point, which is the **tenths' place.**

Practice

a. What is the place value of the 5 in 12.345?

b. Which digit in 5.4321 is in the tenths' place?

c. In 0.0123, what is the digit in the thousandths' place?

d. What is the value of the place held by zero in 50.375?

Problem set 34

1. *(22)* Three eighths of the 24 choir members were tenors. How many tenors were in the choir? Draw a diagram to illustrate the problem.

2. *(15)* Mom wants to triple a recipe for cheesecake. If the recipe calls for 8 ounces of cream cheese, how many ounces of cream cheese should she put in the mix?

3. *(32)* What time is two and one half hours after 10:40 a.m.?

4. *(33)* Write 80% as a reduced fraction.

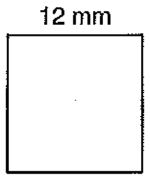
5. *(24)* Compare: $\frac{100}{100} \bigcirc \frac{10}{10}$

6. *(32)* Write $(6 \times 100) + (5 \times 1)$ in standard notation.

7. *(12)* Which digit is in the ones' place in $42,876.39?

8. *(31)* How many square millimeters is the area of the square?

9. *(8)* How many millimeters is the perimeter of the square?



10. *(30)* What is the least common multiple of 6 and 8?

11. *(2)* $5.60 ÷ 10

12. *(29)* $\frac{9}{10} \times \frac{9}{10}$

13. *(16)* Estimate the quotient when 898 is divided by 29.

14. Round 36,847 to the nearest hundred.
(16)

15. $6d = 144$
(4)

16. $\frac{d}{6} = 144$
(4)

17. Compare: $\frac{5}{2} + \frac{5}{2} \bigcirc 2 \times \frac{5}{2}$
(29)

18. $\frac{3}{8} + \frac{3}{8}$
(24)

19. $\frac{11}{12} - \frac{1}{12}$
(24)

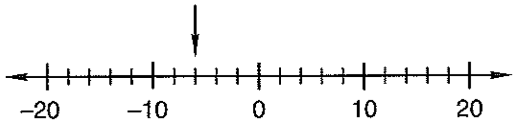
20. $\frac{5}{4} \times \frac{3}{2}$
(29)

21. What number is missing in this sequence?
(10)

6, 12, ______, 24, 30, ...

22. $4.37 × 86
(2)

23. To what number on the number line is the arrow pointing?
(14)



24. $(80 \div 40) - (8 \div 4)$
(5)

25. Which digit is in the thousandths' place in 2,345.678?
(34)

26. Draw a circle and shade $\frac{2}{3}$ of it.
(21)

27. Divide 5225 by 12 and write the quotient as a mixed number.
(26)

28. The first glass contained 12 ounces of water. The second glass contained 11 ounces of water. The third glass contained 7 ounces of water. If some water was poured from the first and second glasses into the third glass until all three glasses contained the same amount of water, then how many ounces of water would be in each glass?
(18)

29. The letters r, t, and d represent three different numbers. The product of r and t is d.
(2)

$$rt = d$$

Arrange the letters to form another multiplication fact and two division facts.

30. Instead of dividing 75 by 5, Sandy mentally doubled both numbers and divided 150 by 10. Find the quotient of $75 \div 5$ and the quotient of $150 \div 10$.
(2)

LESSON 35

Writing Decimal Numbers as Fractions, Part 1 • Reading and Writing Decimal Numbers

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count by 3's from 3 to 60. Count by 7's from 7 to 84.

a. $(4 \times 300) + (4 \times 25)$ **b.** 8×43 **c.** $37 + 39$
d. $\$7.50 + \$7.50$ **e.** $\frac{1}{3}$ of 360 **f.** $\frac{3600}{10}$
g. $5 \times 5, - 1, \div 3, \times 4, + 1, \div 3, + 1, \div 3$

Problem Solving: Copy this problem and fill in the missing digits.

$$\begin{array}{r} \_\,\_\,\_ \\ +\quad 1 \\ \hline \_\,\_\,\_\,\_ \end{array}$$

Writing decimal numbers as fractions, part 1

Decimal numbers are actually fractions with denominators of 10, 100, 1000, or other numbers in this sequence. The denominator of a decimal fraction is not written. Instead, the denominator is indicated by the number of decimal places.

One decimal place indicates that the denominator is 10.

$$0.\underline{3} = \frac{3}{10}$$

Two decimal places indicates that the denominator is 100.

$$0.\underline{0}\underline{3} = \frac{3}{100}$$

Three decimal places indicates that the denominator is 1000.

$$0.\underline{0}\underline{0}\underline{3} = \frac{3}{1000}$$

Notice that the number of zeros in the denominator equals the number of decimal places in the decimal number.

Example 1 Write 0.23 as a fraction.

Solution The decimal number 0.23 has two decimal places, so the denominator is 100. The numerator is 23.

$$0.23 = \mathbf{\frac{23}{100}}$$

Example 2 Write $\frac{9}{10}$ as a decimal number.

Solution The denominator is 10, so the decimal number has one decimal place.

$$0.\_\_$$

We write the digit 9 in this place.

$$\frac{9}{10} = \mathbf{0.9}$$

Reading and writing decimal numbers

We read numbers to the right of a decimal point the same way we read whole numbers, and then we say the place value of the last digit. We read 0.23 as "twenty-three hundredths" because the last digit is in the hundredths' place. To read a mixed decimal number like 20.04, we read the whole number part, say "and," and then read the decimal part.

Example 3 Write 0.023 with words.

Solution We see 23 and three decimal places. We write **twenty-three thousandths.**

Example 4 Use words to write 20.04.

Solution The decimal point separates the whole number part of the number from the decimal part of the number. We name the whole number part, write "and," and then name the decimal part.

Twenty and four hundredths

Example 5 Write twenty-one hundredths

(a) as a fraction, and

(b) as a decimal number.

Solution The same words name both a fraction form and a decimal form of the number.

(a) The word "hundredths" indicates that the denominator is 100.

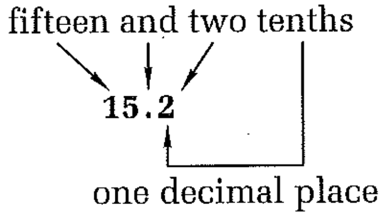
$$\mathbf{\frac{21}{100}}$$

(b) The word "hundredths" indicates that the decimal number has two decimal places.

0.21

Example 6 Write fifteen and two tenths as a decimal number.

Solution The whole number part is fifteen. The fractional part is two tenths, which we write in decimal form.



Practice* Write each decimal number as a fraction:

a. 0.1 **b.** 0.21 **c.** 0.321

Write each fraction as a decimal number:

d. $\frac{3}{10}$ **e.** $\frac{17}{100}$ **f.** $\frac{123}{1000}$

Use words to write these numbers:

g. 0.05

h. 0.015

i. 1.2

Write these numbers first as fractions, then as decimal numbers:

j. Seven tenths

k. Thirty-one hundredths

l. Seven hundred thirty-one thousandths

Write each of these numbers as a decimal number:

m. Five and six tenths

n. Eleven and twelve hundredths

o. One hundred twenty-five thousandths

Problem set 35

1. *(29)* What is the product of three fourths and three fifths?

2. *(22)* Bugs planted 360 carrot seeds in his garden. Three fourths of them grew. How many carrots grew? Draw a diagram to illustrate the problem.

3. *(32)* Jan's birthday cake must bake for 2 hours and 15 minutes. If it is put into the oven at 11:45 a.m., at what time will it be done?

4. Write twenty-three hundredths
(35)
(a) as a fraction.
(b) as a decimal number.

5. Write 20.04 with words.
(35)

6. Write ten and five tenths as a decimal number.
(35)

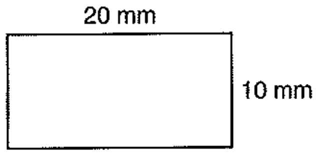
7. Write 75% as a reduced fraction.
(33)

8. Write the following in standard notation:
(32)

$$(5 \times 1000) + (6 \times 100) + (4 \times 10)$$

9. Which digit in 1.23 is in the same place as the 5 in 0.456?
(34)

10. What is the area of the rectangle?
(31)



11. What is the perimeter of the rectangle?
(8)

12. There are 100 centimeters in a meter. How many centimeters are in 10 meters?
(7)

13. Arrange these numbers in order from least to greatest:
(34)

0.001, 0.1, 1.0, 0.01

14. A meter is about one **big** step. About how many meters wide is a door?
(7)

15. $\frac{3}{5} + \frac{2}{5}$ (24)

16. $\frac{5}{8} - \frac{5}{8}$ (24)

17. $\frac{2}{3} \times \frac{3}{4}$ (29)

18. (a) How many $\frac{2}{5}$'s are in 1?
(30)
(b) Use the answer from part (a) to find the number of $\frac{2}{5}$'s in 2.

19. *(29)* Convert $\frac{20}{6}$ to a mixed number; then reduce the fraction.

20. *(29)* $\frac{100\%}{6}$

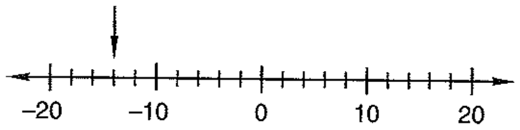
21. *(27)* $3\frac{4}{4} - 1\frac{1}{4}$

22. *(29)* Compare: 5 ○ $4\frac{4}{4}$

23. *(25)* One sixth of a circle is what percent of a circle?

24. *(9)* Compare: $3 \times 18 \div 6$ ○ $3 \times (18 \div 6)$

25. *(14)* To what number on the line is the arrow pointing?



26. *(2)* Which of these division problems has the greatest quotient?

A. $\frac{6}{2}$ B. $\frac{60}{20}$ C. $\frac{12}{4}$ D. $\frac{25}{8}$

27. *(35)* Write 0.3 and 0.7 as fractions. Then multiply the fractions. What is the product?

28. *(33)* Write 21% as a fraction. Then write the fraction as a decimal number.

29. *(2)* Instead of dividing 400 by 50, Sandy doubled each number and divided 800 by 100. Find both quotients.

30. *(26)* A 30-inch-long ribbon was cut into four smaller ribbons of equal length. How long was each of the smaller ribbons?

LESSON 36 Subtracting Fractions and Mixed Numbers from Whole Numbers

Facts Practice: 90 Division Facts (Test F in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 3.

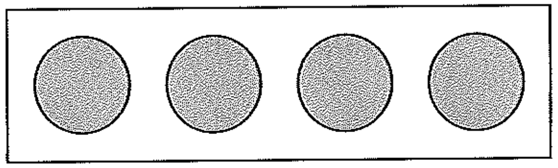
- **a.** $(4 \times 400) + (4 \times 25)$
- **b.** $2500 + 375$
- **c.** $86 - 39$
- **d.** $\$15.00 - \$2.50$
- **e.** $\frac{1}{2}$ of 320
- **f.** $\frac{4800}{100}$
- **g.** $2 \times 4, \times 5, + 10, \times 2, - 1, \div 9, \times 3, - 1, \div 4$

Problem Solving: Nathan has seven coins that total exactly one dollar. Name three sets of coins that he could have.

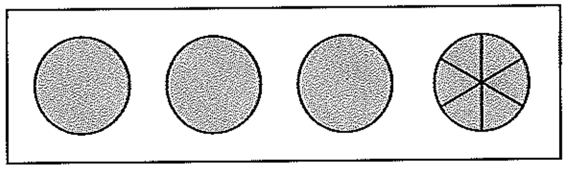
Read this "some went away" story about pies.

> *There were four pies on the shelf. The server sliced one of the pies into sixths and took $2\frac{1}{6}$ pies from the shelf. Then how many pies were on the shelf?*

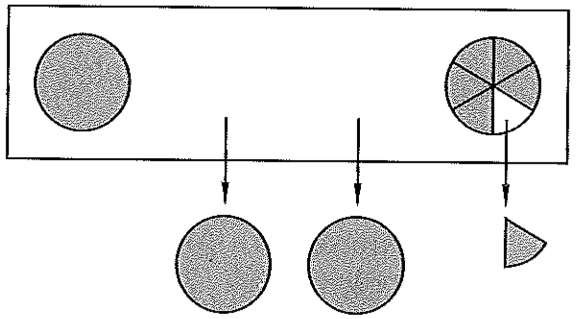
We will illustrate this story with circles. There were four pies on the shelf.



The server sliced one of the pies into sixths. (Then there were $3\frac{6}{6}$ pies, which is another name for 4 pies.)



The server took $2\frac{1}{6}$ pies from the shelf.



We see $1\frac{5}{6}$ pies left on the shelf.

Now we will show the arithmetic for subtracting $2\frac{1}{6}$ from 4.

$$\begin{array}{rl} 4 & \text{pies} \\ -\ 2\frac{1}{6} & \text{pies} \\ \hline \end{array}$$

Just as the server sliced one of the pies into sixths, so we change four wholes into three wholes plus six sixths. Then we subtract.

$$\begin{array}{r} \overset{3}{\cancel{4}}\frac{6}{6} \\ -\ 2\frac{1}{6} \\ \hline 1\frac{5}{6} \end{array} \quad \text{Change 4 to } 3\tfrac{6}{6}.$$

Example $5 - 1\frac{2}{3}$

Solution To subtract $1\frac{2}{3}$ from 5, we first change 5 to 4 plus $\frac{3}{3}$. Then we subtract.

$$\begin{array}{r} \overset{4}{\cancel{5}}\frac{3}{3} \\ -\ 1\frac{2}{3} \\ \hline \mathbf{3\frac{1}{3}} \end{array} \quad \text{Change 5 to } 4\tfrac{3}{3}.$$

Practice* Show the arithmetic of each subtraction:

a. $4 - 2\frac{1}{4}$

b. $3 - \frac{5}{12}$

c. $10 - 2\frac{1}{2}$

d. $6 - 1\frac{3}{10}$

e. There were four whole pies on the shelf. The server took $1\frac{5}{6}$ pies. Then how many pies were on the shelf?

f. Write a problem similar to problem (e) and find the answer.

Problem set 36

1. (33) Twenty-five percent of the students played musical instruments. What fraction of the students played musical instruments?

2. (36) Jack accidentally sat on his lunch and smashed $\frac{3}{4}$ of his sandwich. What fraction of his sandwich was not smashed?

3. (15) A mile is 5280 feet. There are 3 feet in a yard. How many yards are in a mile?

4. (34) Which digit in 23.47 has the same place value as the 6 in 516.9?

5. (35) Write 1.3 with words.

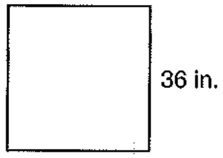
6. (35) Write the decimal number five hundredths.

7. (35) Write thirty-one hundredths
   (a) as a fraction.
   (b) as a decimal number.

8. (32) Write $(4 \times 100) + (3 \times 1)$ in standard notation.

9. (34) Which digit in 4.375 is in the tenths' place?

10. (31) How many 1-inch square tiles are needed to cover this square?



11. (8) What is the perimeter of the square?

12. (27) $3\frac{1}{4} + 2\frac{1}{4}$

13. (36) $3 - 1\frac{1}{4}$

14. (27) $3\frac{1}{3} + 2\frac{2}{3}$

15. (22) $\frac{3}{4}$ of 28

16. (29) $\frac{3}{4} \times \frac{4}{6}$

17. (22) Monte went to the mall with $24.00. He spent $\frac{5}{6}$ of his money in the music store. How much money did he spend in the music store? Draw a diagram to illustrate the problem.

18. (18) What is the average of 42, 57, and 63?

19. (19) The factors of 6 are 1, 2, 3, and 6. List the factors of 20.

20. (30,20) (a) What is the least common multiple of 9 and 6?

(b) What is the greatest common factor of 9 and 6?

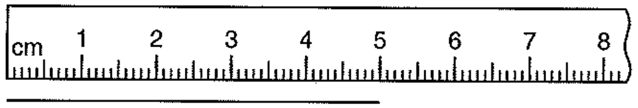
21. (4) $\frac{m}{12} = 6$

22. (4) $\frac{12}{n} = 6$

23. (16) Round 58,742,177 to the nearest million.

24. (16) Estimate the product of 823 and 680.

25. (7) How many millimeters long is the line segment? (1 cm = 10 mm)



26. (25) Using your fraction manipulatives you will find that the sum of $\frac{1}{3}$ and $\frac{1}{6}$ is $\frac{1}{2}$.

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

Arrange these numbers to form another addition fact and two subtraction facts.

27. (35) Write 0.9 and 0.09 as fractions. Then multiply the fractions. What is the product?

28. (35) Write $\frac{81}{1000}$ as a decimal number. (*Hint*: Write a zero in the tenths' place.)

29. (30) (a) How many $\frac{3}{4}$'s are in 1?

(b) Use the answer to part (a) to help you find the number of $\frac{3}{4}$'s in 3.

30. (2) Instead of dividing 350 by 25, Sandy multiplied both numbers by 4 and divided 1400 by 100. Find both quotients.

LESSON 37

Adding and Subtracting Decimal Numbers

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by 25's between 25 and 400.

a. $(4 \times 500) + (4 \times 25)$ **b.** 9×43 **c.** $76 - 29$

d. \$17.50 + \$2.50 **e.** $\frac{1}{2}$ of 520 **f.** $\frac{2500}{10}$

g. $6 \times 8, + 1, \div 7, \times 3, - 1, \div 5, + 1, \div 5$

Problem Solving: Cheryl turned a number cube so that she could see the dots on three faces. The total number of dots she could see was 10. What was the total number of dots on each of the three faces?

When we add or subtract numbers using pencil and paper, it is important to align digits that have the same place value. When we add or subtract whole numbers with pencil and paper we line up the ending digits. When we line up the ending digits, which are in the ones' place, we automatically line up other digits that have the same place value.

$$\begin{array}{r} 23 \\ 241 \\ +\ 317 \\ \hline \end{array}$$

When we line up the ones' place we also align other digits with the same place values.

However, lining up the ending digits of decimal numbers may not properly line up all the digits. We use another method for decimal numbers. **We line up decimal numbers for addition or subtraction by lining up the decimal points.** The decimal point in the answer is in line with the other decimal points. Empty places are treated as zeros.

$$\begin{array}{r@{.}l} 2&3 \\ 2&41 \\ +\ 31&7 \\ \hline & \end{array}$$

When we line up the decimal points, we also align digits that have the same place values.

Example 1 Add: $3.4 + 0.26 + 0.3$

Solution Line up the decimal points in the problem and add. The decimal point in the answer is placed in line with the other decimal points.

$$\begin{array}{r} 3.4 \\ 0.26 \\ +\ 0.3 \\ \hline \mathbf{3.96} \end{array}$$

Example 2 Subtract: $4.56 - 2.3$

Solution Line up the decimal points in the problem and subtract.

$$\begin{array}{r} 4.56 \\ -\ 2.3 \\ \hline \mathbf{2.26} \end{array}$$

Practice Add or subtract. Remember to line up the decimal points.

a. $3.46 + 0.2$

b. $8.28 - 6.1$

c. $0.735 + 0.21$

d. $0.543 - 0.21$

e. $0.43 + 0.1 + 0.413$

f. $0.30 - 0.27$

g. $0.6 + 0.7$

h. $1.00 - 0.24$

i. $0.9 + 0.12$

j. $1.23 - 0.4$

Problem set 37

1. (33) Sixty percent of the students in the class were girls. What fraction of the students in the class were girls?

2. (27) Penny broke 8 pencils on her math test. She broke half that many on her spelling test. How many did she break in all?

3. (3) What number must be added to three hundred seventy-five to total one thousand?

4. (37) $3.4 + 0.62 + 0.3$

5. (37) $4.56 - 3.2$

6. (5) $\$0.37 + \$0.23 + \$0.48$

7. (3) $\$5 - m = 5¢$

8. (10) What is the next number in this sequence?

1, 10, 100, 1000, ______, …

9. (31) Each side of a square is 100 cm long. How many tiles 1 cm on each edge are needed to cover the area?

10. (12) Which digit is in the ten-millions' place in 1,234,567,890?

11. (35) Three of these numbers are equal. Which number is different?

A. $\frac{1}{10}$ B. 0.1 C. $\frac{10}{100}$ D. 0.01

12. (16) Estimate the product of 29, 42, and 39.

13. (2) $3210 \div 3$ **14.** (2) $32{,}100 \div 30$ **15.** (1) \$10,000 − \$345

16. (24) $\frac{3}{4} + \frac{3}{4}$ **17.** (36) $3 - 1\frac{3}{5}$ **18.** (29) $\frac{3}{3} - \frac{2}{2}$

19. (27) $1\frac{1}{3} + 2\frac{1}{3} + 3\frac{1}{3}$

20. (29) Compare: $\frac{1}{4} + \frac{3}{4} \bigcirc \frac{1}{4} \times \frac{3}{4}$

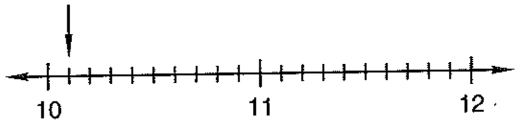
21. (26) Convert the improper fraction $\frac{100}{7}$ to a mixed number.

22. (18) What is the average of 90 lb, 84 lb, and 102 lb?

23. (30) What is the least common multiple of 4 and 5?

24. (14) The temperature changed from 11°C at noon to −4°C at 8:00 p.m. How many degrees did the temperature drop?

25. (17) The arrow points to what mixed number on this number line?



26. *(35)* Write 0.3 and 0.9 as fractions. Then multiply the fractions. Change the product to a decimal number.

27. *(23)* Write three different fractions equal to 1. How can you tell if a fraction is equal to 1?

28. *(2)* Instead of dividing 6 by $\frac{1}{2}$, Sandy doubled both numbers and divided 12 by 1. Do you think both quotients are the same? Write a one or two sentence reason for your answer.

29. *(32)* The movie started at 2:50 p.m. and ended at 4:23 p.m. How long was the movie?

30. *(22)* Three fifths of the 25 students in the class were girls. How many girls were in the class? Draw a diagram to illustrate the problem.

LESSON
38

Adding and Subtracting Decimal Numbers and Whole Numbers • Squares

Facts Practice: 64 Addition Facts (Test A in Test Masters)

Mental Math: Count by 6's from 6 to 72. Count by 7's from 7 to 84.

- **a.** (4 × 600) + (4 × 25)
- **b.** 875 − 125
- **c.** 56 − 19
- **d.** \$10.00 − \$6.25
- **e.** $\frac{1}{2}$ of 150
- **f.** $\frac{\$40.00}{10}$
- **g.** 10 + 10, − 2, ÷ 3, × 4, + 1, × 4, ÷ 2, + 6, ÷ 7

Problem Solving: Teresa wanted to paint each face of a cube so that the faces that were next to each other were different colors. She wanted to use less than six different colors. What is the fewest number of different colors she could use? Describe how the cube could be painted.

Adding and subtracting decimal numbers and whole numbers

Here we show two ways to write three dollars.

\$3 \$3.00

We see that we may write three dollars with or without a decimal point. We may also write whole numbers with or without a decimal point. Here are several ways we may write the whole number three.

3 3. 3.0 3.00

A decimal point follows the ones' place. A whole number may be written with a decimal point after the ones' place. With a calculator, when we enter a whole number a decimal point is displayed. It may be helpful to write a whole number with a decimal point when adding and subtracting decimal numbers with pencil and paper.

Example 1 12 + 7.5

Solution We line up decimal points when adding decimal numbers so that we add digits with the same place values. The whole number 12 may be written with a decimal point to the right of the digit 2. We line up the decimal points and add.

$$\begin{array}{r} 12. \\ +\ 7.5 \\ \hline \mathbf{19.5} \end{array}$$

Example 2 12.75 − 5

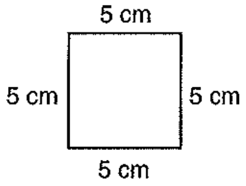
Solution We write the whole number 5 with a decimal point to its right. Then we line up the decimal points and subtract.

$$\begin{array}{r} 12.75 \\ -\ \ 5. \\ \hline \mathbf{7.75} \end{array}$$

Squares We know that the four sides of a square are equal in length. Therefore we can find the perimeter and area of a square if we know the length of one side. Likewise, if we know the perimeter of a square we can find the length of each side and then the area of the square. We divide the perimeter of a square by four to find the length of one side.

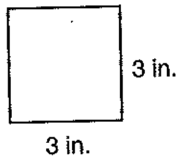
Example 3 The perimeter of a square is 20 cm. What is the length of each side?

Solution Each side of a square is $\frac{1}{4}$ of the perimeter. By dividing the perimeter 20 cm by 4, we find the length of each side of the square is **5 cm,** as we show below.



Example 4 The perimeter of a square is 12 inches. What is the area of the square?

Solution This is a two-step problem. First we find the length of each side. Then we can find the area. We divide the perimeter 12 inches by 4 and find that the length of each side is 3 inches.



Now we multiply 3 inches by 3 inches and find that the area of the square is **9 square inches.**

Practice

a. 4 + 2.1

b. 4.3 − 2

c. 3 + 0.4

d. 43.2 − 5

e. 0.23 + 4 + 3.7

f. 6.3 − 6

g. 12.5 + 10

h. 75.25 − 25

i. The perimeter of a square is 40 inches. How long is each side?

j. The perimeter of a square is 24 inches. What is the area of the square?

Problem set 38

1. (20) What is the largest factor of both 54 and 45?

2. (15) Roberto began saving $3 each week for summer camp, which costs $126. How many weeks will it take to save that amount of money?

3. (13) Ghandi was born in 1869. How old was he when he was assassinated in 1948?

4. (38) 3 + 1.2

5. (38) 3.6 + 4

6. (37) 5.63 − 1.2

7. (37) 5.376 + 0.24

8. (37) 4.75 − 0.6

9. (3) $4 − w = 4¢

10. (35) Write forty-seven hundredths

(a) as a fraction.

(b) as a decimal number.

11. (32) Write $(9 \times 1000) + (4 \times 10) + (3 \times 1)$ in standard notation.

12. (34) Which digit is in the hundredths' place in $123.45?

13. (38) The perimeter of a square is 100 inches. How long is each side?

14. (30) What is the least common multiple of 2, 3, and 4?

15. (27) $1\frac{2}{3} + 2\frac{2}{3}$

16. (36) $5 - 1\frac{1}{4}$

17. (29) $\frac{3}{4}$ of $\frac{4}{5}$

18. (29) $\frac{7}{10} \times \frac{11}{10}$

19. (30) (a) How many $\frac{2}{3}$'s are in 1?

(b) Use the answer to part (a) to help you find the number of $\frac{2}{3}$'s in 2.

20. (29) Six of the nine players got on base. What fraction of the players got on base?

21. (19) List the factors of 30.

22. (33) Write 35% as a reduced fraction.

23. (16) Round 186,497 to the nearest thousand.

24. (30) $\frac{1}{3}m = 1$

25. (5) $\frac{22 + 23 + 24}{3}$

26. (9) Compare: $24 \div 8 \bigcirc 240 \div 80$

27. (35) Write 0.7 and 0.21 as fractions. Then multiply the fractions. Change the product to a decimal number.

28. (15) Peter bought ten carrots for $0.80. What was the cost for each carrot?

29. (31) Which of these fractions is closest to 1?

A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{3}{5}$ D. $\frac{4}{5}$

30. (38) If you know the perimeter of a square, you can find the area of the square in two steps. Describe the two steps.

LESSON
39

Multiplying Decimal Numbers

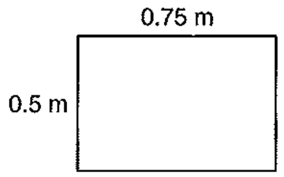
Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 3.

a. (4 × 700) + (4 × 25) **b.** 6 × 45 **c.** 67 − 29
d. $8.75 + $0.75 **e.** $\frac{1}{2}$ of 350 **f.** $\frac{2500}{100}$
g. 8 × 5, ÷ 2, + 1, ÷ 7, × 3, + 1, ÷ 10, ÷ 2

Problem Solving: Sam has many red socks, white socks, and blue socks in a drawer. In the dark, Sam pulled out two socks that did not match. How many more socks does Sam need to pull from the drawer to be certain to have a matching pair?

To find the area of a rectangle that is 0.75 meter long and 0.5 meter wide, we multiply 0.75 m and 0.5 m.



One way to multiply these numbers would be to write each decimal number as a fraction and multiply the fractions.

$$\begin{array}{ccccc} 0.75 & \times & 0.5 & & \\ \downarrow & & \downarrow & & \\ \dfrac{75}{100} & \times & \dfrac{5}{10} & = & \dfrac{375}{1000} \end{array}$$

The product $\frac{375}{1000}$ can be written as the decimal number 0.375. We find the area of the rectangle is 0.375 square meter.

Notice that the product 0.375 has three decimal places and that the factors 0.75 and 0.5 have a **total** of three decimal places. Whenever we multiply decimal numbers, the product has the same number of decimal places as there are in all of the factors. This fact allows us to multiply decimal numbers the same way we multiply whole numbers. After multiplying, we count the total number of decimal places in the factors and place the decimal point in the answer so that the product has the same number of decimal places.

$$\begin{array}{rrl} \text{Three decimal places in the factors} & \left\{\begin{array}{r} 0.\underline{75} \\ \times\ 0.\underline{5} \end{array}\right. & \text{We do not align decimal points. We just multiply, then count decimal places.} \\ \hline & 0.\underbrace{\underline{375}}_{\text{Three decimal places in the product}} & \end{array}$$

Example 1 Multiply: 0.25 × 0.7

Solution We set up the arithmetic as though we were multiplying whole numbers, ignoring the decimal points until after we multiply. Next, we count the number of digits to the right of the decimal point in the two factors. There are three. Then we put a decimal point in the product three places from the right-hand end. We write .175 as **0.175.**

$$\left.\begin{array}{r} 0.25 \\ \times\ 0.7 \end{array}\right\}\text{3 places}$$
$$\begin{array}{r} \hline 0.\underbrace{175} \end{array}$$

Example 2 Multiply: 1.6 × 3

Solution We multiply as though we were multiplying whole numbers. Then we count decimal places in the factors. There is only one. We count one place in the product and place a decimal point. The answer is **4.8.**

$$\begin{array}{r} \left.\begin{array}{r} 1.6 \\ \times\ \ 3 \end{array}\right\} \text{1 place} \\ \hline 4.8 \end{array}$$

Practice

a. 15 × 0.3

b. 1.5 × 3

c. 1.5 × 0.3

d. 0.15 × 3

e. 1.5 × 1.5

f. 0.15 × 10

g. 0.25 × 0.5

h. 0.025 × 100

Problem set 39

1. (12) Mount Everest, the world's tallest mountain, is twenty-nine thousand, twenty-eight feet high. Use digits to write that number.

2. (7) There are three feet in a yard. How many yards high is Mt. Everest?

3. (22) Bam Bam says his pet dinosaur weighs $\frac{3}{4}$ as much as a garbage truck. If the truck weighs 12 tons, how much does his dinosaur weigh?

4. (39) 0.25 × 0.5

5. (2) \$1.80 × 10

6. (39) 63 × 0.7

7. (38) 1.23 + 4 + 0.5

8. (37) 12.34 − 5.6

9. (38) \$3.00 − \$3

10. (35) Write ten and three tenths

(a) as a decimal number.

(b) as a mixed number.

11. (17) Think of two different fractions that are greater than zero but less than one. Multiply the two fractions to form a third fraction. For your answer to this problem, write the three fractions in order from least to greatest.

12. *(35)* Write the decimal number one hundred twenty-three thousandths.

13. *(32)* Write $(6 \times 100) + (4 \times 10)$ in standard form.

14. *(38)* The perimeter of a square is 40 inches. How many square tiles 1 inch on each edge are needed to cover its area?

15. *(30)* What is the least common multiple (LCM) of 2, 3, and 6?

16. *(26)* Convert $\frac{20}{8}$ to a mixed number and reduce the fraction.

17. *(24)* $\left(\frac{1}{3} + \frac{2}{3}\right) - 1$ **18.** *(29)* $\frac{3}{5} \times \frac{2}{3}$ **19.** *(29)* $\frac{8}{9} \times \frac{9}{8}$

20. *(36)* A pie was cut into six equal slices. Two slices were eaten. Then what fraction of the pie was left? (Reduce answers when possible.)

21. *(32)* What time is $2\frac{1}{2}$ hours before 1 a.m.?

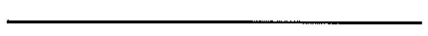
22. *(18)* On Tim's last four assignments he had 26, 29, 28, and 25 correct answers, respectively. He averaged how many correct answers on these papers?

23. *(16)* Estimate the quotient when 7987 is divided by 39.

24. *(14)* Compare: $365 - 364$ ◯ $364 - 365$

25. *(34)* Which digit in 3.675 has the same place value as the 4 in 14.28?

26. *(17)* Use your ruler to find the length of the segment to the nearest sixteenth of an inch.

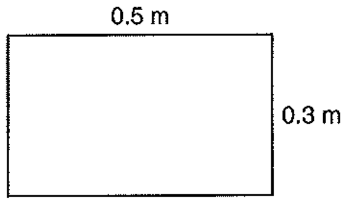


27. *(15)* Morton bought 12 pencils for \$0.96. What was the cost for each pencil?

28. (a) How many $\frac{3}{5}$'s are in 1?
(30)
(b) Use the answer to part (a) to help you find the number of $\frac{3}{5}$'s in 2.

29. Instead of dividing 390 by 15, Sandy divided both numbers by 3 to make the problem $130 \div 5$. Then she multiplied both of those numbers by two to make $260 \div 10$. Find all three quotients.
(2)

30. Find the area of this rectangle.
(39)



LESSON 40 Using Zero as a Place Holder • Circle Graphs

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count by 8's from 8 to 96.

a. (4 × 800) + (4 × 25) **b.** 1500 + 750 **c.** 74 − 39
d. \$8.25 − \$1.50 **e.** Double 240 **f.** $\frac{480}{10}$
g. 4 × 4, − 1, ÷ 5, × 6, + 2, × 2, + 2, ÷ 6

Problem Solving: Copy this problem and fill in the missing digits.

$$\begin{array}{r} \_\,\_\,\_ \\ \times \quad 9 \\ \hline \_\,\_\,2 \end{array}$$

Using zero as a place holder

When subtracting, multiplying, and dividing decimal numbers, we often find a decimal place with no digit in it, like these.

$$\begin{array}{r} 0.5\_ \\ -\ 0.32 \\ \hline \end{array} \qquad \begin{array}{r} 0.2 \\ \times\ 0.3 \\ \hline 0.\_6 \end{array} \qquad \begin{array}{r} \$0.\_4 \\ 3\overline{)\$0.12} \end{array}$$

We may fill an empty decimal place with zero.

In Subtraction

In order to subtract it is sometimes necessary to attach zeros to the top number.

$$\begin{array}{r} 0.50 \\ -\ 0.32 \\ \hline \end{array}$$

Example 1 3 − 0.4

Solution We place the decimal on the back of the whole number and line up the decimal points. We fill the empty place with zero and subtract.

$$\begin{array}{r} 3.0 \\ -\ 0.4 \\ \hline \mathbf{2.6} \end{array}$$

In Multiplication

When multiplying, we may need to insert one or more zeros between the multiplication answer and the decimal point to hold the other digits in their proper places.

$$\begin{array}{r} 0.2 \\ \times\ 0.3 \\ \hline 0.06 \end{array}$$

Example 2 0.12 × 0.3

Solution We multiply and count three places. We fill the empty place with zero.

$$\begin{array}{r} 0.12 \\ \times\ 0.3 \\ \hline \mathbf{0.036} \end{array}$$

Example 3 Use digits to write the decimal number twelve thousandths.

Solution The word "thousandths" tells us that there are three places to the right of the decimal point.

$$.\underline{\ }\,\underline{\ }\,\underline{\ }$$

We fit the two digits of twelve in the last two places.

$$.\underline{\ }\,\underline{1}\,\underline{2}$$

Then we fill the empty place with zero.

0.012

Circle graphs

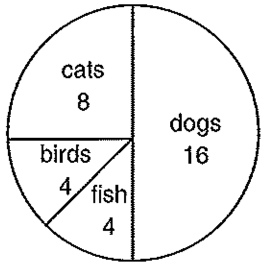
Circle graphs, which are sometimes called **pie graphs** or **pie charts,** display quantitative information in fractions of a circle. In the chart in Example 4, we see that dogs represent half of the pets belonging to the students in Brett's classroom. Circle graphs often express the portions of a graph in percent form. Instead of stating the number of dogs as 16, the graph might have stated the portion of pets that are dogs as 50%.

Example 4 Brett collected information from his classmates about their pets. He displayed the information about the number of pets in a circle graph. Use the information in this graph to answer the following questions:

(a) How many pets are represented in the graph?

(b) What fraction of the pets are birds?

(c) What percent of the pets are dogs?



Solution (a) We add the number of dogs, cats, birds, and fish. The total is **32.**

(b) Birds are 4 of the 32 pets. The fraction $\frac{4}{32}$ reduces to $\frac{1}{8}$. So the circle graph was divided in a way to make the bird portion of the circle $\frac{1}{8}$ of the circle.

(c) Dogs are 16 of the 32 pets, which means that $\frac{1}{2}$ of the pets are dogs. From our fraction manipulatives we know that $\frac{1}{2}$ is equivalent to **50%.**

Practice*

a. 0.2×0.3

b. $4.6 - 0.46$

c. 0.1×0.01

d. $0.4 - 0.32$

e. 0.12×0.4

f. $1 - 0.98$

g. Write the decimal number ten and eleven thousandths.

h. Refer to the circle graph in this lesson. What percent of the pets are cats?

Problem set 40

1. *(40)* Refer to the circle graph in this lesson and to your fraction manipulatives. What percent of the pets are birds?

2. *(13)* The first slaves were taken to the colony of Virginia in 1619. African slave trade ended in 1871. How many years did the slave trade last?

3. *(32)* White Rabbit is three and a half hours late for a very important date. If the time is 2:00 p.m., what was the time of his date?

4. *(40)* $3 - 0.3$

5. *(40)* $1.2 - 0.12$

6. *(40)* $1 - 0.1$

7. *(40)* 0.12×0.2

8. *(40)* 0.01×0.1

9. *(39)* 4.8×0.23

10. *(40)* Write one and two hundredths as a decimal number.

11. *(32)* Write $(6 \times 10{,}000) + (8 \times 100)$ in standard form.

12. *(38)* A square room has a perimeter of 32 feet. How many floor tiles 1 foot on a side are needed to cover the floor of the room?

13. *(30)* What is the least common multiple (LCM) of 2, 4, and 8?

14. *(27)* $6\frac{2}{3} + 4\frac{2}{3}$

15. *(36)* $5 - 3\frac{3}{8}$

16. *(29)* $\frac{5}{8} \times \frac{2}{3}$

17. *(27)* $2\frac{5}{6} + 5\frac{2}{6}$

18. *(29)* Compare: $\frac{1}{2} \times \frac{2}{2}$ ○ $\frac{1}{2} \times \frac{3}{3}$

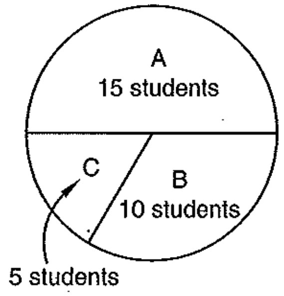
19. *(3)* $1000 - w = 567$

20. *(29)* Eighteen of the thirty students in the class received A's. What fraction of the class received A's?

21. *(19)* How many whole numbers are factors of 100?

22. *(16)* Round $4167 to the nearest hundred dollars.

The circle graph below gives us some information about the test scores of some students who took a test. Use the information in this graph to answer questions 23–26.



23. *(40)* How many students took the test?

24. *(40)* What fraction of the students received a grade of C on the test?

25. (40) What percent of the students received an A on the test?

26. (40) Write a "larger-smaller-difference" question that refers to this graph. Then answer the question.

27. (40) In Example 2 of this lesson is the following multiplication fact:

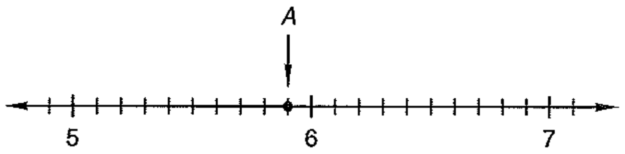
$$0.12 \times 0.3 = 0.036$$

Arrange these numbers to form another multiplication fact and two division facts.

28. (2) Instead of dividing 240 by 15, Sandy divided both numbers by 3 to make $80 \div 5$. Then she doubled both numbers to make $160 \div 10$. Find all three quotients.

29. (33) Forty percent of the 25 students in the class are boys. Write 40% as a reduced fraction. Then find the numbers of boys in the class.

30. (17) What mixed number is represented by point *A* on this number line?



LESSON 41

Renaming Fractions by Multiplying by 1

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count by 12's from 12 to 96.

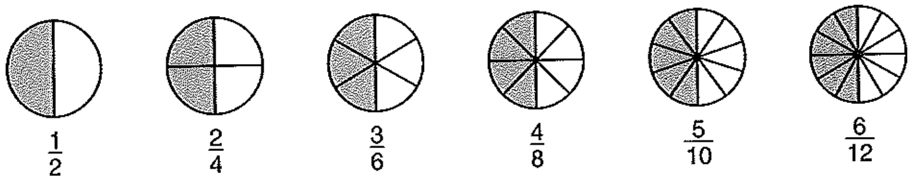
a. 4×125 **b.** $825 + 50$ **c.** $67 + 8$

d. $\$6.75 + \$2.50$ **e.** $\frac{1}{2}$ of 1000 **f.** $\frac{580}{10}$

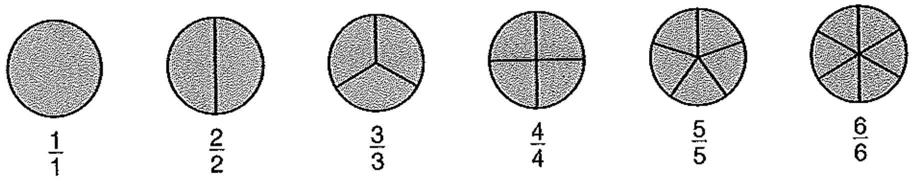
g. $3 \times 4, - 2, \times 5, - 2, \div 6, + 1, \div 3$

Problem Solving: The average of two numbers is 25. If one of the numbers is 19, what is the other number?

With our fraction manipulatives we have seen that the same fraction may be named many different ways. Here we show six ways to name the fraction $\frac{1}{2}$.



In this lesson we will practice renaming a fraction by multiplying the fraction by a fraction equal to 1. Here we show six ways to name 1 as a fraction.



We know that when we multiply a number by 1, the product equals the number multiplied. So if we multiply $\frac{1}{2}$ by 1, the answer is $\frac{1}{2}$.

$$\frac{1}{2} \times 1 = \frac{1}{2}$$

However, if we multiply $\frac{1}{2}$ by a fraction equal to 1, the answer will be a different name for $\frac{1}{2}$. We multiply $\frac{1}{2}$ by $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$ to make three different fractions equal to $\frac{1}{2}$.

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4} \qquad \frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \qquad \frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$$

The fractions $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are all equivalent to $\frac{1}{2}$.

Example 1 Write a fraction equal to $\frac{1}{2}$ that has a denominator of 20.

$$\frac{1}{2} = \frac{?}{20}$$

Solution To rename a fraction, we multiply the fraction by a fraction equal to 1. The denominator of $\frac{1}{2}$ is 2. We want to make an equivalent fraction with a denominator of 20.

$$\frac{1}{2} = \frac{?}{20}$$

Since we need to multiply the denominator by 10, we multiply $\frac{1}{2}$ by $\frac{10}{10}$.

$$\frac{1}{2} \times \frac{10}{10} = \mathbf{\frac{10}{20}}$$

Example 2 Write $\frac{1}{2}$ and $\frac{1}{3}$ as fractions with denominators of 6. Then add the renamed fractions.

Solution We multiply each fraction by a fraction equal to one to form fractions that have a denominator of 6.

$$\frac{1}{2} = \frac{?}{6} \qquad \frac{1}{3} = \frac{?}{6}$$

We multiply $\frac{1}{2}$ by $\frac{3}{3}$. We multiply $\frac{1}{3}$ by $\frac{2}{2}$.

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \qquad \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

The renamed fractions are $\frac{3}{6}$ and $\frac{2}{6}$. We are told to add these fractions.

$$\frac{3}{6} + \frac{2}{6} = \mathbf{\frac{5}{6}}$$

Practice Complete each equivalent fraction by multiplying each fraction by a fraction equal to 1.

a. $\frac{1}{3} = \frac{?}{12}$ **b.** $\frac{2}{3} = \frac{?}{6}$

c. $\frac{3}{4} = \frac{?}{8}$ **d.** $\frac{3}{4} = \frac{?}{12}$

e. Write $\frac{2}{3}$ and $\frac{1}{4}$ as fractions with denominators of 12. Then add the renamed fractions.

f. Write $\frac{1}{6}$ as a fraction with 12 as the denominator. Subtract the renamed fraction from $\frac{5}{12}$. Reduce the subtraction answer.

Problem set 41

1. (41) Write $\frac{1}{2}$ and $\frac{2}{3}$ as fractions with denominators of 6. Then add the renamed fractions. Write the answer as a mixed number.

2. (12) Our own galaxy, the Milky Way, may contain two hundred billion stars. Write that number.

3. (31) The rectangular school yard is 120 yards long and 40 yards wide. How many square yards is its area?

4. (33) Write 40% as a reduced fraction.

In problems 5 and 6, multiply $\frac{1}{2}$ by a fraction equal to 1 to complete each equivalent fraction.

5. (41) $\frac{1}{2} = \frac{?}{8}$ **6.** (41) $\frac{1}{2} = \frac{?}{10}$

7. (36) 4.32 + 0.6 + 11 **8.** (37) 6.3 − 0.54

9. 0.15×0.15
(40)

10. What is the reciprocal of $\frac{6}{7}$?
(30)

11. Which digit in 12,345 has the same place value as the 6 in 67.89?
(34)

12. What is the least common multiple of 3, 4, and 6?
(30)

13. $5\frac{3}{5} + 4\frac{4}{5}$
(27)

14. $6 - 4\frac{2}{3}$
(27)

15. $\frac{8}{3} \times \frac{1}{2}$
(29)

16. $\frac{6}{5} \times 3$
(29)

17. $1 - \frac{1}{4}$
(27)

18. $\frac{10}{10} - \frac{5}{5}$
(28)

19. Make three different fractions that are equal to $\frac{1}{3}$ by multiplying $\frac{1}{3}$ by three different fraction names for 1.
(41)

20. List the factors of 35.
(19)

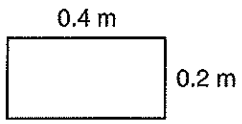
21. In three games Alma's scores were 12,643, 9870, and 14,261. What was her average score per game?
(18)

22. Estimate the quotient of $\frac{8176}{41}$.
(16)

23. How many doughnuts are in $\frac{2}{3}$ of a dozen? Draw a diagram to illustrate the problem.
(22)

24. Write $\frac{3}{4}$ with a denominator of 8. Subtract the renamed fraction from $\frac{7}{8}$.
(41)

25. What is the perimeter of this rectangle?
(8)



26. What is the area of this rectangle?
(31)

27. (1) The regular price (r) minus the discount (d) equals the sale price (s).

$$r - d = s$$

Arrange these letters to form another subtraction fact and two addition facts.

28. (2) Here we show the same division problem written three different ways. Identify which number is the divisor, which is the dividend, and which is the quotient.

$$\frac{20}{4} = 5 \qquad 4\overline{)20}^{\,5} \qquad 20 \div 4 = 5$$

29. (32) What time is $2\frac{1}{2}$ hours after 11:45 a.m.?

30. (30) (a) How many $\frac{5}{6}$'s are in 1?

(b) Use your answer to part (a) to help you find the number of $\frac{5}{6}$'s in 3.

LESSON 42

Equivalent Division Problems • Missing Number Problems with Fractions and Decimals

Facts Practice: 100 Subtraction Facts (Test C in Test Masters)

Mental Math: Count by 6's to 72. Count by 8's to 96.

a. 4×225 **b.** $720 - 200$ **c.** $37 + 28$
d. $\$200 - \$175$ **e.** $\frac{1}{2}$ of 1200 **f.** $\frac{\$70.00}{10}$
g. $8 \times 4, - 2, \times 2, + 3, \div 7, \times 2, \div 3$

Problem Solving: You can "roll" six different numbers with one number cube (1–6). You can roll eleven different numbers with two number cubes (2–12). How many different numbers can you roll with three number cubes?

Equivalent division problems

The following two division problems have the same quotient. We call them **equivalent division problems.** Which problem seems easier to perform mentally?

(a) $700 \div 14$

(b) $350 \div 7$

We can change problem (a) to problem (b) by dividing both 700 and 14 by 2.

$700 \div 14$

↓

Divide both 700 and 14 by 2.

↓

$350 \div 7$

By dividing both the dividend and divisor by the same number, in this case 2, we formed an equivalent division problem that was easier to divide mentally.

We may also form equivalent division problems by multiplying the dividend and divisor by the same number. Consider the following equivalent problems:

(c) $7\frac{1}{2} \div \frac{1}{2}$

(d) $15 \div 1$

We changed problem (c) to problem (d) by doubling both $7\frac{1}{2}$ and $\frac{1}{2}$, that is, by multiplying both numbers by 2.

$$\begin{array}{c} 7\frac{1}{2} \div \frac{1}{2} \\ \downarrow \\ \text{Multiply both } 7\frac{1}{2} \text{ and } \frac{1}{2} \text{ by 2.} \\ \downarrow \\ 15 \div 1 \end{array}$$

Example 1 Make an equivalent division problem; then calculate the quotient.

$$1200 \div 16$$

Solution Instead of dividing 1200 by the two-digit number 16, we can divide both the dividend and the divisor by 2 to form the equivalent division of 600 divided by 8.

$$16\overline{)1200} \xrightarrow{\text{Divide both numbers by 2.}} \begin{array}{r} 75 \\ 8\overline{)600} \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Both quotients are 75, but dividing by 8 is easier than dividing by 16.

Notice that there are often equivalent problems that can be made.

$$1200 \div 16 \rightarrow 600 \div 8 \rightarrow 300 \div 4 \rightarrow 150 \div 2$$

All of these problems have the same quotient.

Example 2 Find an equivalent problem for this division problem and calculate the quotient.

$$7\frac{1}{2} \div 2\frac{1}{2}$$

Solution Instead of performing the division with these mixed numbers, we will double both numbers to make a whole number division problem.

$$7\frac{1}{2} \div 2\frac{1}{2}$$

$\downarrow$

Multiply both $7\frac{1}{2}$ and $2\frac{1}{2}$ by 2.

$\downarrow$

$$\mathbf{15 \div 5 = 3}$$

Missing number problems with fractions and decimals Since Lessons 3 and 4 we have practiced finding missing numbers in whole number arithmetic problems. Beginning in this lesson we will find missing numbers in fraction and decimal problems. If you encounter a problem and you feel unsure how to find the solution, try making up a similar, easier problem to help you think of how to find the answer.

Example 3 $d - 5 = 3.2$

Solution This problem is similar to $d - 5 = 3$. We remember that we find the first number of a subtraction problem by adding the other two numbers.

$$\begin{array}{r} 5 \\ +\ 3.2 \\ \hline 8.2 \end{array}$$

We find that d equals **8.2.**

Example 4 $f + \frac{1}{5} = \frac{4}{5}$

Solution This problem is like $f + 1 = 4$. We can find a missing addend by subtracting the known addend from the sum.

$$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

We find that f equals $\frac{3}{5}$.

Example 5 $\frac{3}{5}n = 1$

Solution Two numbers are multiplied and the product is 1. This is only true when the two factors are reciprocals. So the problem is to find the reciprocal of the known factor, $\frac{3}{5}$. Reversing the terms of $\frac{3}{5}$ makes the fraction $\frac{5}{3}$.

$$\frac{3}{5} \cdot \frac{5}{3} = \frac{15}{15}, \text{ which equals } 1$$

We find that n equals $\frac{5}{3}$.

Practice

a. Make an equivalent division problem for $5 \div \frac{1}{3}$ by multiplying both the dividend and divisor by 3. Then find the quotient.

b. Make an equivalent division problem for $266 \div 14$ that has a one-digit divisor. Then find the quotient.

c. $5 - d = 3.2$

d. $f - \frac{1}{5} = \frac{4}{5}$

e. $m + 1\frac{1}{5} = 4$

f. $\frac{3}{8}w = 1$

Problem set 42

1. (3) What number must be added to six thousand, eighty-four to get a sum of ten thousand?

2. (11) If one hundred fifty knights could sit at the Round Table and only one hundred twenty-eight knights were seated, then how many empty places were at the table?

3. (32) Frank started running the marathon at 11:50 a.m. and finished 2 hours and 11 minutes later. At what time did he finish?

4. (42) $3\frac{7}{8} - 1\frac{3}{8}$

5. (42) $6 - w = 1\frac{4}{5}$

6. (42) $m - 4\frac{1}{4} = 6\frac{3}{4}$

7. (29) $\frac{5}{8} \times \frac{1}{5}$

8. (29) $\frac{3}{4} \times 5$

9. (42) $\frac{2}{3}n = 1$

10. (41) $\frac{2}{3} = \frac{?}{6}$

11. (41) $\frac{1}{2} = \frac{?}{6}$

12. (29) Compare: $\frac{2}{2} \bigcirc \frac{2}{2} \times \frac{2}{2}$

13. (14) The temperature was 8°F at midnight but dropped 15°F by morning. What was the morning temperature?

14. (35) Write the decimal number for nine and twelve hundredths.

15. (16) Round 67,492,384 to the nearest million.

16. (38) $46.37 + 5.93 + 14$

17. (42) $12 - d = 1.43$

18. (39) 0.37×100

19. (40) $0.6 \times 0.4 \times 0.2$

20. (38) The perimeter of a square room is 80 feet. The area of the room is how many square feet?

21. (26) Divide 100 by 16 and write the answer as a mixed number. Reduce the fraction part of the mixed number.

22. (42) Instead of dividing 100 by 16, Sandy divided the dividend and divisor by 4. What new division problem did Sandy make? What is the quotient?

23. *(42)* Make an equivalent division problem for $4\frac{1}{2} \div \frac{1}{2}$ by doubling both the dividend and divisor and then find the quotient.

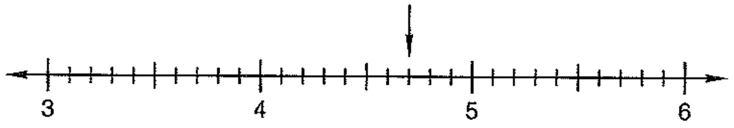
24. *(30)* What is the least common multiple (LCM) of 4, 6, and 8?

25. *(17)* What are the next three numbers in this sequence?

$$\frac{1}{16}, \frac{1}{8}, \frac{3}{16}, \frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{7}{16}, \_\_\_\_, \_\_\_\_, \_\_\_\_, \ldots$$

26. *(17)* Find the length of this segment to the nearest eighth of an inch.

27. *(17)* To what mixed number on this number line is the arrow pointing?



28. *(41)* Write $\frac{1}{2}$ and $\frac{1}{5}$ as fractions with denominators of 10. Then add the renamed fractions.

29. *(22,33)* Forty percent of the 20 seats on the bus were occupied. Write 40% as a reduced fraction. Then find the number of seats that were occupied. Draw a diagram to illustrate the problem.

30. *(30)* Describe how to form the reciprocal of a fraction.

LESSON 43

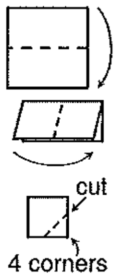
Simplifying Decimal Numbers • Comparing Decimal Numbers

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 3.

a. 4 × 325
b. 426 + 35
c. 28 + 57
d. \$8.50 + \$2.75
e. $\frac{1}{2}$ of 1400
f. $\frac{\$15.00}{100}$
g. 6 × 8, − 3, ÷ 5, + 1, × 6, + 3, ÷ 9

Problem Solving: Jeana folded a square paper in half from top to bottom. Then she folded the folded paper in half from left to right so that the four corners were together at the lower right. Then she cut off the lower right corners as shown. What will the paper look like when it is unfolded?



Simplifying decimal numbers

Perform these two subtractions with a calculator. Which calculator answer differs from the printed answer?

$$\begin{array}{r} 425 \\ -\ 125 \\ \hline 300 \end{array} \qquad \begin{array}{r} 4.25 \\ -\ 1.25 \\ \hline 3.00 \end{array}$$

A calculator automatically simplifies decimal numbers. Zeros at the end of a decimal number are removed. A decimal point at the end of a whole number is shown on a calculator although we usually remove the decimal point if no digits follow it. So 3.00 simplifies to 3. on a calculator. We remove the decimal point and write 3 only.

Example 1 Multiply 0.25 and 0.04 and simplify the product.

Solution We multiply.

$$\begin{array}{r} 0.25 \\ \times\ \ 0.04 \\ \hline 0.0100 \end{array}$$

If we perform this multiplication on a calculator the answer 0.01 is displayed. The calculator simplifies the answer by removing zeros at the end of a decimal number.

0.0100 simplifies to **0.01**

Decimal answers in this book are printed in simplified form unless otherwise stated.

Comparing decimal numbers

Zeros at the end of a decimal number do not affect the value of the decimal number. Each of these decimal numbers has the same value because the 3 is in the tenths' place.

0.3   0.30   0.300

Although 0.3 is the simplified form, sometimes it is useful to attach extra zeros to a decimal number. For instance, comparing decimal numbers can be easier if the numbers being compared have the same number of decimal places.

Example 2 Compare: 0.3 ○ 0.303

Solution When comparing decimal numbers it is important to pay close attention to place values. Writing both numbers with the same number of decimal places can make the job of comparing easier. We will attach two zeros to 0.3 so that it has three decimal places like 0.303.

0.3 ○ 0.303
↓
0.300 ○ 0.303

We see that 300 thousandths is less than 303 thousandths. We write our answer this way.

0.3 < 0.303

Example 3 Arrange these numbers in order from least to greatest:

0.3   0.042   0.24   0.235

Solution We will write each number with three decimal places.

0.300 0.042 0.240 0.235

Then we arrange the numbers in order, omitting ending zeros.

0.042 0.235 0.24 0.3

Practice Write these numbers in simplified form:

a. 0.0500

b. 50.00

c. 1.250

d. 4.000

Compare these decimal numbers:

e. 0.2 ○ 0.15

f. 12.5 ○ 1.25

g. 0.012 ○ 0.12

h. 0.31 ○ 0.039

i. 0.4 ○ 0.40

j. Write these numbers in order from least to greatest.
0.12, 0.125, 0.015, 0.2

Problem set 43

1. (26) What is the sum of the third multiple of four and the third multiple of five?

2. (15) One mile is 5280 feet. How many feet is five miles?

Mt. Everest is 29,028 feet high. Mt. Whitney is 14,495 feet high. Use this information to answer questions 3 and 4.

3. (13) Mt. Everest is how many feet higher than Mt. Whitney?

4. (13) How many feet more than 5 miles high is Mt. Everest? (Refer to question 2.)

5. (42) $5\frac{1}{3} - w = 4$

6. (42) $m - 6\frac{4}{5} = 1\frac{3}{5}$

7. (27) $7\frac{3}{4} - 1\frac{1}{4}$

8. (30) What is the reciprocal of $\frac{5}{16}$?

9. (35) Write thirty-two thousandths as a decimal number.

10. (2) $6\overline{)24,042}$

11. (2) $10\overline{)\$36.00}$

12. (43) Compare: 0.25 ○ 0.125

13. (32) Write the standard numeral for $(6 \times 100) + (4 \times 1)$.

14. (42) Write a division problem that is equivalent to $8\frac{1}{2} \div \frac{1}{2}$ by doubling the dividend and divisor. Then find the quotient.

15. (30) (a) How many $\frac{5}{8}$'s are in 1?

(b) Use your answer to part (a) to help you find the number of $\frac{5}{8}$'s in 3.

16. (30) What is the least common multiple of 2, 3, 4, and 6?

17. (42) $6.74 + 0.285 + f = 11.025$

18. (42) $0.4 - d = 0.33$

19. (39) 1.6×4.2

20. (41) $\frac{3}{4} = \frac{?}{12}$

21. (41) $\frac{2}{3} = \frac{?}{12}$

22. (18) Find the average of 26, 37, 42, and 43.

23. (16) Round 364,857 to the nearest thousand.

24. (10) Write the next two numbers in this sequence.

1, 6, 4, 9, 7, 12, _____, _____, ...

25. List the factors of 100.
(19)

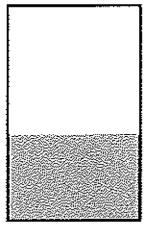
26. Write 9% as a fraction. Then write the fraction as a decimal number.
(33)

27. Write $\frac{3}{4}$ and $\frac{2}{3}$ as fractions with denominators of 12. Then add the renamed fractions.
(41)

28. Which percent best describes the shaded portion of this rectangle and why?
(28)

A. 80% B. 40%

C. 60% D. 20%

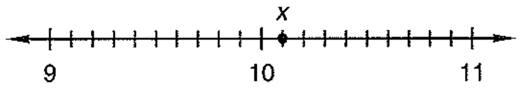


29. Walt started working at 10:30 a.m. and finished working at 2:15 p.m. How many hours and minutes did Walt work?
(32)

30. Which of these numbers is closest to 1?
(43)

A. 0.1 B. 0.8 C. 1.1 D. 1.2

31. What mixed number corresponds to point x on this number line?
(17)



LESSON
44

Dividing a Decimal Number by a Whole Number

Facts Practice: 90 Division Facts (Test F in Test Masters)

Mental Math: Count up and down by 25's between 25 and 300.

a. 4×425
b. $375 + 500$
c. $77 + 18$
d. $\$12.00 - \$1.25$
e. $\frac{1}{2}$ of 1500
f. $\frac{\$40.00}{10}$
g. $4 \times 8, - 2, \div 3, + 2, \div 3, \times 5, + 1, \div 3$

Problem Solving: Sheldon used six blocks to build this three-step shape. How many blocks would be in a six-step shape?

Dividing a decimal number by a whole number is like dividing dollars and cents by a whole number.

$$\begin{array}{r} \$0.45 \\ 5\overline{)\$2.25} \end{array} \qquad \begin{array}{r} 0.45 \\ 5\overline{)2.25} \end{array}$$

Notice that the decimal point in the quotient is directly above the decimal point in the dividend. Decimal division answers are not written with remainders. Instead, we attach zeros to the end of the number we are dividing and continue dividing.

Example 1 $3\overline{)4.2}$

Solution The decimal point in the quotient is straight up from the decimal point in the dividend.

$$\begin{array}{r} \mathbf{1.4} \\ 3\overline{)4.2} \\ \underline{3} \\ 1\,2 \\ \underline{1\,2} \\ 0 \end{array}$$

Example 2 $3\overline{)0.24}$

Solution The decimal point in the quotient is straight up. We fill the empty place with zero.

$$\begin{array}{r} \mathbf{0.08} \\ 3\overline{)0.24} \\ \underline{24} \\ 0 \end{array}$$

Example 3 $5\overline{)0.6}$

Solution The decimal point in the quotient is straight up. To complete the division, we attach a zero to 0.6 making the equivalent decimal number 0.60. Then we continue dividing.

$$\begin{array}{r} \mathbf{0.12} \\ 5\overline{)0.60} \\ \underline{5} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

Practice*

a. The distance from Margaret's house to school and back is 3.6 miles. How far does Margaret live from school?

b. The perimeter of a square is 6.4 meters. How long is each side of the square?

Find each quotient:

c. $\frac{4.5}{3}$ **d.** $0.6 \div 4$ **e.** $2\overline{)0.14}$

f. $0.4 \div 5$ **g.** $4\overline{)0.3}$ **h.** $\frac{0.012}{6}$

i. $10\overline{)1.4}$ **j.** $\frac{0.7}{5}$ **k.** $0.1 \div 4$

> **Teacher Note:** As a class activity, conduct a survey of class members and members of their families to find the favorite movies of the population. Set the rules for the survey in class. Students should collect responses at home. Assemble and display the data collected at a later class meeting.

Problem set 44

1. By what fraction must $\frac{5}{3}$ be multiplied to have a product equal to 1?
(30)

2. How many twenty-dollar bills equal one thousand dollars?
(15)

3. Cindy made $\frac{2}{3}$ of her 24 shots at the basket. Each basket was worth 2 points. How many points did she make?
(29)

4. *(44)* $3\overline{)4.5}$ **5.** *(44)* $8\overline{)0.24}$ **6.** *(44)* $5\overline{)0.8}$

7. *(30)* What is the least common multiple (LCM) of 2, 4, 6, and 8?

8. *(42)* $6 - m = 2\frac{3}{10}$ **9.** *(42)* $g - 2\frac{2}{5} = 5\frac{4}{5}$

10. *(27)* $4\frac{3}{8} - 2\frac{1}{8}$

11. *(16)* Estimate the product of 694 and 412.

12. *(38)* $5.36 + 9 + 0.742$ **13.** *(42)* $m - 1.56 = 1.44$

14. *(39)* $0.7 \times 0.6 \times 0.5$ **15.** *(40)* 0.46×0.17

16. *(26)* Convert the improper fraction $\frac{40}{6}$ to a mixed number with the fraction reduced.

17. *(15)* Brenda's car traveled 177.6 miles on 8 gallons of gas. Her car traveled an average of how many miles per gallon? Use an "equal groups" pattern.

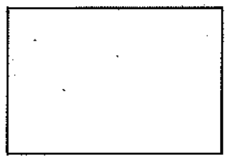
18. *(29)* $\frac{3}{8}$ of 6 **19.** *(29)* $\frac{9}{4} \times \frac{2}{3}$

20. *(41)* Write a fraction equal to $\frac{5}{6}$ that has 12 as the denominator. Then subtract $\frac{7}{12}$ from the fraction. Reduce the answer.

21. *(38)* The perimeter of a square is 24 feet. The area of the square is how many square feet?

22. *(33)* Write 27% as a fraction. Then write the fraction as a decimal number.

23. *(17)* Use your ruler to find the length of this rectangle to the nearest eighth of an inch.



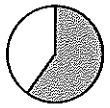
24. (22,23) Seventy-five percent of the 20 answers were correct. Write 75% as a reduced fraction. Then find the number of answers that were correct. Illustrate the fractional part problem.

25. (29) The product of $\frac{1}{2}$ and $\frac{2}{3}$ is $\frac{1}{3}$.

$$\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

Arrange these numbers to form another multiplication fact and two division facts.

26. (23) Which percent best describes the shaded portion of this circle? Why?



A. 80%
B. 60%
C. 40%
D. 20%

27. (35) Write nine hundredths

(a) as a fraction.

(b) as a decimal number.

28. (42) Write a division problem that is equivalent to $5 \div \frac{1}{3}$ by multiplying both the dividend and divisor by three. Then find the quotient.

29. (18) The average number of students in three classrooms was 24. Altogether, how many students were in the three classrooms?

30. (44) Ask your family members about their favorite movies. Record the information to add to the class survey.

LESSON
45

Writing Decimal Numbers in Expanded Notation • Other Multiplication Forms

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count by 12's from 12 to 108.

a. 4 × 525 **b.** 567 − 120 **c.** 38 + 17
d. \$5.75 + \$2.50 **e.** $\frac{1}{2}$ of 950 **f.** $\frac{2000}{100}$
g. 9 × 7, + 1, ÷ 8, × 3, + 1, × 2, − 1, ÷ 7

Problem Solving: Copy this problem and fill in the missing digits.

$$\begin{array}{rcccc} & & \_ & \_ & \_ \\ + & & & & \_ \\ \hline & \_ & \_ & \_ & 8 \end{array}$$

Writing decimal numbers in expanded notation

We may use expanded notation to write decimal numbers just as we use expanded notation to write whole numbers. The values of some decimal places are shown in this table.

Decimal Place Values

1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
___	●	___	___	___
ones'		tenths'	hundredths'	thousandths'

We write 4.025 in expanded notation this way:

$$(4 \times 1) + \left(2 \times \frac{1}{100}\right) + \left(5 \times \frac{1}{1000}\right)$$

The zero that serves as a place holder is usually not included in expanded notation.

Example 1 Write 5.06 in expanded notation.

Solution The 5 is in the ones' place and the 6 is in the hundredths' place.

$$\mathbf{(5 \times 1) + \left(6 \times \frac{1}{100}\right)}$$

Example 2 Write $\left(4 \times \frac{1}{10}\right) + \left(5 \times \frac{1}{1000}\right)$ as a decimal number.

Solution We write a decimal number with a 4 in the tenths' place and a 5 in the thousandths' place. No digits in the ones' place or hundredths' place are indicated, so we write zeros in those places.

0.405

Other multiplication forms To indicate multiplication we may use a times sign, a dot, or write the numbers side by side without a sign. Each of these expressions means that l and w are multiplied.

(1) $l \times w$ (2) $l \cdot w$ (3) lw

Notice that the multiplication dot in form (2) is elevated and is not in the position of a decimal point. Form (3) may be used to show the multiplication of two or more letters, of a number and a letter, or of two numbers.

lwh $4s$ $3(5)$

When two numbers like 3 and 5 are multiplied, one or more sets of parentheses may be used so that 3 times 5 is not confused with 35. Each of these ways is a proper use of parentheses to indicate the multiplication of 3 and 5, although the first form is most commonly used.

3(5) (3)(5) (3)5

Recall that the order of two factors may be reversed without changing the product.

$$3 \cdot 5 = 5 \cdot 3$$

This property of multiplication is known as the **commutative property of multiplication.** We may use this property to rearrange factors in an expression.

$$2 \cdot \underset{\circlearrowleft}{3 \cdot 2} \cdot \underset{\circlearrowleft}{5 \cdot 3} \cdot 5 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5$$

Example 3 If l equals 8 and w equals 5, then what number does lw equal?

Solution The expression lw means l times w. We are given that $l = 8$ and $w = 5$. So lw equals 8 times 5, which is **40.**

Example 4 Use the commutative property of multiplication to arrange these factors in order from least to greatest.

$$2 \cdot 5 \cdot 2 \cdot 7 \cdot 2 \cdot 3 \cdot 2 \cdot 3$$

Solution We are not told to multiply the factors, just to rearrange the factors.

$$\mathbf{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7}$$

Practice* Write these numbers in expanded notation:

a. 2.05

b. 20.5

c. 0.205

Write these numbers in decimal form:

d. $(7 \times 10) + \left(8 \times \frac{1}{10}\right)$

e. $\left(6 \times \frac{1}{10}\right) + \left(4 \times \frac{1}{100}\right)$

Multiply as indicated:

f. 4(2.5) **g.** $7 \cdot 5$ **h.** $\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)$

i. If $l = 6$, $w = 5$, and $h = 4$, then what number does lwh equal?

j. Use the commutative property to arrange the factors in the numerator and the denominator in order from least to greatest. (Do not move factors between the numerator and denominator.)

$$\frac{3 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 2 \cdot 7}$$

Teacher Note: Conclude the Favorite Movie Survey by tallying the responses in a frequency table and by displaying the results in a graph.

Problem set 45

1. (26) When a fraction with a numerator of 30 and a denominator of 8 is converted to a mixed number and reduced, what is the result?

2. (13) Normal body temperature is 98.6° on the Fahrenheit scale. A person with a temperature of 100.2°F would have a temperature how many degrees above normal? Use a "larger-smaller-difference" pattern.

3. (15) Four and twenty blackbirds is how many dozen?

4. (45) Write $(5 \times 10) + \left(6 \times \frac{1}{10}\right) + \left(7 \times \frac{1}{1000}\right)$ in decimal form.

5. (33) Twenty-one percent of the earth's atmosphere is oxygen. Write 21% as a fraction. Then write the fraction as a decimal number.

6. (33) Twenty-one percent is slightly more than 20%. Twenty percent is equivalent to what reduced fraction?

7. (44) $5\overline{)6.35}$ **8.** (44) $4\overline{)0.5}$ **9.** (44) $8\overline{)1.0}$

10. (42) $x + 3\frac{5}{8} = 9$ **11.** (27) $16\frac{1}{4} + 4\frac{3}{4}$

12. (42) $y - 1\frac{7}{8} = 2\frac{3}{8}$

13. (42) $5.63 + 26.9 + 12 + w = 44.53$

14. (42) $1 - q = 0.235$

15. (39) 3.7×0.25

16. (41) $\frac{3}{4} = \frac{?}{8}$

17. (41) $\frac{3}{4} = \frac{?}{12}$

18. (30) What is the least common multiple of 3, 4, and 8?

19. (43) Compare: $\frac{1}{10}$ ○ 0.1

20. (34) Which digit is in the thousandths' place in 1,234.5678?

21. (16) Estimate the quotient when 3967 is divided by 48.

22. (38) The area of a square is 100 square centimeters. How long is each side?

23. (7) There are 100 centimeters in 1 meter and 1000 meters in 1 kilometer. How many centimeters are in 2 kilometers?

24. (29) $\frac{1}{2} \cdot \frac{4}{5}$

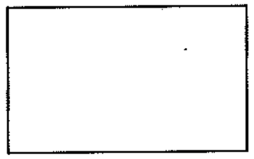
25. (45) $\left(\frac{3}{4}\right)\left(\frac{5}{3}\right)$

26. (45) If $b = 8$ and $h = 4$, then what number does bh equal?

27. (45) Use the commutative property of multiplication to rearrange these factors in order from least to greatest.

$$3 \cdot 7 \cdot 2 \cdot 5 \cdot 2 \cdot 3 \cdot 3 \cdot 5$$

28. (17) Use your ruler to find the width of this rectangle to the nearest eighth of an inch.



29. (30) (a) How many $\frac{3}{8}$'s are in 1?

(b) Use your answer to part (a) to find the number of $\frac{3}{8}$'s in 3.

30. (41) Rename $\frac{1}{2}$ and $\frac{1}{3}$ so that the denominators of the renamed fractions are 6. Then add the renamed fractions.

LESSON 46

Mentally Multiplying Decimal Numbers by 10 and by 100

Facts Practice: 100 Multiplication Facts (Test E in Test Masters)

Mental Math: Count by 9's from 9 to 108.

a. 4×925 **b.** 3×87 **c.** $56 - 19$

d. $9.00 − $1.25 **e.** $\frac{1}{2}$ of $12.50 **f.** $\frac{\$25.00}{10}$

g. $6 \times 8, + 2, \times 2, -10, \div 9, + 5, \div 3, + 1, \div 6$

Problem Solving: The average of two numbers is 44. If one of the numbers is 34, what is the other number?

When we multiply whole numbers by 10 or by 100 we may mentally attach zeros to the number we are multiplying to find the product.

$$24 \times 10 = 240$$

$$24 \times 100 = 2400$$

It may seem that we are just attaching zeros, but we are actually shifting the digits to the left. When we multiply 24 by 10, the digits shift one place to the left. When we multiply 24 by 100, the digits shift two places to the left. The zeros hold the 2 and the 4 in their proper places.

	1000s	100s	10s	1s	
			2	4	24
one-place shift		2	4	0	24 × 10
two-place shift	2	4	0	0	24 × 100

When we multiply a decimal number by 10, the digits shift one place to the left. When we multiply by 100, the digits shift two places to the left. Here we show the products when 0.24 is multiplied by 10 and by 100.

10s	1s	.	$\frac{1}{10}$s	$\frac{1}{100}$s	
	0	.	2	4	0.24
	2	.	4		0.24 × 10
2	4	.			0.24 × 100

Although it is the digits that are shifting one or two places to the left, we get the same effect by shifting the decimal point one or two places to the right.

$0.24 \times 10 = 2.4$ one-place shift $0.24 \times 100 = 24$ two-place shift

Example 1 3.75×10

Solution Since we are multiplying by 10, the product will have the same digits as 3.75, but the digits will be shifted one place. The product will be 10 times as large, so we mentally shift the decimal point one place to the right.

$3.75 \times 10 = \mathbf{37.5}$ (one-place shift)

We do not need to attach any zeros because the decimal point serves to hold the digits in their proper places.

Example 2 3.75×100

Solution Multiplying by 100, we mentally shift the decimal point two places to the right.

$3.75 \times 100 = 375.$ (two-place shift)

We do not need to attach zeros. Since there are no decimal places we may remove the decimal point to simplify the answer.

Example 3 $\frac{1.2}{0.4} \times \frac{10}{10}$

Solution Multiplying 1.2 and 0.4 by 10 shifts both decimal points one place.

$$\frac{1.2}{0.4} \times \frac{10}{10} = \frac{12}{4}$$

The expression $\frac{12}{4}$ means to divide 12 by 4.

$$\frac{12}{4} = \mathbf{3}$$

Practice Mentally calculate the product of each multiplication:

a. 0.35×10 **b.** 0.35×100

c. 2.5×10 **d.** 2.5×100

e. 0.125×10 **f.** 0.125×100

Answer "true" or "false" to the following statements:

g. If 0.04 is multiplied by 10, the product is a whole number.

h. If 0.04 is multiplied by 100, the product is a whole number.

Multiply as shown. Then complete the division.

i. $\frac{1.5}{0.5} \times \frac{10}{10}$ **j.** $\frac{2.5}{0.05} \times \frac{100}{100}$

Problem set 46

1. (10) The first positive odd number is 1. The second is 3. What is the tenth positive odd number?

2. (15) Giant tidal waves can travel 500 miles per hour. How long would it take a tidal wave traveling at that speed to cross 3000 miles of ocean?

3. (15) José bought Carmen one dozen red roses, two for each month he had known her. How long had he known her?

4. (42) $m - 1.25 = 3.75$ **5.** (39) $(0.5)(0.12)$

6. *(45)* If $s = \frac{1}{2}$, then what number does $4s$ equal?

7. *(45)* Write 6.25 in expanded notation.

8. *(33)* Write 99% as a fraction. Then write the fraction as a decimal number.

9. *(44)* $12\overline{)0.18}$

10. *(44)* $10\overline{)12.30}$

11. *(4)* $w \div 12 = 36$

12. *(42)* $5y = 1.25$

13. *(29)* Three of the twelve months begin with the letter *J*. What fraction of the months begin with *J*?

14. *(42)* $n + 5\frac{11}{12} = 10$

15. *(42)* $m - 6\frac{2}{5} = 3\frac{3}{5}$

16. *(27)* $8\frac{3}{4} + 5\frac{3}{4}$

17. *(29)* $\frac{5}{3} \times \frac{5}{4}$

18. *(41)* $\frac{3}{4} = \frac{?}{20}$

19. *(41)* $\frac{3}{5} = \frac{?}{20}$

20. *(18)* Bob's scores on his first five tests were 18, 20, 18, 20, and 20. His average score is closest to which of these numbers?

A. 17 B. 18 C. 19 D. 20

21. *(19)* Which factors of 20 are also factors of 30?

22. *(46)* Mentally calculate the product of 6.25 and 10.

23. *(46)* Multiply as shown. Then complete the division.

$$\frac{1.25}{0.5} \cdot \frac{10}{10}$$

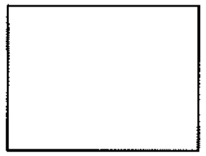
24. *(5)* What number is equal to "threescore and ten"? (Remember, a score is 20.)

25. *(13)* Use the chart to find out how many more days it takes Mars to go around the sun than it takes the earth to go around the sun.

Planet	Earth days to orbit the sun
Mercury	88
Venus	225
Earth	365
Mars	687

26. *(15)* In the time it takes Mars to travel around the sun once, Venus travels around the sun about how many times?

27. *(17)* Use your ruler to find the length and width of this rectangle.



28. *(8)* Calculate the perimeter of the rectangle in problem 27.

29. *(41)* Rename $\frac{2}{5}$ so the denominator of the renamed fraction is 10. Then subtract the renamed fraction from $\frac{9}{10}$. Remember to reduce the answer.

30. *(46)* When we mentally multiply 15 by 10, we may think of attaching a zero to 15 to make the product 150. When we multiply 1.5 by 10, why can't we attach a zero to make the product 1.50?

LESSON 47

Subtracting Mixed Numbers with Regrouping, Part 1

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 3.

a. 8×25 **b.** $630 - 50$ **c.** $62 + 19$

d. \$4.50 + 75¢ **e.** $\frac{1}{2}$ of \$15.00 **f.** $\frac{\$25.00}{100}$

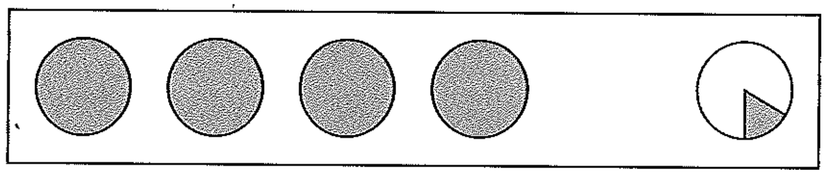
g. $4 \times 7, - 1, \div 3, \times 4, \div 6, \times 3, \div 2$

Problem Solving: Brad's first two test scores were 85 and 85. What score does he need on his third test to have an average score of 90 on his first three tests?

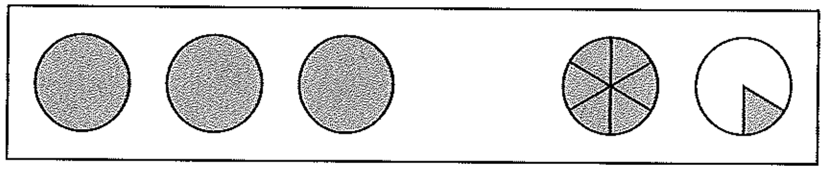
Here is another "some went away" story about pies.

> *There were $4\frac{1}{6}$ pies on the restaurant shelf. The server sliced one of the whole pies into sixths. Then the server removed $1\frac{2}{6}$ pies. How many pies were left on the shelf?*

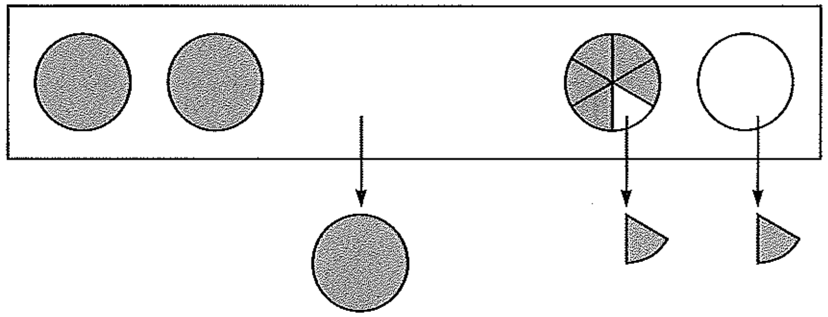
We will illustrate this story with circles. There were $4\frac{1}{6}$ pies on the shelf.



The server sliced one of the pies into sixths. This makes $3\frac{7}{6}$ pies, which equals $4\frac{1}{6}$ pies.



The server removed $1\frac{2}{6}$ pies. So $2\frac{5}{6}$ pies were left on the shelf.



Now we will show the arithmetic for subtracting $1\frac{2}{6}$ from $4\frac{1}{6}$.

$$\begin{array}{r} 4\frac{1}{6} \text{ pies} \\ -1\frac{2}{6} \text{ pies} \\ \hline \end{array}$$

We cannot subtract $\frac{2}{6}$ from $\frac{1}{6}$, so we will rename $4\frac{1}{6}$. Just as the server sliced one of the pies into sixths, so we will take one of the four wholes and change it to $\frac{6}{6}$. This makes three wholes plus $\frac{6}{6}$ plus $\frac{1}{6}$. We combine the $\frac{6}{6}$ and $\frac{1}{6}$, which makes $3\frac{7}{6}$.

$$\begin{array}{r} 4\frac{1}{6} \\ -\ 1\frac{2}{6} \\ \hline \\ \end{array} \xrightarrow{3 + \frac{6}{6} + \frac{1}{6}} \begin{array}{rl} 3\frac{7}{6} & \text{pies} \\ -\ 1\frac{2}{6} & \text{pies} \\ \hline 2\frac{5}{6} & \text{pies} \end{array}$$

Example $5\frac{1}{3} - 2\frac{2}{3}$

Solution We cannot subtract $\frac{2}{3}$ from $\frac{1}{3}$, so we will rename $5\frac{1}{3}$. We take one of the five wholes and make $\frac{3}{3}$. Then we combine $\frac{3}{3}$ and $\frac{1}{3}$ to make $4\frac{4}{3}$. Now we subtract.

$$\begin{array}{r} 5\frac{1}{3} \\ -\ 2\frac{2}{3} \\ \hline \\ \end{array} \xrightarrow{4 + \frac{3}{3} + \frac{1}{3}} \begin{array}{r} 4\frac{4}{3} \\ -\ 2\frac{2}{3} \\ \hline \mathbf{2\frac{2}{3}} \end{array}$$

Practice*

a. $\begin{array}{r} 4\frac{1}{3} \\ -\ 1\frac{2}{3} \\ \hline \end{array}$ b. $\begin{array}{r} 3\frac{2}{5} \\ -\ 2\frac{3}{5} \\ \hline \end{array}$ c. $\begin{array}{r} 5\frac{2}{4} \\ -\ 1\frac{3}{4} \\ \hline \end{array}$

d. $\begin{array}{r} 5\frac{1}{8} \\ -\ 2\frac{4}{8} \\ \hline \end{array}$ e. $\begin{array}{r} 7\frac{3}{12} \\ -\ 4\frac{10}{12} \\ \hline \end{array}$ f. $\begin{array}{r} 6\frac{1}{4} \\ -\ 2\frac{3}{4} \\ \hline \end{array}$

Problem set 47

1. (18) The average of two numbers is ten. What is the sum of the two numbers?

2. (46) What would be the cost of 10.0 gallons of gasoline priced at $1.449 per gallon?

3. (32) The movie started at 11:45 a.m. and ended at 1:20 p.m. The movie was how many hours and minutes long?

4. (43) Three of these numbers are equal to each other. Which number is different?

A. $\frac{1}{2}$ B. 0.2 C. 0.5 D. $\frac{10}{20}$

5. (43) Arrange these numbers in order from least to greatest:

1.02 0.102 0.12 1.20

6. (37) $0.1 + 0.2 + 0.3 + 0.4$ **7.** (39) $(8)(0.125)$

8. (11) The hike to the waterfall was 3 miles. After hiking 2.1 miles, how many more miles was it to the waterfall?

9. (16) Estimate the sum of 4967, 8142, and 6890.

10. (44) $8\overline{)0.144}$ **11.** (44) $6\overline{)0.9}$ **12.** (44) $4\overline{)0.9}$

13. (46) What is the price of 100 pens at 39¢ each?

14. (45) Write $(5 \times 10) + (6 \times \frac{1}{10}) + (4 \times \frac{1}{100})$ as a standard number.

15. (30) What is the least common multiple of 6 and 8?

16. (42) $w - 7\frac{7}{12} = 5\frac{5}{12}$

17. (42) $12 - m = 5\frac{2}{3}$

18. (42) $n + 2\frac{3}{4} = 5\frac{1}{4}$

19. (45) $\left(\frac{9}{10}\right)\left(\frac{3}{2}\right)$

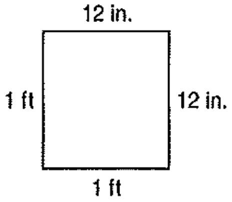
20. (22) What number is $\frac{5}{6}$ of 60?

21. (14) The temperature rose from –12°F to 5°F. How many degrees did the temperature rise?

22. (18) What number is halfway between 440 and 660?

23. (41) $\frac{3}{8} = \frac{?}{24}$

24. (8) The perimeter of this square is four feet. What is the perimeter in inches?



25. (31) The area of this square is one square foot. What is the area in square inches?

26. (45) If $r = 60$ and $t = 4$, then what does rt equal?

27. (33) Seventy-five percent of the 32 chairs in the room were occupied. Write 75% as a reduced fraction. Then find the number of chairs that were occupied.

28. (41) Rename $\frac{1}{3}$ and $\frac{1}{4}$ as fractions with denominators of 12. Then add the renamed fractions.

29. (46) Multiply as shown. Then simplify the answer.

$$\frac{3.5}{0.7} \cdot \frac{10}{10}$$

30. (47) There were $3\frac{1}{6}$ pies on the shelf. Explain how the server can take $1\frac{5}{6}$ pies from the shelf.

LESSON 48

Dividing by a Decimal Number

Facts Practice: 72 Mixed Multiplication and Division (Test H in Test Masters)

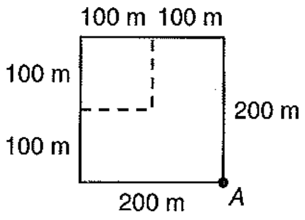
Mental Math: Count by 12's from 12 to 120.

a. $(8 \times 100) + (8 \times 25)$ **b.** $290 + 50$ **c.** $58 - 19$

d. \$5.00 − \$3.25 **e.** $\frac{1}{2}$ of \$30.00 **f.** $\frac{4000}{100}$

g. $5 \times 10, \div 2, + 5, \div 2, + 5, \div 2, \div 2$

Problem Solving: The P.E. class ran around the school block, starting and finishing at point *A*. Instead of running all the way around the block, Brad took what he called his "shortcut," shown by the dotted line in the diagram. How many meters of running did Brad save by taking his "shortcut"?



When the **divisor** of a division problem is a decimal number, we change the problem so that the divisor is a whole number.

$$\frac{1.24}{0.4} \qquad 0.4\overline{)1.24}$$

The divisor is a decimal number. We will change the problem before we divide.

One way to change the problem is to multiply the divisor and the dividend by 10. Notice that multiplying both numbers by 10 does not change the division answer.

$$\begin{array}{r} 2 \\ 4\overline{)8} \end{array} \longrightarrow \begin{array}{r} 2 \\ 40\overline{)80} \end{array}$$

Multiplying 4 and 8 by 10 does not change the answer.

If we multiply the divisor and dividend by 10 in $\frac{1.24}{0.4}$, the new problem has a whole number divisor.

$$\text{decimal divisor} \longrightarrow \frac{1.24}{0.4} \times \frac{10}{10} = \frac{12.4}{4} \longleftarrow \text{whole number divisor}$$

We divide 12.4 by 4 to find the quotient.

$$\begin{array}{r} 3.1 \\ 4\overline{)12.4} \end{array}$$

Example 1 $\frac{1.24}{0.04}$

Solution The divisor, 0.04, is a decimal number with two decimal places. To make the divisor a whole number, we will multiply $\frac{1.24}{0.04}$ by $\frac{100}{100}$ to shift the decimal point two places.

$$\frac{1.24}{0.04} \times \frac{100}{100} = \frac{124}{4}$$

We divide and find the quotient is **31.**

$$\begin{array}{r} 31 \\ 4\overline{)124} \\ \underline{12} \\ 04 \\ \underline{4} \\ 0 \end{array}$$

Example 2 $0.6\overline{)1.44}$

Solution The divisor, 0.6, has one decimal place. If we multiply the divisor and dividend by 10, we will shift the decimal point one place in both numbers.

$$0\underset{\smile}{.6}\overline{)1\underset{\smile}{.4}4}$$

This makes a new problem with a whole number divisor.

$$\begin{array}{r} \mathbf{2.4} \\ 6\overline{)14.4} \\ \underline{12} \\ 2\,4 \\ \underline{2\,4} \\ 0 \end{array}$$

Some people think of the phrase "over, over, and up" to remind themselves of how to keep track of the decimal points when dividing by decimal numbers.

$$\begin{array}{r} \text{up} \\ \cdot \\ \underset{\text{over}}{0.6}\overline{)\underset{\text{over}}{1.44}} \end{array}$$

Practice*

a. We would multiply the divisor and dividend of $\frac{1.44}{1.2}$ by what number to make the divisor a whole number?

b. We would multiply the divisor and dividend of $0.12\overline{)0.144}$ by what number to make the divisor a whole number?

Change each problem so that the divisor is a whole number. Then divide.

c. $\frac{0.24}{0.4}$

d. $\frac{9}{0.3}$

e. $0.05\overline{)2.5}$

f. $0.3\overline{)12}$

g. $0.24 \div 0.8$

h. $0.3 \div 0.03$

i. $0.05\overline{)0.4}$

j. $0.2 \div 0.4$

Problem set 48

1. (39) When the product of 0.2 and 0.3 is subtracted from the sum of 0.2 and 0.3, what is the difference?

2. *(22)* Four fifths of a dollar is how many cents? Draw a diagram to illustrate the problem.

3. *(32)* Dolores went to sleep at 9:15 p.m. and woke up at 7:15 a.m. How many hours did she sleep?

4. *(38)* If each side of a square is 2.4 cm, then what is the perimeter of the square?

5. *(43)* Compare: 0.31 ○ 0.301

6. *(38)* $0.67 + 2 + 1.33$

7. *(39)* $12(0.25)$

8. *(48)* $0.07\overline{)3.5}$

9. *(48)* $0.5\overline{)12}$

10. *(44)* $8\overline{)0.14}$

11. *(42)* $m + 7\frac{1}{4} = 15$

12. *(42)* $n - 6\frac{1}{8} = 4\frac{3}{8}$

13. *(41)* $\frac{5}{6} = \frac{?}{24}$

14. *(42)* $5 - m = 1.37$

15. *(39)* $(0.012)(1.5)$

16. *(35)* Write the decimal number one and twelve thousandths.

17. *(27)* $5\frac{7}{10} + 4\frac{9}{10}$

18. *(29)* $\frac{5}{2} \cdot \frac{5}{3}$

19. *(48)* $\frac{0.125}{0.05}$

20. *(29)* There are 24 hours in a day. Jim sleeps 8 hours each night. Eight hours is what fraction of a day?

21. *(19)* List the factors that 12 and 18 have **in common.** (That is, list the numbers that are factors of **both** 12 and 18.)

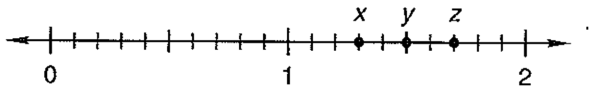
22. *(18)* What is the average of 1.2, 1.3, and 1.7?

23. *(16)* Estimate the difference of 5670 and 3940 to the nearest thousand.

24. *(30)* (a) How many $\frac{3}{4}$'s are in 1?

(b) Use your answer to part (a) to find the number of $\frac{3}{4}$'s in 4.

25. *(17)* Refer to this number line to answer the following questions about points x, y, and z.



(a) Which point is halfway between 1 and 2?

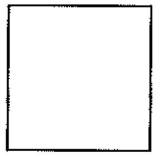
(b) Which point is closer to 1 than 2?

(c) Which point is closer to 2 than 1?

26. *(45)* Multiply and divide as indicated: $\frac{2 \cdot 3 \cdot 2 \cdot 5 \cdot 7}{2 \cdot 5 \cdot 7}$

27. *(48)* We can find the number of quarters in three dollars by dividing \$3.00 by \$0.25. Show this division using the pencil-and-paper method taught in this lesson.

28. *(8)* Use your ruler to find the length of each side of this square to the nearest eighth of an inch. Then calculate the perimeter of the square.



29. *(33)* Ninety percent of the 20 answers on the test were correct. Write 90% as a reduced fraction. Then find the number of correct answers on the test.

30. Sam was given the following division problem:
(48)

$$\frac{2.5}{0.5}$$

Instead of multiplying the numerator and denominator by 10, he accidentally multiplied by 100.

$$\frac{2.5}{0.5} \times \frac{100}{100} = \frac{250}{50}$$

Then he divided 250 by 50 and found that the quotient was 5. Did Sam find the correct answer to 2.5 ÷ 0.5? Why or why not?

LESSON 49

Decimal Number Line (Tenths) • Dividing by a Fraction

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count by 7's from 7 to 84.

a. (8 × 200) + (8 × 25)
b. 565 − 250
c. 58 + 27
d. \$1.45 + 99¢
e. $\frac{1}{2}$ of \$25.00
f. $\frac{5000}{10}$
g. 8 × 9, + 3, ÷ 3, − 1, ÷ 3, + 1, ÷ 3, ÷ 3

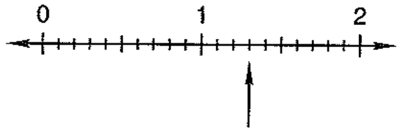
Problem Solving: Sheldon began building stair-step patterns with blocks. He used one block for a one-step pattern, three blocks for a two-step pattern, and six blocks for a three-step pattern. He wrote the information in a table. Copy the table and complete it through a ten-step pattern.

Blocks needed to make pattern

Steps	Blocks
1	1
2	3
3	6
4	
5	
6	
7	
8	
9	
10	

Decimal number line (tenths)

We can locate different kinds of numbers on the number line. We have learned to locate whole numbers, negative numbers, and fractions on the number line. We can also locate decimal numbers on the number line.



The distance between the whole numbers has been divided into 10 equal lengths. Each length is $\frac{1}{10}$ of the distance between the whole numbers. The arrow is pointing to a mark three spaces beyond the 1. The mark it is pointing to is $1\frac{3}{10}$. We can rename $\frac{3}{10}$ as the decimal 0.3, so we can say that the arrow is pointing to the mark 1.3. When a unit has been divided into 10 spaces, we normally use the decimal form instead of the fractional form to name the mark.

Example 1 What decimal number is represented by point y on this number line?

Solution The distance from 7 to 8 has been divided into ten smaller segments. Point y is four segments to the right of the whole number 7. So point y represents $7\frac{4}{10}$. We write $7\frac{4}{10}$ as the decimal number **7.4.**

Dividing by a fraction

The following question can be answered by dividing by a decimal number or by dividing by a fraction.

How many quarters are in three dollars?

If we think of a quarter as $\frac{1}{4}$ of a dollar, we have this division problem.

$$3 \div \frac{1}{4} \quad \text{"How many quarters are in three?"}$$

We take two steps to solve this type of problem. First we answer this question, "How many quarters are in one dollar?" The answer to this question is the reciprocal of $\frac{1}{4}$, which is $\frac{4}{1}$ or just 4.

$$1 \div \frac{1}{4} = \frac{4}{1}, \text{ which is } 4$$

Then we use the answer to this question to find the number of quarters in three dollars. There are four quarters in one dollar, so there are three times as many quarters in three dollars. For the second step, we multiply 3 times 4 and find there are 12 quarters in three dollars.

$$3 \times \overset{\text{number of quarters in one dollar}}{4} = \underset{\text{number of quarters in three dollars}}{12}$$

We will review the steps we took to solve the problem.

Original problem: How many quarters are in $3? $3 \div \frac{1}{4}$

Step 1: Find the number of quarters in $1. $1 \div \frac{1}{4} = 4$

Step 2: Use the number of quarters in 1$ to find the number in $3. $3 \times 4 = 12$

Example 2 The diameter of a penny is $\frac{3}{4}$ of an inch. How many pennies are needed to make a row of pennies 6 inches long?

Solution In effect, this problem asks, "How many $\frac{3}{4}$-inch segments are in 6 inches?" We can write the question this way.

$$6 \div \frac{3}{4}$$

We will take two steps. First we will find the number of pennies—the number of $\frac{3}{4}$-inch segments—in one inch. The number of $\frac{3}{4}$'s in 1 is the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$1 \div \frac{3}{4} = \frac{4}{3}$$

Four thirds of a penny is $1\frac{1}{3}$ pennies. There are $1\frac{1}{3}$ pennies in an inch. However, we will not convert $\frac{4}{3}$ to the mixed number $1\frac{1}{3}$. Instead, we will use $\frac{4}{3}$ in the second step of the solution. Since there are $\frac{4}{3}$ pennies in 1 inch, there are six times as many in 6 inches. So we multiply 6 times $\frac{4}{3}$. We find the number of pennies in a 6-inch row is **8.**

$$6 \times \frac{4}{3} = \frac{24}{3}, \text{ which is } 8$$

We will review the steps of the solution.

Original problem: How many $\frac{3}{4}$'s are in 6? $6 \div \frac{3}{4}$

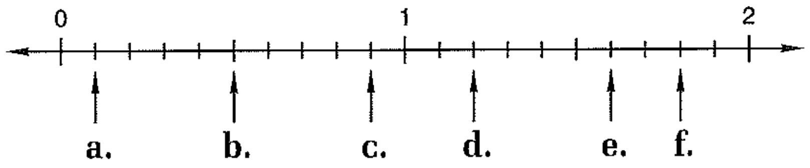
Step 1: Find the number of $\frac{3}{4}$'s in 1. $1 \div \frac{3}{4} = \frac{4}{3}$

Step 2: Use the number of $\frac{3}{4}$'s in 1 to find the number in 6. Then simplify the answer. $6 \times \frac{4}{3} = \frac{24}{3}$

$\frac{24}{3} = 8$

Practice

To which decimal number is each arrow pointing?



g. Write and solve a fraction division problem to find the number of quarters in four dollars. Follow this pattern.

Original problem
Step 1
Step 2

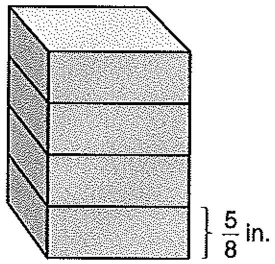
h. Write and solve a fraction division problem for this question:

Pads of writing paper were stacked 12 inches high on a shelf. The thickness of each pad was $\frac{3}{8}$ of an inch. How many pads were in a 12-inch stack?

Problem set 49

1. (10) The first three positive odd numbers are 1, 3, and 5. Their sum is 9. The first five positive odd numbers are 1, 3, 5, 7, and 9. Their sum is 25. What is the sum of the first ten positive odd numbers?

2. *(49)* Jack keeps each of his cassette tapes in a plastic box that is $\frac{5}{8}$ of an inch high. How many boxes are in a stack 10 inches high?



3. *(15)* The boxing match ended after two minutes of the 12th round. Each of the first eleven rounds lasted three minutes. For how many minutes did the contenders box?

4. *(43)* Compare: 3.4 ○ 3.389

5. *(43)* Compare: 0.60 ○ 0.600

6. *(42)* $7.25 + 2 + w = 10$

7. *(39)* $(3.75)(2.4)$

8. *(38)* $1 - 0.97$

9. *(48)* $0.12\overline{)7.2}$

10. *(48)* $0.4\overline{)7}$

11. *(44)* $6\overline{)0.138}$

12. *(42)* $w + \frac{5}{12} = 1$

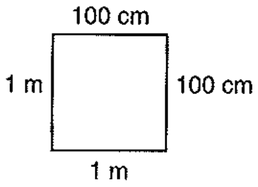
13. *(42)* $6\frac{1}{8} - x = 1\frac{7}{8}$

14. *(41)* $\frac{3}{4} = \frac{?}{24}$

15. *(33)* Write 7% as a fraction. Then change the fraction to a decimal number.

16. *(34)* Which digit in 4.637 is in the same place as the 2 in 85.21?

17. *(31)* One hundred centimeters equal one meter. How many square centimeters equal one square meter?



18. *(30)* What is the least common multiple of 6 and 9?

19. *(27)* $6\frac{5}{8} + 4\frac{5}{8}$

20. *(29)* $\frac{8}{3} \cdot \frac{3}{1}$

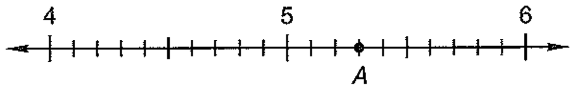
21. *(29)* $\frac{2}{3} \cdot \frac{3}{4}$

22. *(29)* If you sleep 8 hours each day, what fraction of the day do you **not** sleep?

23. Find the average of 2.4, 6.3, and 5.7.
(18)

24. What factors do 18 and 24 have in common?
(19)

25. What decimal number corresponds to point A on this number line?
(49)

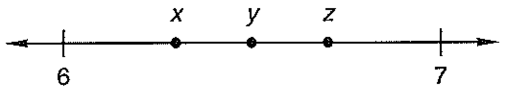


26. $\dfrac{2 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 5}$
(45)

27. 0.375×100
(46)

28. Rename $\frac{1}{3}$ as a fraction with 6 as the denominator. Then subtract the renamed fraction from $\frac{5}{6}$. Remember to reduce your answer.
(41)

29. Points x, y, and z are three points on this number line. Refer to the number line to answer the following questions.
(49)



(a) Which point is halfway between 6 and 7?

(b) Which point corresponds to $6\frac{7}{10}$?

(c) Which point corresponds to a number that would be closest to 6?

30. Which of these numbers is divisible by both 2 and 5?
(19)

A. 552 B. 255 C. 250 D. 525

LESSON 50

Rounding Decimal Numbers

Facts Practice: 72 Mixed Multiplication and Division (Test H in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 3.

a. 8 × 125 **b.** 4 × 68 **c.** 64 − 29

d. \$4.64 + 99¢ **e.** $\frac{1}{2}$ of \$150.00 **f.** $\frac{\$100.00}{100}$

g. 8 × 8, − 4, ÷ 2, + 2, ÷ 4, + 2, ÷ 5, × 10

Problem Solving: Copy this problem and fill in the missing digits.

```
      9_
9 ) _9_
    __
     __
     __
      0
```

It is often necessary to round decimal numbers. For instance, money is usually rounded to two places after the decimal point because we do not have a coin smaller than one hundredth of a dollar.

Example 1 Dan wanted to buy a tape for \$6.89. The sales tax rate was 8%. Dan calculated the sales tax. He knew that 8% equaled the fraction $\frac{8}{100}$ and the decimal 0.08. To figure the amount of tax, he multiplied the price \$6.89 by the sales tax rate 0.08.

$$\begin{array}{r} \$6.89 \\ \times\ 0.08 \\ \hline \$0.5512 \end{array}$$

How much tax would Dan need to pay?

Solution Sales tax is rounded to the nearest cent, which is two places to the right of the decimal point. We will mark the places that will be included in the answer.

$$\underline{\$0.55}|12$$

Next we decide the possible answers. We see that \$0.5512 is a little more than \$0.55 but less than \$0.56. We will decide whether \$0.5512 is closer to \$0.55 or \$0.56 by looking at the next digit. If the next digit is 5 or more, we round up to $0.56. If the next digit is less than 5, we round down to $0.55. Since the next digit is 1, we round $0.5512 down. If Dan buys the tape he will need to pay **$0.55** sales tax.

Example 2 Sheila pulled into the gas station and filled the car's tank with 10.381 gallons of gasoline. Round the amount of gasoline she purchased to the nearest tenth of a gallon.

Solution Tenths is one place to the right of the decimal point. We will mark the places that will be included in the answer.

$$\underline{10.3}|81$$

Next we determine what our possible answers may be. The number we are rounding is more than 10.3 but less than 10.4. Our answer will be one of these two numbers. We decide that 10.381 is closer to 10.4 because the digit in the next place is 8, and we round up when the next digit is 5 or more. Sheila bought about **10.4** gallons of gasoline.

Example 3 Estimate the product of 6.85 and 4.2 by rounding the numbers to the nearest whole number before multiplying.

Solution We mark the whole-number places.

$$\underline{6}|85 \qquad \underline{4}|2$$

We see that 6.85 is more than 6 but less than 7. The next digit, 8, shows that 6.85 is closer to 7. The number 4.2 is more than 4 but less than 5. The next digit, 2, shows that 4.2 is closer to 4. So 6.85 rounds to 7, and 4.2 rounds to 4.

We multiply the rounded numbers.

$$7 \cdot 4 = 28$$

We estimate that the product of 6.85 and 4.2 is about **28.**

Practice* Round to the nearest cent:

a. $6.6666 **b.** $0.4625 **c.** $0.08333

Round to the nearest tenth:

d. 0.12 **e.** 12.345 **f.** 2.375

Round to the nearest whole number:

g. 16.75 **h.** 4.875 **i.** 73.29

Problem set 50

1. *(26)* When the third multiple of 8 is subtracted from the fourth multiple of 6, what is the difference?

2. *(37)* From Brad's home to school is 3.5 miles. How far does Brad travel riding from home to school and back home?

3. *(13)* Napoleon I was born in 1769. How old was he when he was crowned emperor of France in 1804?

4. *(50)* Round $0.1625 to the nearest cent.

5. *(50)* Round 2.375 to the nearest tenth.

6. *(50)* Explain how to round 12.75 to the nearest whole number.

7. *(37)* $0.125 + 0.25 + 0.375$ **8.** *(42)* $0.399 + w = 0.4$

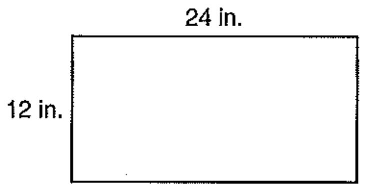
9. *(48)* $\dfrac{4}{0.25}$ **10.** *(44)* $4\overline{)0.5}$ **11.** *(44)* $3.25 \div 10$

12. *(47)* $3\frac{5}{12} - 1\frac{7}{12}$ **13.** *(41)* $\frac{5}{8} = \frac{?}{24}$ **14.** *(47)* $20 - 17\frac{3}{4}$

15. *(39)* $(0.19)(0.21)$ **16.** *(35)* Write 0.01 as a fraction.

17. *(45)* Write $(6 \times 10) + (7 \times \frac{1}{100})$ as a decimal number.

18. *(31)* How many square inches are needed to cover the area of the rectangle?



19. *(30)* What is the least common multiple of 2, 3, and 4?

20. *(27)* $5\frac{3}{10} + 6\frac{9}{10}$

21. *(29)* $\frac{10}{3} \times \frac{1}{2}$

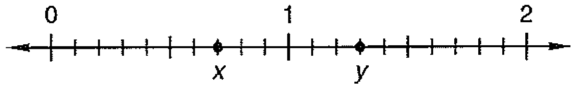
22. *(49)* A collection of paperback books was stacked 12 inches high. Each book in the stack was $\frac{3}{4}$ inch thick. Use the method described in Lesson 49 to find the number of books in the stack.

23. *(16)* Estimate the quotient when 4876 is divided by 98.

24. *(19)* What factors do 16 and 24 have in common?

25. *(50)* Estimate the product of 11.8 and 3.89 by rounding the factors to the nearest whole number before multiplying.

26. *(49)* Find the average of the decimal numbers that correspond to points x and y on this number line.



27. *(45)* $\frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5}$

28. *(46)* Mentally calculate the total price of ten pounds of bananas at $0.79 per pound.

29. *(41)* Rename $\frac{2}{3}$ and $\frac{3}{4}$ as fractions with 12 as the denominator. Then add the renamed fractions. Write the sum as a mixed number.

30. *(48)* How many $0.40 pens can Sam buy for $10.00? Show the division using the method taught in Lesson 48.

LESSON 51

Mentally Dividing Decimal Numbers by 10 and by 100

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count by 12's from 12 to 132.

a. 4×250 **b.** $368 - 150$ **c.** $250 + 99$
d. $15.00 + $7.50 **e.** $\frac{1}{2}$ of 5 **f.** 20×40
g. $5 \times 10, + 4, \div 6, \times 8, + 3, \div 3$

Problem Solving: Debbie averaged 88% on her first three tests. What score does she need on her fourth test to have a four-test average of 90%?

When we divide a decimal number by 10 or by 100, the answer (quotient) has the same digits as the number that is divided (dividend). However, the position of the digits is shifted. Here we show 12.5 divided by 10 and by 100.

$$10\overline{)12.50}\quad 1.25 \qquad\qquad 100\overline{)12.500}\quad .125$$

When we divide by 10, the digits shift one place to the right. When we divide by 100, the digits shift two places to the right. Although it is the digits that are shifting places, we produce the shift by moving the decimal point. When we divide by 10, the decimal point moves one place to the left. When we divide by 100, the decimal point moves two places to the left.

Example 1 $37.5 \div 10$

Solution Dividing by 10, the answer will be less than 37.5, so we mentally shift the decimal point one place to the left.

$$37.5 \div 10 = \mathbf{3.75}$$

Example 2 $3.75 \div 100$

Solution We mentally shift the decimal point two places to the left. This creates an empty place between the decimal point and the 3, which we fill with a zero. We also write a zero in the ones' place.

0.0375

Practice* Mentally calculate each quotient. Write each answer as a decimal number.

a. $2.5 \div 10$ **b.** $2.5 \div 100$

c. $87.5 \div 10$ **d.** $87.5 \div 100$

e. $0.5 \div 10$ **f.** $0.5 \div 100$

g. $25 \div 10$ **h.** $25 \div 100$

Problem set 51

1. (29) What is the product of one half and two thirds?

2. (13) A piano has 88 keys. Fifty-two of the keys are white. How many more white keys are there than black keys?

3. (12) The deepest part of the Atlantic Ocean is thirty thousand, two hundred forty-six feet. Write that number.

4. (46) 3.75×10 **5.** (51) $3.75 \div 10$

6. (45) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ **7.** (26) Convert and reduce: $\frac{150}{12}$

8. (39) Multiply and simplify: $(0.125)(4)$

9. (48) $\frac{(1 + 0.2)}{(1 - 0.2)}$ **10.** (42) $5\frac{1}{3} - m = 1\frac{2}{3}$

11. (29) $\frac{5}{2} \times \frac{4}{1}$ **12.** (42) $m - 5\frac{1}{3} = 1\frac{2}{3}$

13. (1) $10 − $0.10

14. (50) Round $6.789 to the nearest cent.

15. (50) Round 12.475 to the nearest tenth.

16. (43) Arrange these numbers in order from least to greatest:

1.02, 1.2, 0.21, 0.201

17. (10) What is the missing number in this sequence?

1, 2, 4, 7, 11, ______, 22, ...

18. (38) The perimeter of a square room is 80 feet. How many floor tiles 1 foot square would be needed to cover the area of the room?

19. (22) One foot is 12 inches. What fraction of a foot is 3 inches?

20. (22) How many cents is $\frac{2}{5}$ of a dollar?

21. (49) The diameter of a penny is $\frac{3}{4}$ of an inch. How many pennies are needed to make a row of pennies 12 inches long? (Write and solve a fraction division problem to answer this question.)

22. (30) What is the least common multiple of 2, 4, and 6?

23. (29) $\frac{4}{4} - \frac{2}{2}$

24. (7) A meter is about one **big** step. About how many meters above the floor is the top of the chalkboard?

25. (49) To what decimal number is the arrow pointing?

26. (45) $\frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 2}$

27. (41) Rename $\frac{1}{2}$ and $\frac{2}{3}$ as fractions with denominators of 6. Then add the renamed fractions. Write the sum as a mixed number.

28. (33) Seventy-eight percent of the earth's atmosphere is nitrogen. Write 78% as an unreduced fraction. Then write the fraction as a decimal number.

29. (32) What time was $12\frac{1}{2}$ hours before 7 a.m.?

30. (38) Draw a square with a perimeter of 4 inches. Then shade 50% of the square.

LESSON 52

Decimals Chart • Simplifying Fractions

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count by 9's from 9 to 108.

a. 8×225 **b.** $256 + 34$ **c.** $250 - 99$
d. \$25.00 − \$12.50 **e.** Double $2\frac{1}{2}$ **f.** $\frac{800}{20}$
g. $10 \times 10, - 20, + 1, \div 9, \times 5, - 1, \div 4$

Problem Solving: Here is part of the multiplication table. What number is missing?

36	42	48
42	?	56
48	56	64

Decimals chart

For many lessons we have been developing our decimal arithmetic skills. We find that arithmetic with decimal numbers is similar to arithmetic with whole numbers. However, in decimal number arithmetic, we need to keep track of the decimal point. The chart on the next page summarizes the rules for arithmetic with decimal numbers by providing keywords to help you keep track of the decimal point.

Across the top of the chart are the four operation signs (+, −, ×, ÷). Below each sign is the rule or memory cue to follow when performing that operation. (There are two kinds of division problems, so there are two different cues.)

Decimals Chart

+ −	×	÷ by whole	÷ by decimal
Line up the decimal points.	Multiply. Then count decimal places.	Decimal point is up.	over, over, up
1. Place a decimal point to the right of a whole number. 2. Fill empty places with zeros.			

The bottom of the chart contains two rules that apply to more than one operation.

Simplifying fractions

We simplify fractions in two ways. We reduce fractions to lowest terms and we convert improper fractions to mixed numbers. Sometimes a fraction can be reduced and converted to a mixed number.

Example $\frac{2}{3} + \frac{5}{6}$

Solution We rename $\frac{2}{3}$ to $\frac{4}{6}$. We add $\frac{4}{6}$ and $\frac{5}{6}$. The sum $\frac{9}{6}$ can be simplified.

$$\begin{array}{rcccr} \frac{2}{3} & \times & \frac{2}{2} & = & \frac{4}{6} \\ +\frac{5}{6} & & & = & \frac{5}{6} \\ \hline & & & & \frac{9}{6} \end{array}$$

We may reduce first and then convert the fraction to a mixed number, or we may convert first and then reduce.

REDUCE FIRST:	CONVERT FIRST:
1. Reduce $\frac{9}{6} = \frac{3}{2}$	1. Convert $\frac{9}{6} = 1\frac{3}{6}$
2. Convert $\frac{3}{2} = \mathbf{1\frac{1}{2}}$	2. Reduce $1\frac{3}{6} = \mathbf{1\frac{1}{2}}$

Practice

a. Discuss how the rules in the decimals chart apply to each of these problems.

5 − 4.2 0.4 × 0.2 0.12 ÷ 3 5 ÷ 0.4

b. Draw the decimals chart on your paper.

Add and simplify:

c. $\frac{5}{6} + \frac{5}{12}$ **d.** $\frac{9}{10} + \frac{3}{5}$ **e.** $\frac{2}{3} + \frac{7}{12}$

Teacher Note: Plan a survey about a subject the class selects. Students should collect information about the subject from other students in the school.

Problem set 52

1. (52) The decimals chart in this lesson shows that we line up the decimal points when we add or subtract decimal numbers. Why do we do that?

2. (32) The turkey must cook for 4 hours and 45 minutes. What time must it be put in the oven in order to be done by 3:00 p.m.?

3. (49) Billy won the contest by eating $\frac{1}{4}$ of a berry pie in 7 seconds. At this rate, how long would it take Billy to eat a whole berry pie?

4. (18) In four games the basketball team scored 47, 52, 63, and 66 points. What was the average number of points scored per game?

5. (42) $0.375x = 37.5$

6. (51) 0.375 ÷ 10

7. (33) Write 1% as a fraction. Then write the fraction as a decimal number.

8. (38) 3.6 + 4 + 0.39

9. (48) $\frac{36}{0.12}$

10. (44) $\frac{0.15}{4}$

11. (47) $6\frac{1}{4} - 3\frac{3}{4}$

12. (29) $\frac{2}{3} \times \frac{3}{5}$

13. (27) $5\frac{5}{8} + 7\frac{7}{8}$

14. (34) Which digit in 3456 has the same place value as the 2 in 28.7?

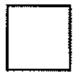
15. (50) Round 0.416 to the nearest hundredth.

16. (49) Which number is closest to 1?

A. 1.2 B. 0.9 C. 0.1 D. $\frac{1}{2}$

17. (10) What is the sum of the first eight positive odd numbers?

18. (38) What is the perimeter of the square?



$\frac{3}{8}$ in.

19. (29) A yard is 36 inches. What fraction of a yard is 3 inches?

20. (4) $8m = 1000$

21. (19) List the factors of 11.

22. (30) What is the smallest number that is a multiple of both 6 and 9?

23. (30) The product of $\frac{2}{3}$ and $\frac{3}{2}$ is 1.

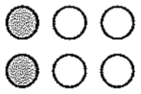
$$\frac{2}{3} \cdot \frac{3}{2} = 1$$

Use these numbers to form another multiplication fact and two division facts.

24. (45) $\frac{2 \cdot 3 \cdot 2 \cdot 5 \cdot 2 \cdot 5}{2 \cdot 5 \cdot 2 \cdot 5}$

25. (41) $\frac{5}{6} = \frac{?}{24}$

26. (29) What fraction of the circles are shaded?



27. (22,33) Thirty percent of the 350 students ride the bus to Thompson School. Write 30% as a reduced fraction. Then find the number of students who ride the bus.

28. (41) Rename $\frac{1}{4}$ and $\frac{1}{6}$ as fractions with denominators of 12. Then add the renamed fractions.

29. (49) The number that corresponds to point A is how much less than the number that corresponds to point B?

30. (49) The classroom set of Huckleberry Finn books fills a shelf that is 24 inches long. Each book is $\frac{3}{4}$ of an inch thick. How many books are in the classroom set? (Write and solve a fraction division problem to answer the question.)

LESSON 53

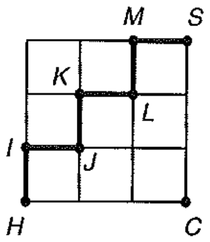
More on Reducing • Dividing Fractions

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 3.

a. 6×250 **b.** $736 - 400$ **c.** $375 + 99$
d. \$8.75 + \$5.00 **e.** $\frac{1}{2}$ of 9 **f.** 30×30
g. 8 × 8, − 1, ÷ 9, × 7, + 1, ÷ 5, × 10

Problem Solving: Ned walked from his home (H) to school (S) following the path from H to I to J to K to L to M to S. After school he walked home from S to C to H. Compare the distance of his walk to school and the distance of his walk home.

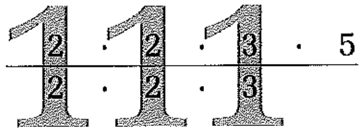


More on reducing

The factors in the problem below are arranged in order from least to greatest. Notice that some factors appear in both the dividend and the divisor.

$$\frac{2 \cdot 2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3}$$

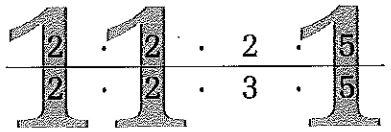
Since 2 ÷ 2 is 1 and 3 ÷ 3 is 1, we will mark the combinations of factors equal to 1 in this problem.



Looking at the factors this way, the problem becomes 1 · 1 · 1 · 5, which is 5.

Example 1 Reduce this fraction: $\dfrac{2 \cdot 2 \cdot 2 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5}$

Solution We will mark combinations of factors equal to 1.

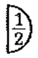


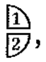
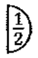
Grouping factors equal to 1, the problem becomes $1 \cdot 1 \cdot 1 \cdot \frac{2}{3}$, which is $\frac{2}{3}$.

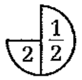
Dividing fractions When we divide 10 by 5, we are answering the question "How many 5's are in 10?" When we divide $\frac{3}{4}$ by $\frac{1}{2}$, we are answering the same type of question, "How many $\frac{1}{2}$'s are in $\frac{3}{4}$?" While it is easy to see how many 5's are in 10, it is not as easy to see how many $\frac{1}{2}$'s are in $\frac{3}{4}$. When the divisor is a fraction, we take two steps to find the answer. First we find how many of the divisors are in 1. This is the reciprocal of the divisor. Then we use the reciprocal to find the answer to the original division problem by multiplying, as we show in these examples.

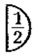
Example 2 How many $\dfrac{1}{2}$'s are in $\dfrac{3}{4}$? $\left(\dfrac{3}{4} \div \dfrac{1}{2}\right)$

Solution Before we show the two-step process we will solve the problem with our fraction manipulatives. The question in the example can be stated this way:

How many of these  are needed to make this ?

We see that the answer is more than one but less than two. If we take one  and cut another one into two parts , then we can fit the  and one of the parts  together to make three fourths.



We see that we need $1\frac{1}{2}$ of these  to make this .

Now we will show $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$ by arithmetic. The original problem asks, "How many $\frac{1}{2}$'s are in $\frac{3}{4}$?"

$$\frac{3}{4} \div \frac{1}{2}$$

The first step is to find the number of $\frac{1}{2}$'s in 1.

$$1 \div \frac{1}{2} = 2$$

The number of $\frac{1}{2}$'s in 1 is 2, which is the reciprocal of $\frac{1}{2}$. So the number of $\frac{1}{2}$'s in $\frac{3}{4}$ should be $\frac{3}{4}$ of 2. We find $\frac{3}{4}$ of 2 by multiplying.

$$\frac{3}{4} \text{ of } 2 = \frac{6}{4}, \text{ which equals } 1\frac{1}{2}$$

We simplified $\frac{6}{4}$ by reducing $\frac{6}{4}$ to $\frac{3}{2}$ and by converting $\frac{3}{2}$ to $\mathbf{1\frac{1}{2}}$. We will review the steps we took to solve the problem.

Original problem: How many $\frac{1}{2}$'s are in $\frac{3}{4}$? $\quad \frac{3}{4} \div \frac{1}{2}$

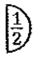
Step 1: Find the number of $\frac{1}{2}$'s in 1. $\quad 1 \div \frac{1}{2} = 2$

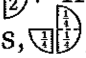
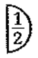
Step 2: Use the number of $\frac{1}{2}$'s in 1 to find the number of $\frac{1}{2}$'s in $\frac{3}{4}$. $\quad \frac{3}{4} \times 2 = \frac{6}{4}$

Then simplify the answer. $\quad = 1\frac{1}{2}$

Example 3 How many $\frac{3}{4}$'s are in $\frac{1}{2}$? $\left(\frac{1}{2} \div \frac{3}{4}\right)$

Solution Using our fraction manipulatives, the question can be stated this way:

How much of  is needed to make ?

The answer is less than 1. We need to cut off part of  to make . If we cut off one of the three parts of three fourths, , then two of the three parts equal . So $\frac{2}{3}$ of $\frac{3}{4}$ is needed to make $\frac{1}{2}$.

Now we will show the arithmetic. The original problem asks, "How many $\frac{3}{4}$'s are in $\frac{1}{2}$?"

$$\frac{1}{2} \div \frac{3}{4}$$

First we find the number of $\frac{3}{4}$'s in 1. The number is the reciprocal of $\frac{3}{4}$.

$$1 \div \frac{3}{4} = \frac{4}{3}$$

The number of $\frac{3}{4}$'s in 1 is $\frac{4}{3}$. So the number of $\frac{3}{4}$'s in $\frac{1}{2}$ should be $\frac{1}{2}$ of $\frac{4}{3}$. We find $\frac{1}{2}$ of $\frac{4}{3}$ by multiplying.

$$\frac{1}{2} \times \frac{4}{3} = \frac{4}{6}, \text{ which equals } \frac{2}{3}$$

The product, $\frac{4}{6}$, reduces to $\frac{2}{3}$. Again we will review the steps we took to solve the problem.

Original problem: How many $\frac{3}{4}$'s are in $\frac{1}{2}$? $\quad \frac{1}{2} \div \frac{3}{4}$

Step 1: Find the number of $\frac{3}{4}$'s in 1. $\quad 1 \div \frac{3}{4} = \frac{4}{3}$

Step 2: Use the number of $\frac{3}{4}$'s in 1 to find the number of $\frac{3}{4}$'s in $\frac{1}{2}$. $\quad \frac{1}{2} \times \frac{4}{3} = \frac{4}{6}$

Then simplify the answer. $\quad = \frac{2}{3}$

Practice

a. $\frac{2 \cdot 2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 5}$

b. $\frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5 \cdot 5}$

c. How many $\frac{3}{8}$'s are in $\frac{1}{2}$? $\left(\frac{1}{2} \div \frac{3}{8}\right)$

d. How many $\frac{1}{2}$'s are in $\frac{3}{8}$? $\left(\frac{3}{8} \div \frac{1}{2}\right)$

Problem set 53

1. Draw the decimals chart from Lesson 52.
(52)

2. If 0.4 is the dividend and 4 is the divisor, what is the quotient?
(44)

3. In 1900 the U.S. population was 76,212,168. In 1950 the population was 151,325,798. **Estimate** the increase in population between 1900 and 1950 to the nearest million.
(16)

4. Mark was $59\frac{3}{4}$ inches tall when he turned 11 and $61\frac{1}{4}$ inches tall when he turned 12. How many inches did he grow during the year?
(47)

5. $1000 - (100 - 1)$
(5)

6. $\frac{1000}{24}$
(26)

7. What number is halfway between 37 and 143?
(18)

8. $\$3 - n = 24¢$
(3)

9. $(1.2 \div 0.12)(1.2)$
(52)

10. $4.2 \div 100$
(44)

11. $m + 3\frac{4}{5} = 6\frac{2}{5}$
(42)

12. $\frac{4}{3} \cdot \frac{4}{3}$
(29)

13. $\frac{4}{3} = \frac{?}{18}$
(41)

14. Which digit is in the hundred-thousands' place in 123,456,789?
(12)

15. Round \$26.777 to the nearest cent.
(50)

16. Use your rulers to compare:
(9)

One centimeter ○ one inch

17. What is the twelfth number in this sequence?
(10)

1, 3, 5, 7, 9, ...

18. How many square feet of tile would be needed to cover the area of a room 14 feet long and 12 feet wide?
(31)

19. Nine of the 30 students received A's on the test. What fraction of the students received A's?
(29)

20. $\frac{5}{6} = \frac{?}{24}$
(41)

21. What is the least common multiple of 3, 4, and 6?
(30)

22. How many $\frac{1}{2}$'s are in $\frac{2}{3}$? $\left(\frac{2}{3} \div \frac{1}{2}\right)$
(53)

23. Eighty percent of the 30 questions were correct. Write 80% as a reduced fraction; then find the number of questions that were correct.
(22,33)

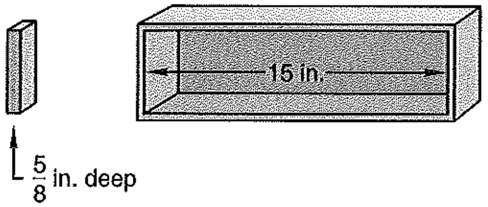
24. One inch equals 2.54 centimeters. A line 100 inches long would be how many centimeters long?
(46)

25. $\frac{2 \cdot 3 \cdot 5 \cdot 3 \cdot 2}{2 \cdot 3 \cdot 2 \cdot 5}$
(53)

26. $\frac{2 \cdot 3 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}$
(53)

27. Rename $\frac{2}{3}$ and $\frac{1}{2}$ as fractions with denominators of 6. Then add the renamed fractions and convert the answer to a mixed number.
(41)

28. A music store sells cassette tapes in clear plastic boxes that are $4\frac{1}{2}$ inches long, $2\frac{3}{4}$ inches wide, and $\frac{5}{8}$ inch deep. The store also sells a carrying case for cassettes that has an inside length of 15 inches. How many cassettes in boxes can the carrying case hold?
(49)



29. Draw a rectangle that is $1\frac{1}{2}$ inches long and 1 inch wide. What is the perimeter of the rectangle?
(27)

30. Instead of dividing $2\frac{1}{2}$ by $\frac{1}{4}$, Sandra made an equivalent division problem with whole numbers by multiplying the dividend and the divisor by 4. What equivalent problem did she make, and what is the quotient?
(42)

LESSON 54

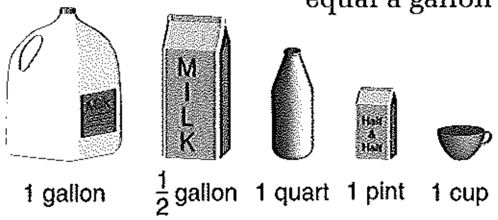
Common Denominators, Part 1

Facts Practice: 28 Fractions to Simplify (Test I in Test Masters)

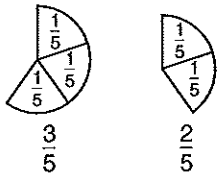
Mental Math: Count by 7's from 7 to 84.

a. 8×325 **b.** $329 + 50$ **c.** $375 - 99$
d. $12.50 − $5.00 **e.** Double $3\frac{1}{2}$ **f.** $\frac{600}{20}$
g. $8 \times 5, + 2, \div 6, \times 7, + 7, \div 8, \times 4, \div 7$

Problem Solving: Half of a gallon is a half gallon. Half of a half gallon is a quart. Half of a quart is a pint. Half of a pint is a cup. How many cups of water equal a gallon of water?

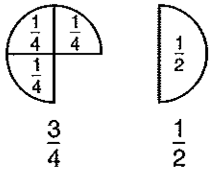


When the denominators of two or more fractions are equal, we say that the fractions have **common denominators.** The fractions $\frac{3}{5}$ and $\frac{2}{5}$ have common denominators.



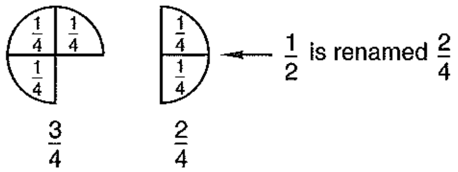
The common denominator is 5.

The fractions $\frac{3}{4}$ and $\frac{1}{2}$ do not have common denominators because the denominators 4 and 2 are not equal.



These fractions do not have common denominators.

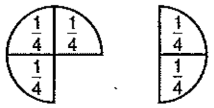
Fractions that do not have common denominators can be renamed to make fractions that do have common denominators. Since $\frac{2}{4}$ equals $\frac{1}{2}$, we may rename $\frac{1}{2}$ as $\frac{2}{4}$. The fractions $\frac{3}{4}$ and $\frac{2}{4}$ have common denominators.



$\frac{1}{2}$ is renamed $\frac{2}{4}$

The common denominator is 4.

Fractions that have common denominators can be added by counting the number of parts, that is, by adding the numerators.



$$\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$$

To add or subtract fractions that do not have common denominators, we rename one or more fractions to make fractions that do have common denominators. Then we add or subtract. Recall that we rename fractions by multiplying the fractions to be renamed by a fraction equal to 1. Here we rename $\frac{1}{2}$ by multiplying $\frac{1}{2}$ by $\frac{2}{2}$. This forms the equivalent fraction $\frac{2}{4}$, which can be added to $\frac{3}{4}$.

Rename $\frac{1}{2}$.

$$\begin{array}{rcccl} & \frac{1}{2} \times \frac{2}{2} & = & \frac{2}{4} & \\ + & \frac{3}{4} & = & \frac{3}{4} & \\ \hline & & & \frac{5}{4} & = 1\frac{1}{4} \end{array}$$

Then add.

Simplify your answer, if possible.

To find the common denominator of two fractions, we find a common multiple of the denominators. The least common multiple of denominators is the lowest common denominator of the fractions.

Example $\frac{1}{2} - \frac{1}{6}$

Solution The denominators are 2 and 6. The least common multiple of 2 and 6 is 6. So 6 is a common denominator of the two fractions. We change halves to sixths by multiplying by $\frac{3}{3}$. We do not need to rename $\frac{1}{6}$.

Rename $\frac{1}{2}$.

$$\begin{array}{rcccl} \frac{1}{2} & \times \frac{3}{3} & = & \frac{3}{6} & \\ -\frac{1}{6} & & = & \frac{1}{6} & \\ \hline & & & \frac{2}{6} & = \frac{1}{3} \end{array}$$

Subtract $\frac{1}{6}$ from $\frac{3}{6}$.

Reduce.

Practice

a. $\begin{array}{r} \frac{1}{2} \\ +\frac{3}{8} \\ \hline \end{array}$

b. $\begin{array}{r} \frac{3}{8} \\ +\frac{1}{4} \\ \hline \end{array}$

c. $\begin{array}{r} \frac{3}{4} \\ +\frac{1}{8} \\ \hline \end{array}$

d. $\begin{array}{r} \frac{1}{2} \\ -\frac{1}{4} \\ \hline \end{array}$

e. $\begin{array}{r} \frac{5}{8} \\ -\frac{1}{4} \\ \hline \end{array}$

f. $\begin{array}{r} \frac{3}{4} \\ -\frac{3}{8} \\ \hline \end{array}$

Problem set 54

1. (52) In the decimals chart, the memory cue for dividing by a whole number is "up." What does that mean?

2. (49) How many $\frac{3}{8}$-inch-thick CD holders will fit on a 12-inch-long shelf? (Write and solve a fraction division problem to answer the question.)

3. (15) The average pumpkin weighs 6 pounds. The Great Pumpkin weighs 324 pounds. The Great Pumpkin weighs as much as how many average pumpkins?

4. (54) $\frac{1}{8} + \frac{1}{2}$

5. (54) $\begin{array}{r} \frac{1}{2} \\ -\frac{1}{8} \\ \hline \end{array}$

6. (54) $\begin{array}{r} \frac{2}{3} \\ -\frac{1}{6} \\ \hline \end{array}$

7. (38) $6.28 + 4 + 0.13$

8. (48) $81 \div 0.9$

9. (51) $0.2 \div 10$

10. (46) $(0.17)(100)$

11. (42) $x + \frac{3}{4} = 3\frac{1}{4}$

12. (29) $\frac{5}{6} \cdot \frac{2}{3}$

13. (41) $\frac{5}{8} = \frac{?}{24}$

14. (32) Write the following in standard notation:

$$(6 \times 10{,}000) + (4 \times 100) + (2 \times 10)$$

15. (50) Multiply 0.14 and 0.8 and round the product to the nearest hundredth.

16. (29) Compare: $\frac{2}{3} \bigcirc \frac{2}{3} \times \frac{2}{2}$

17. (17) Which of these fractions is closest to 1?

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{3}{4}$

18. (38) A 20-foot rope was used to make a square. How many square feet of area are enclosed by the rope?

19. (29) What fraction of a dollar is six dimes?

20. (30) What is the least common multiple (LCM) of 3 and 4?

21. (19) List the factors of 23.

22. (15) How many 12's are in 1212?

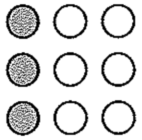
23. (30) By what fraction should $\frac{2}{5}$ be multiplied to make the product 1?

24. (7) Compare: 2 cm ○ 1 in.

25. (53) How many $\frac{2}{5}$'s are in $\frac{1}{2}$? $\left(\frac{1}{2} \div \frac{2}{5}\right)$

26. *(31)* Draw a rectangle that is $1\frac{1}{2}$ inches long and 1 inch wide. What is the area of the rectangle?

27. *(29)* What fraction of this group of circles is shaded?



28. *(53)* Reduce: $\frac{2 \cdot 3 \cdot 2 \cdot 5 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 5}$

29. *(32)* The performance began at 7:45 p.m. and concluded at 10:25 p.m. How many hours and minutes long was the performance?

30. *(52)* Draw the decimals chart from Lesson 52.

LESSON 55

Common Denominators, Part 2

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by 25's between 25 and 300.

a. 8×250 **b.** $462 - 350$ **c.** $150 + 49$

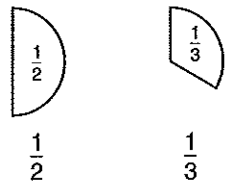
d. \$3.75 + \$4.50 **e.** $\frac{1}{2}$ of 15 **f.** 30×40

g. $10 \times 8, + 1, \div 9, \times 3, + 1, \div 4, \times 6, - 2, \div 5$

Problem Solving: Copy this problem and fill in the missing digits.

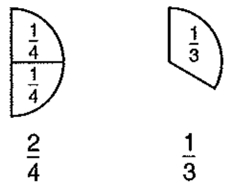
$$\begin{array}{r} \_56 \\ + \;\; \_\_ \\ \hline \_000 \end{array}$$

In Lesson 54 we added and subtracted fractions which required us to rename one of the fractions so that the fractions would have common denominators. In this lesson we will rename both fractions before we add or subtract. To add $\frac{1}{2}$ and $\frac{1}{3}$ we cannot simply count the number of parts because the parts are not the same size—the denominators are different.



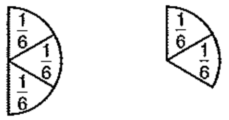
These fractions do not have common denominators.

Renaming $\frac{1}{2}$ as $\frac{2}{4}$ does not help us to add because the parts still are a different size.



These fractions do not have common denominators.

We need to rename both fractions in order to have two fractions whose parts are the same size.



$\frac{3}{6}$ $\frac{2}{6}$

The common denominator is 6.

Often both fractions of an addition or subtraction problem need to be renamed. The least common multiple of the denominators may be used as the common denominator of the renamed fractions.

Example 1 $\frac{1}{2} + \frac{1}{3}$

Solution The denominators are 2 and 3. The least common multiple of 2 and 3 is 6. We rename each fraction so that 6 is the common denominator. Then we add. The sum is $\frac{5}{6}$.

$$\xrightarrow{\text{Rename } \frac{1}{2} \text{ and } \frac{1}{3}.}$$

$$\left.\begin{array}{r} \frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \\ \frac{1}{3} \times \frac{2}{2} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}\right\downarrow \text{Add.}$$

Example 2 $\begin{array}{r} \frac{3}{4} \\ -\frac{2}{3} \\ \hline \end{array}$

Solution The least common multiple of 4 and 3 is 12. We rename both fractions so that their denominators are 12. Then we subtract. The difference is $\frac{1}{12}$.

$$\xrightarrow{\text{Rename } \frac{3}{4} \text{ and } \frac{2}{3}.}$$

$$\left.\begin{array}{r} \frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \\ \frac{2}{3} \times \frac{4}{4} = \frac{8}{12} \\ \hline \frac{1}{12} \end{array}\right\downarrow \text{Subtract.}$$

Practice a. $\begin{array}{r} \frac{2}{3} \\ -\frac{1}{2} \\ \hline \end{array}$ b. $\begin{array}{r} \frac{1}{4} \\ +\frac{2}{5} \\ \hline \end{array}$ c. $\begin{array}{r} \frac{3}{4} \\ -\frac{1}{3} \\ \hline \end{array}$ d. $\begin{array}{r} \frac{2}{3} \\ +\frac{1}{4} \\ \hline \end{array}$ e. $\begin{array}{r} \frac{1}{3} \\ -\frac{1}{4} \\ \hline \end{array}$

Problem set 55

1. (55) Add $\frac{1}{4}$ and $\frac{1}{3}$. Make the common denominator 12.

2. (55) Subtract $\frac{1}{3}$ from $\frac{1}{2}$. Make the common denominator 6.

3. (29) Of the 88 keys on a piano, 52 are white. What fraction of a piano's keys are white?

4. (29) If $4\frac{1}{2}$ apples are needed to make an apple pie, how many apples would be needed to make two apple pies?

5. (55) Subtract $\frac{1}{4}$ from $\frac{2}{3}$. Make the common denominator 12.

6. (54) Add $\frac{1}{3}$ and $\frac{1}{6}$. Reduce your answer.

7. (54) Subtract $\frac{1}{2}$ from $\frac{5}{6}$. Reduce your answer.

8. (5) \$3 + \$1.75 + 65¢

9. (39) (0.625)(0.4)

10. (48) $24 \div 0.08$

11. (47) $3\frac{1}{8} - 1\frac{7}{8}$

12. (29) $\frac{5}{8} \cdot \frac{2}{3}$

13. (22,33) Moe answered 40% of the 100 questions correctly. Write 40% as a reduced fraction. How many questions did Moe answer correctly?

14. (45) Write the following as a decimal number:

$$(8 \times 10) + \left(6 \times \frac{1}{10}\right) + \left(5 \times \frac{1}{100}\right)$$

15. (16) Estimate the sum of 3627 and 4187 to the nearest hundred.

47

16. (49) Which of these numbers is between 1 and 2?

A. 1.875 B. 2.01 C. 0.15

17. (18) What is the average of 1.2, 1.3, 1.4, and 1.5?

18. (38) The perimeter of a square is 36 inches. What is its area?

19. (29) Returning from the store, Dad found that four of the dozen eggs were cracked. What fraction of the eggs were cracked?

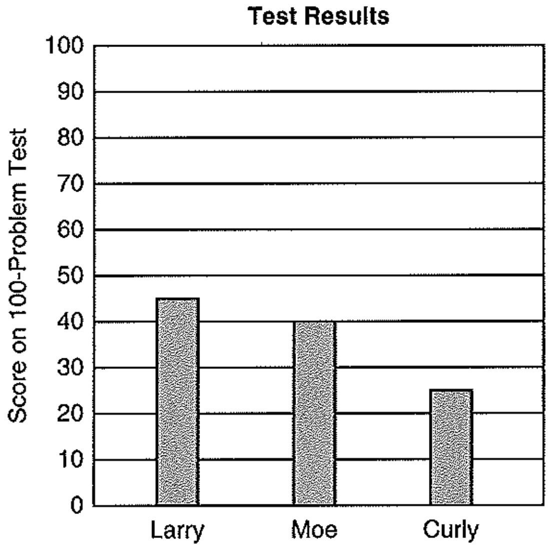
Find a number for each letter to make each equation true:

20. (42) $\frac{2}{3}w = 0$

21. (30) $\frac{2}{3}m = 1$

22. (42) $\frac{2}{3} - n = 0$

Use this graph of test results to answer questions 23, 24, 25, and 26.



23. (16) How many problems did Moe miss on the test?

24. (16) How many more problems did Larry have right on the test than Curly?

25. (29) What fraction of the questions on the test did Curly answer correctly?

26. (29) Write a percent question that refers to the bar graph and answer the question.

27. (53) $\frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 5 \cdot 7}$

28. (53) How many $\frac{2}{3}$'s are in $\frac{1}{2}$? $\left(\frac{1}{2} \div \frac{2}{3}\right)$

29. (7) Draw three rectangles that are two centimeters long and one centimeter wide. Show three different ways to divide the rectangle in half. Then shade half of each rectangle.

30. (31) What is the area of the shaded part of one of the rectangles you drew in problem 29?

LESSON 56

Adding and Subtracting Fractions: Three Steps

Facts Practice: 72 Mixed Multiplication and Division (Test H in Test Masters)

Mental Math: Count by 12's from 12 to 144.

a. 8 × 425
b. 465 + 250
c. 150 − 49
d. \$9.75 − \$3.50
e. Double $4\frac{1}{2}$
f. $\frac{600}{30}$
g. 2 × 2, × 2, × 2, × 2, × 2, ÷ 8, ÷ 8

Problem Solving: Brad was thinking of two different two-digit odd numbers whose average was 15. Find two pairs of numbers of which Brad could have been thinking.

To solve a fraction problem there are three steps to consider:

Step 1. We need to be sure that the problem is in the right **shape** or form. When adding or subtracting fractions, the correct form is with common denominators.

Step 2. We perform the **operation** indicated—we add, subtract, multiply, or divide.

Step 3. We **simplify** the answer, if necessary, by reducing the fraction or by writing an improper fraction as a mixed number.

Example 1 $\frac{1}{2} + \frac{2}{3}$

Solution We will identify the shape, operate, and simplify steps:

Step 1. Shape—Write the fractions with common denominators.

Step 2. Operate—Add the renamed fractions.

Step 3. Simplify—Convert the improper fraction to a mixed number.

$$\begin{array}{rcccc} & \frac{1}{2} \times \frac{3}{3} & = & \frac{3}{6} & \\ + & \frac{2}{3} \times \frac{2}{2} & = & \frac{4}{6} & \\ \hline & & & \frac{7}{6} & = \mathbf{1\frac{1}{6}} \end{array}$$

Example 2 $\frac{1}{2} - \frac{1}{6}$

Solution

Step 1. Shape—Write fractions with common denominators.

Step 2. Operate—Subtract the renamed fractions.

Step 3. Simplify—Reduce the fraction.

$$\begin{array}{rcccc} & \frac{1}{2} \times \frac{3}{3} & = & \frac{3}{6} & \\ - & \frac{1}{6} & = & \frac{1}{6} & \\ \hline & & & \frac{2}{6} & = \mathbf{\frac{1}{3}} \end{array}$$

Practice*

a. $\frac{1}{2} + \frac{1}{6}$

b. $\frac{2}{3} + \frac{3}{4}$

c. $\frac{1}{5} + \frac{3}{10}$

d. $\frac{5}{6} - \frac{1}{2}$

e. $\frac{7}{10} - \frac{1}{2}$

f. $\frac{5}{12} - \frac{1}{6}$

Problem set 56

1. (29) What is the difference between the sum of $\frac{1}{2}$ and $\frac{1}{2}$ and the product of $\frac{1}{2}$ and $\frac{1}{2}$?

2. (13) Thomas Jefferson was born in 1743. How old was he when he was elected president of the United States in 1800?

3. (55) Subtract $\frac{3}{4}$ from $\frac{5}{6}$. Make the common denominator 12.

4. (56) $\frac{1}{2} + \frac{2}{3}$

5. (56) $\frac{1}{2} + \frac{1}{6}$

6. (56) $\frac{5}{6} + \frac{2}{3}$

7. (53) How many $\frac{3}{5}$'s are in $\frac{3}{4}$? $\left(\frac{3}{4} \div \frac{3}{5}\right)$

8. (51) \$32.50 ÷ 10

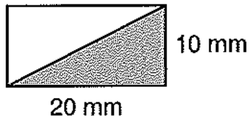
9. (5) $2 - (1 - 0.2)$

10. (48) $6 \div 0.12$

11. (47) $5\frac{3}{8} - 2\frac{5}{8}$

12. (29) $\frac{3}{4} \cdot \frac{5}{3}$

13. (31,33) Fifty percent of this rectangle is shaded. Write 50% as a reduced fraction. What is the area of the shaded part of the rectangle?



14. (34) Name the place value of the 7 in 3.567.

15. (50) Divide 0.5 by 4 and round the quotient to the nearest tenth.

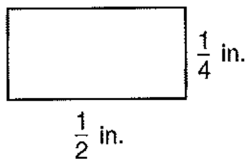
16. (43) Rearrange these numbers in order from least to greatest:

0.3, 3.0, 0.03

17. (10) What is the twentieth number in this sequence?

2, 4, 6, 8, ...

18. (56) What is the perimeter of the rectangle?



19. (31) Multiply the length and width of the rectangle to find the area of the rectangle in square inches.

20. (22) What number is $\frac{5}{8}$ of 80?

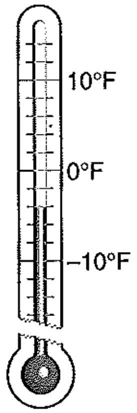
21. (19) List the factors of 29.

22. (30) What is the least common multiple of 12 and 18?

23. (21) Compare: $\frac{5}{8} \bigcirc \frac{7}{8}$

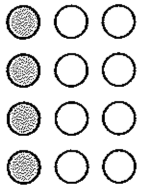
24. (10) What temperature is shown on the thermometer?

25. (14) If the temperature rose from the temperature shown on the thermometer to 12°F, then how many degrees did the temperature rise?



26. (53) Reduce: $\frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 5 \cdot 5 \cdot 7 \cdot 7}$

27. (29) What fraction of the group of circles is shaded?



28. (49) Cheryl has a 6-inch stack of CD's on the shelf. Each CD is in a $\frac{3}{8}$-inch-thick plastic holder. How many CD's are in the 6-inch stack? (Write and solve a fraction division problem to answer the question.)

29. (55) Subtract $\frac{1}{2}$ from $\frac{4}{5}$. Make the common denominator 10.

30. (33,35) Three of these numbers are equivalent. Which one is different?

A. $\frac{1}{5}$ B. $\frac{1}{2}$ C. 0.5 D. 50%

LESSON 57

Comparing Fractions by Renaming with Common Denominators

Facts Practice: 28 Fractions to Simplify (Test I in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 2.

a. 2 × 75 **b.** 315 − 150 **c.** 250 + 199
d. \$7.50 + \$12.50 **e.** $\frac{1}{2}$ of 25 **f.** 20 × 50
g. 10 × 10, − 1, ÷ 11, × 8, + 3, ÷ 3, ÷ 5

Problem Solving: What are the next three numbers in this sequence?

$\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \_\_\_, \_\_\_, \_\_\_, \ldots$

To compare fractions that have common denominators, we simply compare the numerators.

$$\frac{4}{6} < \frac{5}{6}$$

One way to compare fractions that do not have common denominators is to rename one or both fractions so that they do have common denominators.

Example 1 Compare: $\frac{3}{8} \bigcirc \frac{1}{2}$

Solution We rename $\frac{1}{2}$ so that the denominator is 8.

$$\frac{1}{2} \cdot \frac{4}{4} = \frac{4}{8}$$

We see that $\frac{3}{8}$ is less than $\frac{4}{8}$.

$$\frac{3}{8} < \frac{4}{8}$$

Therefore, $\frac{3}{8}$ is less than $\frac{1}{2}$.

$$\mathbf{\frac{3}{8} < \frac{1}{2}}$$

Example 2 Compare: $\frac{2}{3} \bigcirc \frac{3}{4}$

Solution The denominators are 3 and 4. We rename both fractions with a common denominator of 12.

$$\frac{2}{3} \cdot \frac{4}{4} = \frac{8}{12} \qquad \frac{3}{4} \cdot \frac{3}{3} = \frac{9}{12}$$

We see that $\frac{8}{12}$ is less than $\frac{9}{12}$.

$$\frac{8}{12} < \frac{9}{12}$$

Therefore, $\frac{2}{3}$ is less than $\frac{3}{4}$.

$$\mathbf{\frac{2}{3} < \frac{3}{4}}$$

Practice Before comparing the fractions, write each pair of fractions with common denominators.

a. $\frac{2}{3} \bigcirc \frac{1}{2}$ **b.** $\frac{4}{6} \bigcirc \frac{3}{4}$ **c.** $\frac{2}{3} \bigcirc \frac{3}{5}$

Problem set 57

1. *(55)* What is the difference between the sum and product of $\frac{1}{2}$ and $\frac{1}{3}$?

2. *(22)* The flat of eggs held $2\frac{1}{2}$ dozen eggs. How many eggs are in $2\frac{1}{2}$ dozen?

3. (18) In three nights Rumpelstiltskin spun \$44,400 worth of gold thread. What was the average value of thread he spun each night?

4. (57) Compare: $\frac{5}{8} \bigcirc \frac{1}{2}$

5. (57) Compare: $\frac{4}{3} \bigcirc \frac{5}{4}$

6. (42) $m + \frac{3}{8} = \frac{1}{2}$

7. (56) $\frac{2}{3} + \frac{3}{4}$

8. (42) $3 - f = \frac{5}{6}$

9. (46) $\$32.50 \times 10$

10. (39) $(6.2)(0.48)$

11. (48) $1.0 \div 0.8$

12. (48) $120 \div 0.5$

13. (30) $\frac{7}{8} \cdot \frac{8}{7}$

14. (29) $\frac{5}{6} \cdot \frac{3}{4}$

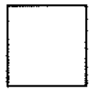
15. (42) Instead of dividing $7\frac{1}{2}$ by $1\frac{1}{2}$, Julie doubled both numbers, then divided mentally. What was the division problem Julie did mentally, and what was the quotient?

16. (50) Round 36.486 to the nearest hundredth.

17. (10) What number is next in this sequence?

0.6, 0.7, 0.8, 0.9, _____, ...

18. (38) The perimeter of this square is 4 cm. What is the area of this square?



19. (53) How many $\frac{3}{5}$'s are in $\frac{3}{4}$? $\left(\frac{3}{4} \div \frac{3}{5}\right)$

20. (42) $0.32w = 32$

21. (42) $x + 3.4 = 5$

22. (19) List the factors of 27.

23. (7) Arrange in order from shortest to longest:

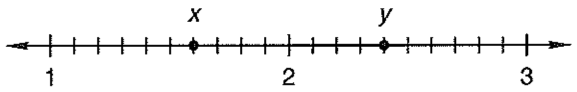
1 in., 3 cm, 20 mm

24. (29,33) Larry correctly answered 45% of the 100 questions. Write 45% as a reduced fraction. How many questions did Larry answer correctly?

25. Describe how to mentally calculate $\frac{1}{10}$ of \$12.50.
(51)

26. Reduce: $\frac{2 \cdot 5 \cdot 2 \cdot 3 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7}$
(53)

27. What is the sum of the decimal numbers represented by points x and y on this number line?
(49)



28. Draw a rectangle that is $1\frac{1}{2}$ inches long and $\frac{3}{4}$ inch wide. Then draw a segment that divides the rectangle into two triangles.
(7)

29. What is the perimeter of the rectangle drawn in problem 28?
(8)

30. If $l = 1.5$ and $w = 0.75$, then what does lw equal?
(45)

LESSON 58

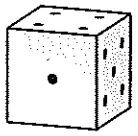
Adding Mixed Numbers

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by 3's between 3 and 36.

a. 4×75 **b.** $279 + 350$ **c.** $250 - 199$
d. \$15.00 − \$7.75 **e.** Double \$1.50 **f.** $\frac{800}{40}$
g. $4 \times 12, \div 6, \times 8, - 4, \div 6, \times 3, \div 2$

Problem Solving: A number cube has six faces. Where two faces meet there is an edge. How many edges does a number cube have?



We have been practicing adding mixed numbers since Lesson 27. In this lesson we will rename the fraction parts of the mixed numbers so that the fractions have common denominators. Then we will add.

Example 1 $2\frac{1}{2} + 1\frac{1}{6}$

Solution

Step 1. Shape—Write the fractions with common denominators.

Step 2. Operate—Add the fractions and add the whole numbers.

Step 3. Simplify—Reduce the fraction.

$$\begin{array}{rcccrcr} 2\frac{1}{2} & \times & \frac{3}{3} & = & 2\frac{3}{6} & & \\ +\ 1\frac{1}{6} & & & = & 1\frac{1}{6} & & \\ \hline & & & & 3\frac{4}{6} & = & \mathbf{3\frac{2}{3}} \end{array}$$

Example 2 $1\frac{1}{2} + 2\frac{2}{3}$

Solution

Step 1. Shape—Write the fractions with common denominators.

Step 2. Operate—Add the fractions and add the whole numbers.

Step 3. Simplify—Convert the improper fraction to a mixed number and combine the mixed number with the whole number.

$$\begin{array}{rcccrcr} 1\frac{1}{2} & \times & \frac{3}{3} & = & 1\frac{3}{6} & & \\ +\ 2\frac{2}{3} & \times & \frac{2}{2} & = & 2\frac{4}{6} & & \\ \hline & & & & 3\frac{7}{6} & = & 3 + 1\frac{1}{6} \\ & & & & & = & \mathbf{4\frac{1}{6}} \end{array}$$

Practice **a.** $1\frac{1}{2} + 1\frac{1}{3}$ **b.** $1\frac{1}{2} + 1\frac{2}{3}$ **c.** $5\frac{1}{3} + 2\frac{1}{6}$

d. $3\frac{3}{4} + 1\frac{1}{3}$

e. $5\frac{1}{2} + 3\frac{1}{6}$

f. $7\frac{1}{2} + 4\frac{5}{8}$

> **Teacher Note:** "Activity Master 5" may be helpful to students in Lesson 61. Refer to the activity in that lesson to determine if copies need to be prepared.

Problem set 58

1. *(40)* What is the product of the decimal numbers four tenths and four hundredths?

2. *(32)* Larry looked at the clock. It was 9:45 p.m. His book report was due the next morning at 8:30. How many hours and minutes were there until Larry's book report was due?

3. *(12)* Pluto's greatest distance from the sun is seven billion, four hundred million kilometers. Write that number.

4. *(58)* $2\frac{1}{2} + 1\frac{1}{6}$

5. *(58)* $1\frac{1}{2} + 2\frac{2}{3}$

6. *(57)* Compare: $\frac{1}{2} \bigcirc \frac{3}{5}$

7. *(57)* Compare: $\frac{2}{3} \bigcirc \frac{6}{9}$

8. *(47)* $8\frac{1}{5} - 3\frac{4}{5}$

9. *(29)* $\frac{3}{4} \cdot \frac{5}{2}$

10. *(53)* How many $\frac{1}{2}$'s are in $\frac{2}{5}$? $\left(\frac{2}{5} \div \frac{1}{2}\right)$

11. *(39)* $(0.875)(40)$

12. *(44)* $0.07 \div 4$

13. *(42)* $30 \div d = 0.6$

14. *(49)* What number is halfway between 0.1 and 0.24?

15. *(16)* Round 36,428,591 to the nearest million.

16. *(14)* What temperature is 23°F less than 8°F?

17. *(10)* What number is missing in this sequence?

320, 160, 80, _____, 20, 10, 5, ...

18. *(38)* How many square inches are needed to cover a square foot?

19. *(51)* One centimeter is what fraction of one meter?

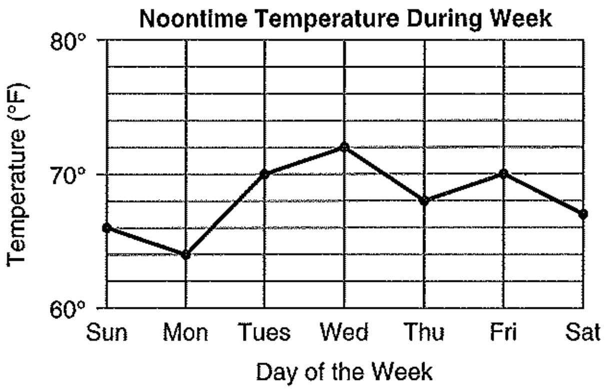
Mentally calculate the answers to problems 20 and 21:

20. *(46)* 6.25×10

21. *(51)* $6.25 \div 10$

22. *(9)* Compare: $32 \div 10 \div 10$ ◯ $32 \div (10 \div 10)$

Use the chart below to answer questions 23, 24 and 25.



23. *(18)* What was the difference in degrees between the highest and lowest noontime temperatures during the week?

24. *(18)* What was the Saturday noontime temperature?

25. *(18)* Write a question that refers to this line graph and answer the question.

26. *(49)* Rumpelstiltskin could pronounce his name in six tenths of a second. At that rate, how many times could he pronounce his name in 15 seconds? (Write and solve a fraction division problem to answer the question.)

27. *(27)* One eighth is equivalent to $12\frac{1}{2}\%$. To what percent is three eighths equivalent?

28. (46) Mentally calculate the total cost of exactly 10 gallons of gas priced at \$1.599 per gallon.

29. (57) Arrange these three numbers in order from least to greatest:

$$\frac{3}{4}, \text{ the reciprocal of } \frac{3}{4}, 1$$

30. (45) If $l = 4$ and $w = 3$, then what does $2l + 2w$ equal?

LESSON 59

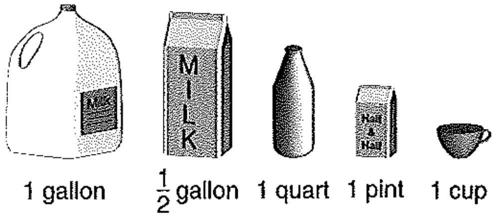
Adding Three or More Fractions

Facts Practice: 28 Fractions to Simplify (Test I in Test Masters)

Mental Math: Count by 7's from 7 to 84.

a. 2×750 **b.** $429 - 250$ **c.** $750 + 199$
d. \$9.50 + \$1.75 **e.** $\frac{1}{2}$ of \$5 **f.** 40×50
g. $12 \times 3, + 4, \times 2, + 20, \div 10, \times 5, \div 2$

Problem Solving: Half of a gallon is a half gallon. Half of a half gallon is a quart. Half of a quart is a pint. Half of a pint is a cup. Into an empty gallon container is poured a half gallon of water, plus a quart of water, plus a pint of water, plus a cup of water. How much more water is needed to fill the gallon container?



To add three or more fractions we find a common denominator for all the fractions being added. The lowest common denominator is the least common multiple of all of the denominators. When we know what the common denominator is we can rename the fractions and add.

Example 1 $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

Solution First we find a common denominator. The LCM of 2, 4, and 8 is 8. We rename all fractions as eighths. Then we add and simplify if possible.

$$\begin{array}{rcccc} & \frac{1}{2} \times \frac{4}{4} & = & \frac{4}{8} \\ & \frac{1}{4} \times \frac{2}{2} & = & \frac{2}{8} \\ + & \frac{1}{8} \times \frac{1}{1} & = & \frac{1}{8} \\ \hline & & & \mathbf{\frac{7}{8}} \end{array}$$

Example 2 $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{1}{6}$

Solution A common denominator is 6. We rename all fractions. We add whole numbers and fractions. We simplify when possible.

$$\begin{array}{rcccccc} & 1\frac{1}{2} \times \frac{3}{3} & = & 1\frac{3}{6} & & \\ & 2\frac{1}{3} \times \frac{2}{2} & = & 2\frac{2}{6} & & \\ + & 3\frac{1}{6} \times \frac{1}{1} & = & 3\frac{1}{6} & & \\ \hline & & & 6\frac{6}{6} & = & \mathbf{7} \end{array}$$

Practice

a. $\frac{1}{2} + \frac{3}{4} + \frac{1}{8}$

b. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

c. $1\frac{1}{2} + 1\frac{1}{3} + 1\frac{1}{4}$

d. $\frac{1}{2} + \frac{2}{3} + \frac{5}{6}$

e. $\frac{1}{2} + \frac{3}{4} + \frac{7}{8}$

f. $1\frac{1}{4} + 1\frac{1}{8} + 1\frac{1}{2}$

Problem set 59

1. *(15)* What is the cost per ounce for a 42-ounce box of oatmeal priced at $1.26?

2. *(13)* There are 30 days in November. How many days are there from November 19 to December 25?

3. *(12)* The smallest three-digit number is 100. What is the largest three-digit number?

4. *(59)* $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$

5. *(59)* $1\frac{1}{2} + 2\frac{2}{3} + 3\frac{1}{6}$

6. *(42)* $m + 1\frac{3}{5} = 5$

7. *(42)* $6\frac{1}{8} - w = 3\frac{5}{8}$

8. *(57)* Compare: $\frac{5}{8}$ ○ $\frac{3}{4}$

9. *(29)* $\frac{3}{5} \cdot \frac{1}{3}$

10. *(53)* How many $\frac{1}{3}$'s are in $\frac{3}{5}$? $\left(\frac{3}{5} \div \frac{1}{3}\right)$

11. *(22)* How much money is $\frac{5}{8}$ of $24.00?

12. *(40)* (0.65)(0.14)

13. *(48)* 65 ÷ 0.05

14. *(34)* What is the place value of the 9 in 46.934?

15. *(50)* Round the product of 0.24 and 0.26 to the nearest hundredth.

16. *(18)* What is the average of 1.3, 2, and 0.81?

17. *(10)* What is the sum of the first seven numbers of this sequence?

1, 3, 5, 7, ...

18. *(38)* How many square feet are needed to cover a square yard?

19. *(51)* Ten centimeters is what fraction of one meter?

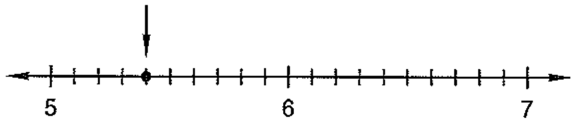
20. *(42)* $3x = 1.2 + 1.2 + 1.2$

21. *(30)* $\frac{4}{3}y = 1$

22. *(19)* The number 37 has how many different factors?

23. *(29)* $\frac{5}{5} \times \left(\frac{4}{4} - \frac{3}{3}\right)$

24. *(49)* To what decimal number is the arrow pointing?



25. *(50)* The decimal number answer to problem 24 rounds to what whole number?

26. *(49)* Mary found that the elm tree in her yard added about $\frac{3}{8}$ of an inch to the diameter of its trunk every year. If the diameter of the tree is about 12 inches, then the tree is about how many years old? (Write and solve a fraction division problem to answer the question.)

27. *(29)* Duncan's favorite T.V. show starts at 8 p.m. and ends at 9 p.m. Duncan timed the commercials and found that there were 12 minutes of commercials between 8 p.m. and 9 p.m. What fraction of the hour was commercial time?

28. *(42)* Instead of dividing 400 by 16, Chip thought of an equivalent division problem that was easier for him to divide. Write an equivalent division problem that has a one-digit divisor and find the quotient.

29. *(50)* Round \$0.6666 to the nearest cent.

30. *(57)* Compare: $3\frac{1}{2}$ ○ $\frac{6}{2} + \frac{1}{2}$

LESSON 60

Writing Mixed Numbers as Improper Fractions

Facts Practice: 72 Mixed Multiplication and Division (Test H in Test Masters)

Mental Math: Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 4.

a. 4×750 **b.** $283 + 250$ **c.** $750 - 199$
d. \$8.25 − \$2.50 **e.** Double $12\frac{1}{2}$ **f.** $\frac{900}{30}$
g. $6 \times 10, \div 3, \times 2, \div 4, \times 5, \div 2, \times 4$

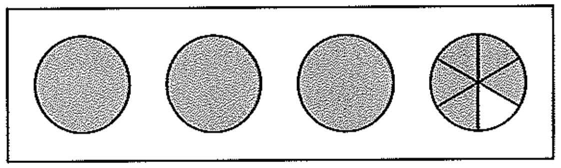
Problem Solving: Copy this problem and fill in the missing digits.

$$\begin{array}{r} \_\,\_ \\ 5\overline{)\_\,\_\,\_} \\ \underline{4\,5} \\ 1\,\_ \\ \underline{\_\,5} \\ 0 \end{array}$$

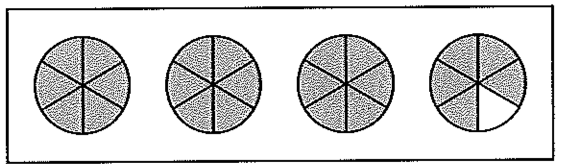
Here is another story about pies. In this story a mixed number is changed to an improper fraction.

> *There were $3\frac{5}{6}$ pies on the shelf. The restaurant manager asked the server to cut the whole pies into serving slices—into sixths. Altogether, how many slices of pie will there be when the server cuts the pies?*

We will illustrate this story with circles. There were $3\frac{5}{6}$ pies on the shelf.



The server will cut the whole pies into sixths. Each whole pie will have six slices.



The three whole pies will make 18 slices ($3 \times 6 = 18$). The five additional slices from the $\frac{5}{6}$ of a pie will make the total 23 slices—23 sixths. This story illustrates that $3\frac{5}{6}$ is equivalent to $\frac{23}{6}$.

Now we will describe the arithmetic for changing a mixed number like $3\frac{5}{6}$ to an improper fraction. Recall that a mixed number has a whole number part and a fraction part.

$$\overset{\text{whole number}\ \downarrow\ \downarrow\ \text{fraction}}{3\frac{5}{6}} \longleftarrow \text{denominator of fraction}$$

The denominator of the mixed number is also the denominator of the improper fraction.

$$\underset{\text{same denominator}}{\underset{\uparrow}{3\frac{5}{6}} = \underset{\uparrow}{\frac{}{6}}}$$

The denominator indicates the size of the fraction "pieces." In this case the fraction pieces are sixths. We change the whole number 3 into sixths. We know that one whole is $\frac{6}{6}$, so three wholes is $3 \times \frac{6}{6}$, which is $\frac{18}{6}$. Now we add $\frac{18}{6}$ and $\frac{5}{6}$, which equals $\frac{23}{6}$.

$$\overbrace{\frac{18}{6} + \frac{5}{6}}^{3\frac{5}{6}} = \frac{23}{6}$$

Example 1 Write $2\frac{3}{4}$ as an improper fraction.

Solution The denominator of the fraction part of the mixed number is fourths, so the denominator of the improper fraction will also be fourths.

$$2\frac{3}{4} = \frac{}{4}$$

We change the whole number 2 into fourths. Since 1 equals $\frac{4}{4}$, the whole number 2 equals $2 \times \frac{4}{4}$, which is $\frac{8}{4}$. We add $\frac{8}{4}$ and $\frac{3}{4}$, which equals $\frac{11}{4}$.

$$2\frac{3}{4} = \mathbf{\frac{11}{4}}$$

Example 2 Write $5\frac{2}{3}$ as an improper fraction.

Solution We see that the denominator of the improper fraction will be thirds.

$$5\frac{2}{3} = \frac{}{3}$$

Some people use a quick, mechanical method to find the numerator of the improper fraction. Looking at the mixed number, they multiply the whole number by the denominator and then add the numerator. The result is the numerator of the improper fraction.

$$5\frac{{}^{+}2}{{}_{\times}3} = \frac{\mathbf{17}}{\mathbf{3}}$$

Practice Write each mixed number as an improper fraction.

a. $2\frac{4}{5}$
b. $3\frac{1}{2}$
c. $1\frac{3}{4}$
d. $6\frac{1}{4}$
e. $1\frac{5}{6}$
f. $3\frac{3}{10}$
g. $2\frac{1}{3}$
h. $12\frac{1}{2}$
i. $3\frac{1}{6}$

Problem set 60

1. *(26)* Convert the improper fraction $\frac{20}{6}$ to a mixed number with the fraction reduced.

2. *(22)* A fathom is 6 feet. How many feet deep is water that is $2\frac{1}{2}$ fathoms?

3. *(18)* After 3 days and 7425 guesses, the queen guessed Rumpelstiltskin's name. What was the average number of names she guessed each day?

4. *(56)* $5\frac{1}{2} - 1\frac{2}{3}$

5. *(56)* $5\frac{1}{3} - 2\frac{1}{2}$

6. (59) $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{1}{4}$

7. (58) $3\frac{3}{4} + 3\frac{1}{3}$

8. (57) Compare: $\frac{2}{3} \bigcirc \frac{3}{5}$

9. (29) $\frac{5}{6} \times 42$

10. (29) $\frac{3}{8} \cdot \frac{2}{3}$

11. (53) How many $\frac{2}{3}$'s are in $\frac{3}{8}$? $\left(\frac{3}{8} \div \frac{2}{3}\right)$

12. (44) $(4 - 0.4) \div 4$

13. (48) $4 - (0.4 \div 4)$

14. (34) Which digit in 49.63 has the same place value as the 7 in 8.7?

15. (16) Estimate the sum of \$642.23 and \$861.17 to the nearest hundred dollars.

16. (41) $\frac{1}{10} = \frac{?}{100}$

17. (10) What is the next number in this sequence?

100, 10, 1, ______, ...

18. (38) The perimeter of a square is 1 foot. How many square inches cover its area?

19. (29) Ten seconds is what fraction of one minute?

20. (4) $15m = 300$

21. (19) List the factors of 50.

22. (41) By what name for 1 must $\frac{2}{3}$ be multiplied to form a fraction with a 15 in the denominator?

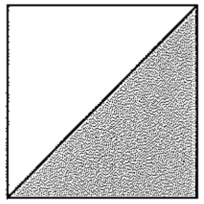
23. (32) What time is 5 hours and 15 minutes after 9:50 a.m.?

24. (60) Write $7\frac{1}{2}$ as an improper fraction.

25. Write $1\frac{1}{3}$ and $1\frac{1}{2}$ as improper fractions and multiply the improper fractions. What is the product?
(60)

26. The sales tax rate was 7%. Write 7% as a fraction. Then write the fraction as a decimal number.
(33)

Refer to the figure to answer problems 27 and 28:



27. The perimeter of this square is 8 cm. How long is each side?
(38)

28. Half of the area of the square is shaded. What is the area of the shaded part of the square?
(38)

29. Rawlings bought a sheet of 100 stamps from the post office for $35.00. What was the price for each stamp?
(51)

30. Describe how to convert $2\frac{1}{3}$ to an improper fraction.
(60)

LESSON 61

Prime Numbers

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count by 12's from 12 to 144.

a. 5 × 40 *(10 × 40 ÷ 2)* **b.** 475 + 1200 **c.** 3 × 84
d. \$8.50 + \$2.50 **e.** $\frac{1}{3}$ of \$36.00 **f.** $\frac{\$25}{10}$
g. 6 × 8, − 4, ÷ 4, × 2, + 2, ÷ 6, ÷ 2
h. Hold your hands one foot apart.

Problem Solving: The average number of students in two classrooms was 27. If the students are separated into three classrooms instead of two classrooms, what will be the average number of students in each of the three classrooms?

Here we list the first ten counting numbers and their factors. Which of the numbers have exactly two factors?

NUMBER	FACTORS
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1, 2, 3, 6
7	1, 7
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10

Numbers that have exactly two factors are **prime numbers.** The first four prime numbers are 2, 3, 5, and 7. The number 1 is not a prime number because it has only one factor, itself. The only factors of a prime number are the number itself and 1. Therefore, to determine whether or not a number is prime, we may ask ourselves the question, "Is this number divisible by any number other than the number itself and 1?" If the number is divisible by any other number, the number is not prime.

Example The first four prime numbers are 2, 3, 5, and 7. What are the next four prime numbers?

Solution We will consider the next several numbers and eliminate those that are not prime.

8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

All even numbers have 2 as a factor. So no even numbers greater than two are prime numbers. We can eliminate the even numbers from the list.

~~8~~, 9, ~~10~~, 11, ~~12~~, 13, ~~14~~, 15, ~~16~~, 17, ~~18~~, 19, ~~20~~

Since 9 is divisible by 3, and 15 is divisible by 3 and by 5, we can eliminate 9 and 15 from the list.

~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~

Each of the remaining four numbers on the list is divisible only by itself and by 1. Thus the next four prime numbers after 7 are **11, 13, 17,** and **19.**

Activity: Prime Numbers

List the counting numbers from 1 to 50 (or use the Hundred Chart, "Activity Master 5," from the Test Masters). Then follow these directions:

a. Draw a line through the number 1. The number 1 is not a prime number.

b. Circle the prime number 2. Draw a line through all the other multiples of 2 (4, 6, 8, etc.).

c. Circle the prime number 3. Draw a line through all the other multiples of 3 (6, 9, 12, etc.).

d. Circle the prime number 5. Draw a line through all the other multiples of 5 (10, 15, 20, etc.).

e. Circle the prime number 7. Draw a line through all the other multiples of 7 (14, 21, 28, etc.).

f. All of the numbers on the list (or chart) from 1 through 50 (or 1 through 100) that do not have a line drawn through them are prime numbers. Circle the prime numbers.

Practice Which number in each group is a prime number?

a. 21, 23, 25

b. 31, 32, 33

c. 43, 44, 45

Which number in each group is not a prime number?

d. 41, 42, 43

e. 31, 41, 51

f. 23, 33, 43

Prime numbers can be multiplied to equal whole numbers that are not prime. To make 12 we multiply $2 \cdot 2 \cdot 3$. To make 15 we multiply $3 \cdot 5$. Show which prime numbers we multiply to make these products.

g. 16 **h.** 18

Problem set 61

1. (53) In music there are whole notes, half notes, quarter notes, and eighth notes. How many quarter notes equal a whole note?

2. (53) How many eighth notes equal a quarter note?

3. (60) Don is 5 feet $2\frac{1}{2}$ inches tall. How many inches tall is that?

4. (61) Which of these numbers is not a prime number?

A. 11 B. 21 C. 31 D. 41

5. *(61)* Which of these numbers is a prime number?

A. 35 B. 41 C. 63 D. 72

6. *(18)* The prices for three pairs of skates were \$36.25, \$41.50, and \$43.75. What was the average price for a pair of skates?

7. *(42)* Instead of dividing 15 by $2\frac{1}{2}$, Solomon doubled both numbers, then divided mentally. What was Solomon's mental division problem and its quotient?

8. *(42)* $m - 4\frac{3}{8} = 3\frac{1}{4}$

9. *(59)* $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$

10. *(54)* $\frac{5}{6} - \frac{1}{2}$

11. *(54)* $\frac{3}{5} - \frac{3}{10}$

12. *(29)* $\frac{1}{2} \cdot \frac{4}{5}$

13. *(53)* $\frac{2}{3} \div \frac{1}{2}$

14. *(40)* $1 - (0.2 - 0.03)$

15. *(38)* $(0.14)(0.16)$

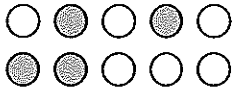
16. *(44)* $\frac{0.456}{6}$

17. *(48)* $\frac{1.5}{0.04}$

18. *(46)* One centimeter equals 10 millimeters. How many millimeters does 2.5 centimeters equal?

19. *(19)* List all of the common factors of 18 and 24 and circle the greatest.

20. *(21)* What fraction of the group is shaded?



21. *(38)* If the perimeter of a square is 40 mm, what is the area of the square?

22. *(14)* At 6 a.m. the temperature was −6°F. At noon the temperature was 14°F. From 6 a.m. to noon the temperature rose how many degrees?

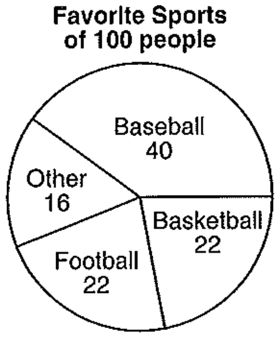
23. Compare: $\frac{3}{3} \times \frac{2}{2} \bigcirc \frac{3}{3} \div \frac{2}{2}$
(49)

Use this chart of favorite sports of 100 people to answer questions 24 through 28.

24. How many more people favored baseball than favored football?
(40)

25. What fraction of the people favored baseball?
(40)

26. Was any sport the favorite sport of the majority of the people surveyed? Write one or two sentences to explain your answer.
(40)



27. Since baseball was the favorite sport of 40 **out of 100** people, it was the favorite sport of 40% of the people asked. What percent of the people answered that football was their favorite sport?
(40)

28. Write a question that refers to this circle graph and answer the question.
(40)

29. Tom thought of one number he could put in each of these boxes to make the equation true. Tom thought of what number?
(38)

$$\square \times \square = 100$$

30. If we multiply the prime numbers $2 \cdot 3 \cdot 3$, the product is 18. Which prime numbers would we multiply to make a product of 20?
(61)

LESSON 62

Subtracting Mixed Numbers with Regrouping, Part 2

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 2.

a. 5 × 140 *(10 × 140 ÷ 2)* **b.** 420 − 50 **c.** 4 × 63
d. \$8.50 − \$2.50 **e.** Double $7\frac{1}{2}$ **f.** $\frac{\$25}{100}$
g. 5 × 10, − 20, + 2, ÷ 4, + 1, ÷ 3, − 3
h. Hold your hands one foot apart; then one inch apart.

Problem Solving: What are the next three numbers in this sequence?

$\frac{1}{12}, \frac{1}{6}, \frac{1}{4}$, ___, ___, ___, ...

Since Lesson 47 we have practiced subtracting mixed numbers with regrouping. In this lesson we will rename the fractions with common denominators before subtracting.

To subtract $1\frac{1}{2}$ from $3\frac{2}{3}$, we first write the fractions with common denominators. Then we subtract the whole numbers and fractions and simplify when possible.

$$\begin{array}{rcccr} & 3\frac{2}{3} & \times & \frac{2}{2} & = 3\frac{4}{6} \\ - & 1\frac{1}{2} & \times & \frac{3}{3} & = 1\frac{3}{6} \\ \hline & & & & 2\frac{1}{6} \end{array}$$

Sometimes when subtracting it is necessary to regroup. We write the fractions with common denominators before regrouping.

Example $5\frac{1}{2} - 1\frac{2}{3}$

Solution We write fractions with common denominators. We regroup if necessary. We simplify if possible.

$$\begin{array}{rcccr} & 5\frac{1}{2} & \times & \frac{3}{3} & = \overset{4}{\cancel{5}}\frac{\overset{9}{\cancel{3}}}{6} \\ - & 1\frac{2}{3} & \times & \frac{2}{2} & = 1\frac{4}{6} \\ \hline & & & & 3\frac{5}{6} \end{array}$$

Practice*

a. $5\frac{1}{2} - 3\frac{1}{3}$ b. $4\frac{1}{4} - 2\frac{1}{3}$ c. $6\frac{1}{2} - 1\frac{3}{4}$

d. $7\frac{2}{3} - 3\frac{5}{6}$ e. $6\frac{1}{6} - 1\frac{1}{2}$ f. $4\frac{1}{3} - 1\frac{1}{2}$

g. $4\frac{5}{6} - 1\frac{1}{3}$ h. $6\frac{1}{2} - 3\frac{5}{6}$ i. $8\frac{2}{3} - 5\frac{3}{4}$

Problem set 62

1. (52) What is the difference between the sum of 0.6 and 0.4 and the product of 0.6 and 0.4?

2. (13) Mt. Whitney, the highest point in California, has an elevation of 14,494 feet above sea level. From there one can see Death Valley, the lowest point, which has an elevation of 282 feet **below** sea level. The floor of Death Valley is how many feet below the peak of Mt. Whitney?

3. (15) The anaconda was 288 inches long. How many feet is 288 inches?

4. (60) Write the mixed number $4\frac{2}{3}$ as an improper fraction.

5. (60) Write $2\frac{1}{2}$ and $1\frac{1}{5}$ as improper fractions. Then multiply the improper fractions and simplify the product.

6. (32) What time is $2\frac{1}{2}$ hours after 10:15 a.m.?

7. (5) $(30 \times 15) \div (30 - 15)$ **8.** (57) Compare: $\frac{5}{8}$ ○ $\frac{2}{3}$

9. (42) $w - 3\frac{2}{3} = 1\frac{1}{2}$ **10.** (54) $\frac{6}{8} - \frac{3}{4}$

11. (62) $6\frac{1}{4} - 5\frac{5}{8}$ **12.** (29) $\frac{3}{4} \times \frac{2}{5}$ **13.** (53) $\frac{3}{4} \div \frac{2}{5}$

14. (39) $(1 - 0.4)(1 + 0.4)$ **15.** (22) $\frac{3}{5}$ of \$45.00

16. (44) 0.4 ÷ 8

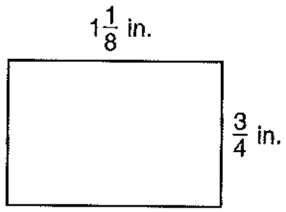
17. (48) 8 ÷ 0.4

18. (10) What number is next in this sequence?

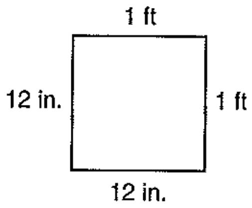
0.2, 0.4, 0.6, 0.8, _____, ...

19. (61) What is the tenth prime number?

20. (8) What is the perimeter of the rectangle?

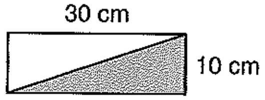


21. (38) This floor tile is one square foot. The kitchen floor was covered with 100 floor tiles. What was the area of the kitchen floor in square inches?



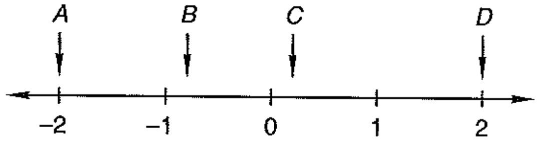
22. (16) Round $678.25 to the nearest ten dollars.

23. (31) What is the area of the shaded part of this rectangle?



24. (15) A ton is 2000 pounds. How many pounds is $2\frac{1}{2}$ tons?

25. (49) Which arrow could be pointing to 0.2 on the number line?



26. (61) Think of a prime number for n that makes this equation true.

$$n \cdot n = 49$$

27. (29,33) Jefferson got a hit 30% of the 240 times he came to bat during the season. Write 30% as a reduced fraction. Then find the number of hits Jefferson got during the season.

28. (42) Dixon has run 11.5 miles of a 26.2-mile race. Find the remaining distance Dixon has to run to complete the race by solving this equation.

$$11.5 \text{ mi} + d = 26.2 \text{ mi}$$

29. (33) Write 7% as a fraction. Then write the fraction as a decimal number.

30. (57) Arrange these numbers in order from least to greatest:

$$1, \tfrac{1}{2}, 0, 1\tfrac{1}{2}, 2$$

LESSON 63

Polygons

Facts Practice: Write 24 Mixed Numbers as Improper Fractions (Test J in Test Masters)

Mental Math: Count up and down by 25's between 25 and 300.

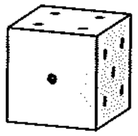
a. 5 × 240 **b.** 4500 + 450 **c.** 7 × 34

d. $7.50 + $7.50 **e.** $\frac{1}{4}$ of $20.00 **f.** $\frac{\$75}{10}$

g. 6 × 8, ÷ 2, + 1, ÷ 5, − 1, × 4, ÷ 2

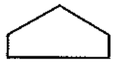
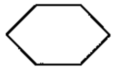
h. Hold your hands one foot apart; then one yard apart.

Problem Solving: A number cube has six faces. Where two faces meet there is an edge. A "corner" where three edges meet is called a *vertex* (plural is *vertices*). A number cube has how many vertices?



Polygons are closed shapes with straight sides. Polygons are named by the number of sides they have. The chart below names some common polygons.

Polygons

Shape	Number of Sides	Name of Polygon
	3	triangle
	4	quadrilateral
	5	pentagon
	6	hexagon
	8	octagon

Two sides of a polygon meet, or **intersect,** at a **vertex** (plural is **vertices**). A polygon has the same number of vertices as it has sides.

Example 1 What is the name of a polygon that has four sides?

Solution The answer is not "square" or "rectangle." Squares and rectangles do have four sides, but not all four-sided polygons are squares or rectangles. The correct answer is **quadrilateral.** A rectangle is one kind of quadrilateral. A square is a rectangle with sides of equal length.

If all the sides of a polygon are the same length and if all the angles are the same measure, then the polygon is called a **regular polygon.** A square is a regular quadrilateral, but a rectangle that is longer than it is wide is not a regular quadrilateral.

Example 2 The area of a square is 25 square centimeters.

(a) What is the length of each side of the square?

(b) What is the perimeter of the square?

Solution (a) We find the area of a square, or of any rectangle, by multiplying the length by the width. With a square, the length and width are equal. Finding the length of a side is like finding the one number that goes into each of these boxes.

$$\square \times \square = 25$$

Since $5 \times 5 = 25$, the length of each side is **5 cm.**

(b) The perimeter of the square is the sum of the lengths of the four sides. We may find the perimeter by adding the lengths of the four sides or by multiplying the length of one side by four.

$$5 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} + 5 \text{ cm} = \mathbf{20\ cm}$$

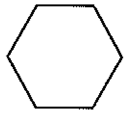
$$4 \times 5 \text{ cm} = \mathbf{20\ cm}$$

Example 3 A **regular octagon** has a perimeter of 96 inches. How long is each side?

Solution An octagon has eight sides. The sides of a regular octagon are the same length. Dividing the perimeter of 96 inches by 8, we find each side is **12 inches.** (Most of the red stop signs on our roads are regular octagons with sides 12 inches long.)

Practice

a. What is the name of this six-sided shape?



b. How many sides does a pentagon have?

c. Can a polygon have 19 sides?

d. The area of a square is 16 square inches. What is its perimeter?

Problem set 63

1. *(52)* When the sum of 1.3 and 1.2 is divided by the difference of 1.3 and 1.2, what is the quotient?

2. *(13)* William Shakespeare was born in 1564 and died in 1616. How many years did he live?

3. *(15)* Robin Hood's arrow hit a target 45 yards away. How many feet did the arrow travel?

4. *(63)* Why is a square a regular quadrilateral?

5. *(63)* A regular hexagon has a perimeter of 36 inches. How long is each side?

6. *(41)* $\frac{1}{4} = \frac{?}{100}$

7. *(5)* $\frac{8 \times 8}{8 + 8}$

8. *(58)* $5\frac{2}{3} + 3\frac{3}{4}$

9. *(59)* $\frac{1}{2} + \frac{2}{3} + \frac{1}{4}$

10. *(56)* $\frac{9}{10} - \frac{1}{2}$

11. *(62)* $6\frac{1}{2} - 2\frac{7}{8}$

12. *(43)* Compare: 2×0.4 ○ $2 + 0.4$

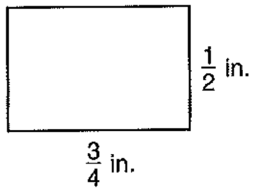
13. *(39)* 4.8×0.35

14. *(48)* $1 \div 0.4$

15. *(15)* How many $0.12 pencils can Mr. Jones buy for $4.80?

16. *(50)* Round the product of 0.33 and 0.38 to the nearest hundredth.

17. Multiply the length by the width to find the area of this rectangle.
(31)



18. Which is the twelfth prime number?
(61)

19. Write $3\frac{4}{5}$ as an improper fraction.
(60)

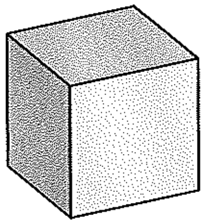
20. The area of a square is 9 sq. cm.
(63)

(a) How long is each side of the square?

(b) What is the perimeter of the square?

21. Five minutes is what fraction of an hour?
(29)

22. The top, bottom, and sides of a box are also called the **faces** of the box. This box has how many faces?
(63)



23. There are 100 centimeters to a meter. How many centimeters equal 2.5 meters?
(15)

24. Write the mixed numbers $1\frac{1}{2}$ and $2\frac{1}{2}$ as improper fractions. Then multiply the improper fractions and simplify the product.
(60)

25. The numbers 2, 3, 5, 7, and 11 are prime numbers. The numbers 4, 6, 8, 9, 10, and 12 are not prime numbers, but they can be formed by multiplying prime numbers.
(61)

$$2 \cdot 2 = 4$$

$$2 \cdot 3 = 6$$

$$2 \cdot 2 \cdot 2 = 8$$

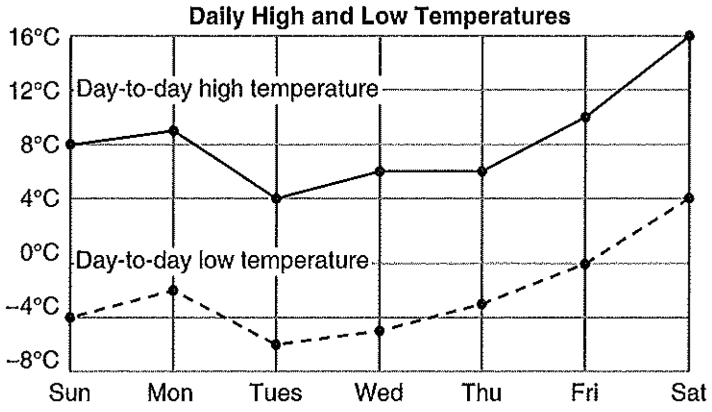
Show how to make 9, 10, and 12 by multiplying prime numbers.

26. Write 75% as an unreduced fraction. Then write the fraction as a decimal number.
(33)

27. Reduce: $\dfrac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 5 \cdot 5}$
(53)

28. 16.6 mi + l = 26.2 mi
(42)

Refer to this double-line graph to find information for problems 29 and 30.



29. The difference between Tuesday's high and low temperatures was how many degrees?
(18)

30. The difference between the lowest temperature of the week and the highest temperature of the week was how many degrees?
(18)

LESSON 64

Prime Factorization • Division by Primes • Factor Tree

Facts Practice: 72 Mixed Multiplication and Division (Test H in Test Masters)

Mental Math: Count by 9's from 9 to 108.

a. 5 × 60
b. 586 − 50
c. 3 × 65
d. \$20.00 − \$2.50
e. Double 75¢
f. $\frac{\$75}{100}$
g. 9 × 9, − 1, ÷ 2, + 2, ÷ 6, + 3, ÷ 10
h. Hold your hands one yard apart; then one meter apart.

Problem Solving: Nelson bought a pizza and ate half of it. Then his sister ate half of what was left. Then his little brother ate half of what his sister had left. What fraction of the pizza did Nelson's little brother eat? Illustrate the portions of the pizza eaten by each family member.

Prime factorization

All whole numbers greater than 1 are either prime numbers or **composite numbers.** A composite number has **more than two** factors. As we studied in Lesson 61, the numbers 2, 3, 5, and 7 are prime numbers. The numbers 4, 6, 8, and 9 are composite numbers. All composite numbers can be made by multiplying prime numbers together.

$$4 = 2 \cdot 2$$

$$6 = 2 \cdot 3$$

$$8 = 2 \cdot 2 \cdot 2 \text{ (also } 2 \cdot 4\text{, but 4 is not prime)}$$

$$9 = 3 \cdot 3$$

When we write a composite number as a product of its prime factors, we have written the **prime factorization** of the number. The prime factorizations of 4, 6, 8, and 9 are shown above.

In this lesson we will show two methods for factoring a composite number. One method is **division by primes,** and the other method is using a **factor tree.** We will illustrate both methods as we factor 60.

Division by primes

The smallest prime number is 2, then 3, then 5, and so on. Since 60 is divisible by 2, we begin by dividing 60 by 2. The quotient is 30.

$$\begin{array}{r} 30 \\ \mathbf{2}\overline{)60} \end{array}$$

Since 30 is also divisible by 2, we divide 30 by 2. The quotient is 15. Notice how we "stack" the divisions.

$$\begin{array}{r} 15 \\ \mathbf{2}\overline{)30} \\ \mathbf{2}\overline{)60} \end{array}$$

Although 15 is not divisible by 2, it is divisible by the next prime number, which is 3. The quotient is 5.

$$\begin{array}{r} 5 \\ \mathbf{3}\overline{)15} \\ \mathbf{2}\overline{)30} \\ \mathbf{2}\overline{)60} \end{array}$$

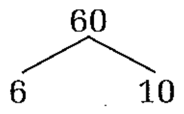
Five is a prime number. The only prime number that divides 5 is 5.

$$\begin{array}{r} 1 \\ \mathbf{5}\overline{)5} \\ \mathbf{3}\overline{)15} \\ \mathbf{2}\overline{)30} \\ \mathbf{2}\overline{)60} \end{array}$$

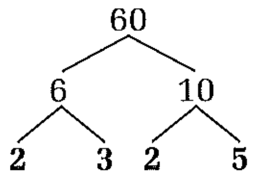
By dividing by prime numbers we have found the prime factorization of 60.

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

Factor tree When using a factor tree, we simply think of any two numbers whose product is 60. Since 6 times 10 equals 60, we will use 6 and 10 as the first two "branches" of the factor tree.



The numbers 6 and 10 are not prime numbers, so we continue the process by factoring 6 into 2 · 3 and by factoring 10 into 2 · 5.

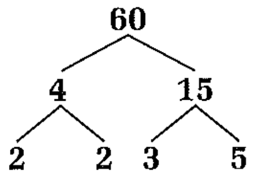


The numbers at the ends of the branches are all prime numbers. We have completed the factor tree. We will arrange the factors in order from least to greatest and write the prime factorization of 60.

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

Example 1 Use a factor tree to find the prime factorization of 60. Use 4 and 15 as the first branches.

Solution Some composite numbers can be divided into many different factor trees. However, when the factor tree is completed, the same prime numbers appear at the ends of the branches.



$$\mathbf{60 = 2 \cdot 2 \cdot 3 \cdot 5}$$

Example 2 Use division by primes to find the prime factorization of 36.

Solution We begin by dividing by the smallest prime number that is a factor of 36, which is 2. We will continue dividing by prime numbers until the quotient is 1.[†]

$$\begin{array}{r} 1 \\ 3\overline{)3} \\ 3\overline{)9} \\ 2\overline{)18} \\ 2\overline{)36} \end{array}$$

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

Practice* **a.** Which of these numbers are composite numbers?

19, 20, 21, 22, 23

b. Write the prime factorization of each composite number in problem (a).

c. Use a factor tree to find the prime factorization of 36.

d. Use division by primes to find the prime factorization of 48.

e. Write 125 as a product of prime factors.

f. Write the prime factorization of 10 and of 100. What similarities do you notice? Can you guess what the prime factorization of 1000 is?

Problem set 64

1. (12) The total land area of the world is about fifty-seven million, two hundred eighty thousand square miles. Write that number.

2. (15) Nelson photographed an African white rhinoceros that stood $6\frac{1}{2}$ feet high. How many inches is $6\frac{1}{2}$ feet?

[†] Some prefer to complete the division when the quotient is a prime number, in which case the final quotient, together with the divisors, is included in the prime factorization of the number.

3. Jenny shot 10 free throws and made 6. What fraction of her shots did she make? What percent of her shots did she make?
(29,41)

4. Draw a factor tree for 40. Then write the prime factorization of 40.
(64)

5. Which of these is a composite number?
(64)

21, 31, 41

6. Write $2\frac{2}{3}$ as an improper fraction and multiply the improper fraction by $\frac{3}{8}$. What is the product?
(60)

7. 10,000 − (10,000 ÷ 10)
(5)

8. $8\frac{1}{2} + 1\frac{1}{3} + 2\frac{1}{6}$
(59)

9. $\frac{1}{12} + \frac{1}{6} + \frac{1}{2}$
(59)

10. $15\frac{3}{4} - m = 2\frac{1}{8}$
(42)

11. Compare: $\frac{1}{2} - \frac{1}{3} \bigcirc \frac{2}{3} - \frac{1}{2}$
(57)

12. $\frac{3}{8} \times \frac{1}{3}$
(29)

13. $\frac{3}{8} \div \frac{1}{2}$
(53)

14. 1 − (0.2 + 0.48)
(38)

15. 0.0144 ÷ 12
(44)

16. What is the total cost of two dozen erasers that are priced at 8¢ each?
(15)

17. The store manager put $20.00 worth of $0.25 pieces in the change drawer. How many $0.25 pieces are in $20.00?
(15)

18. What time is $2\frac{1}{2}$ hours before 1:15 p.m.?
(32)

19. *(64)* Use division by primes to find the prime factorization of 50.

20. *(63)* What is the name of a six-sided polygon? How many vertices does it have?

21. *(60)* Write $3\frac{4}{7}$ as an improper fraction.

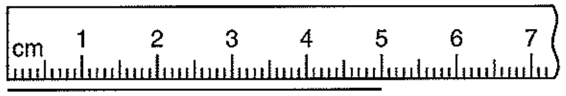
22. *(63)* The area of a square is 36 square inches.

(a) What is the length of each side?

(b) What is the perimeter of the square?

23. *(33)* Write 16% as a reduced fraction.

24. *(7)* How many millimeters long is the line segment?



25. *(7)* A meter is about one **big** step. About how many meters long is an automobile?

26. *(64)* Write the prime factorization of 375 and of 1000.

27. *(53)* Reduce: $\dfrac{3 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5}$

28. *(41)* $\dfrac{4}{25} = \dfrac{?}{100}$

29. *(29,33)* Eighty percent of the 20 answers were correct. Write 80% as a reduced fraction. Then find the number of correct answers.

30. *(63)* The "rect-" part of *rectangle* means "right." A rectangle is a "right-angle" shape. Why is every square also a rectangle?

LESSON 65

Multiplying Mixed Numbers

Facts Practice: Write 24 Mixed Numbers as Improper Fractions (Test J in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 2.

- **a.** 5×160
- **b.** $376 + 99$
- **c.** 8×23
- **d.** \$1.75 + \$1.75
- **e.** $\frac{1}{3}$ of \$60.00
- **f.** $\frac{\$30}{10}$
- **g.** $8 \times 8, - 4, \div 2, + 3, \div 3, + 1, \div 6, \div 2$
- **h.** Hold your hands one meter apart; then one yard apart.

Problem Solving: Copy this problem and fill in the missing digits.

```
    _4_
6 )__6
   __
   __
   __
    __
    3_
     0
```

Recall the three steps to solving an arithmetic problem with fractions.

Step 1. Be sure the problem is in the correct shape.
Step 2. Perform the operation indicated.
Step 3. Simplify the answer if possible.

Remember that the correct shape for adding and subtracting fractions is to write the fraction with common denominators. To multiply or divide fractions we do not need to write the fractions with common denominators. The correct shape for multiplying and dividing fractions is to write the numbers in **fraction form.** This means we will write mixed numbers as improper fractions. We will also write whole numbers in fraction form by writing the whole number as the numerator of a fraction with the denominator 1.

Example 1 $2\frac{2}{3} \times 4$

Solution First, we write $2\frac{2}{3}$ and 4 in fraction form.

$$\frac{8}{3} \times \frac{4}{1}$$

Second, we multiply the numerators to find the numerator of the product, and we multiply the denominators to find the denominator of the product.

$$\frac{8}{3} \times \frac{4}{1} = \frac{32}{3}$$

Third, we simplify the product by converting the improper fraction to a mixed number.

$$\frac{32}{3} = \mathbf{10\frac{2}{3}}$$

Example 2 $2\frac{1}{2} \times 1\frac{1}{3}$

Solution First, write the numbers in fraction form.

$$\frac{5}{2} \times \frac{4}{3}$$

Second, multiply the terms of the fractions.

$$\frac{5}{2} \times \frac{4}{3} = \frac{20}{6}$$

Third, simplify the product.

$$\frac{20}{6} = 3\frac{2}{6}$$

$$3\frac{2}{6} = \mathbf{3\frac{1}{3}}$$

Practice* **a.** $1\frac{1}{2} \times \frac{2}{3}$ **b.** $1\frac{2}{3} \times \frac{3}{4}$ **c.** $1\frac{1}{2} \times 1\frac{2}{3}$

d. $1\frac{2}{3} \times 3$ **e.** $2\frac{1}{2} \times 2\frac{2}{3}$ **f.** $3 \times 1\frac{3}{4}$

g. $3\frac{1}{3} \times 1\frac{2}{3}$ **h.** $2\frac{3}{4} \times 2$ **i.** $2 \times 3\frac{1}{2}$

Problem set 65

1. (29,33) Fifty percent of the 60 questions on the test were multiple choice. Write 50% as a reduced fraction and find the number of multiple choice questions.

2. (53) How many quarter notes equal a half note?

3. (15) Some railroad rails weigh 155 pounds per yard and are 33 feet long. How much would a 33-foot-long rail weigh?

4. (65) $1\frac{1}{2} \times 2\frac{2}{3}$

5. (65) $2\frac{2}{3} \times 2$

6. (18) The sum of five numbers is 200. What is the average of the numbers?

7. (5) $\frac{100 + 75}{100 - 75}$

8. (42) $m - 1\frac{1}{5} = 3\frac{1}{2}$

9. (59) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$

10. (62) $35\frac{1}{4} - 12\frac{1}{2}$

11. (29) $\frac{4}{5} \times \frac{1}{2}$

12. (53) $\frac{4}{5} \div \frac{1}{2}$

13. (44) $0.25 \div 5$

14. (48) $5 \div 0.25$

15. (39) What is the product of the answers to problems 13 and 14?

16. (53) $\frac{1}{2} + \frac{1}{2}$ is equal to which of the following?

A. $\frac{1}{2} - \frac{1}{2}$ B. $\frac{1}{2} \times \frac{1}{2}$ C. $\frac{1}{2} \div \frac{1}{2}$

17. (64) Use a factor tree to find the prime factorization of 30.

18. (15) If three of the items cost a total of 75¢, how much would six of the items cost?

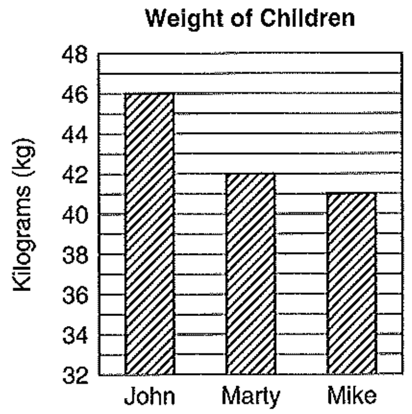
19. (50) Round $1.1675 to the nearest cent.

20. (63) One side of a regular pentagon is 0.8 meter. What is the perimeter?

21. Twenty minutes is what fraction of an hour?
(29)

22. The temperature dropped from 12°C to –8°C. This was a drop of how many degrees?
(14)

Use the chart below to answer questions 23, 24, and 25.



23. John weighs how much more than Mike?
(16)

24. What is the average weight of the three boys?
(16)

25. Write a "larger-smaller-difference" problem that refers to the graph and answer the question.
(16)

26. Use division by primes to find the prime factorization of 400.
(64)

27. Simon covered the floor of a square room with 144 floor tiles. How many floor tiles were along each wall of the room?
(38)

28. The weight of a one-kilogram object on earth is about 2.2 pounds. A large man may be 100 kilograms. About how many pounds is that?
(46)

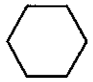
29. Reduce: $\frac{5 \cdot 5 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5}$
(53)

30. Which of these polygons is not a regular polygon?
(63)

A. 

B. 

C. 

D. 

LESSON
66

Using Prime Factorization to Reduce Fractions • Naming Solids

Facts Practice: Write 24 Mixed Numbers as Improper Fractions (Test J in Test Masters)

Mental Math: Count down by 2's from 10 to negative 10.

a. 5 × 260 **b.** 341 − 50 **c.** 3 × 48
d. \$9.25 − 75¢ **e.** Double \$1.25 **f.** $\frac{\$30}{100}$
g. 6 × 6, − 1, ÷ 5, × 2, + 1, ÷ 3, ÷ 2
h. Hold your hands one foot apart; then nine inches apart.

Problem Solving: There are about 520 nine-inch-long noodles in a 1-pound package of spaghetti. Laid end to end, how many **feet** would the noodles in a package of spaghetti reach?

Using prime factorization to reduce fractions

One way to reduce fractions with large terms is to factor the terms and then reduce the common factors. To reduce $\frac{125}{1000}$, we could begin by writing the prime factorization of 125 and of 1000.

$$\frac{125}{1000} = \frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5}$$

We see three pairs of 5's that can be reduced. Each $\frac{5}{5}$ reduces to $\frac{1}{1}$.

$$\frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}}}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}}} = \frac{1}{8}$$

We multiply the remaining factors and find that $\frac{125}{1000}$ reduces to $\frac{1}{8}$.

Example 1 Reduce: $\frac{375}{1000}$

Solution We write the prime factorization of the numerator and of the denominator.

$$\frac{375}{1000} = \frac{3 \cdot 5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5}$$

Then we reduce the common factors and multiply the remaining factors.

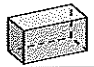
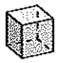
$$\frac{3 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{5}}}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{5}}} = \frac{3}{8}$$

We find that $\frac{375}{1000}$ reduces to $\frac{3}{8}$.

Naming solids

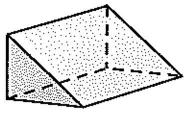
Polygons are two-dimensional shapes. Polygons have length and width. However, polygons do not have height (or depth). The objects that we encounter in the world around us are three-dimensional. These objects have length, width, and height. This table illustrates some three dimensional shapes—shapes that take up space. Three-dimensional shapes are often called **solids.**

Solids

Shape	Name	
	Triangular Prism	Prisms
	Rectangular Prism	
	Cube	
	Pyramid	
	Cylinder	
	Cone	
	Sphere	

You should be able to recognize, name, and draw each of these shapes. Notice that when these shapes are drawn, the edges which are hidden from the viewer can be indicated by using dashed lines.

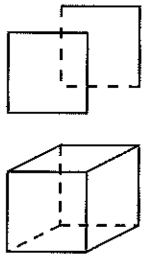
Example 2 Name this shape.



Solution This shape is a **triangular prism.**

Example 3 Draw a cube.

Solution To draw a cube, we first sketch two squares like this. Then we draw lines to connect the vertices using dashed lines to draw the edges that are hidden from view.

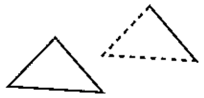


Practice Write the prime factorization of the numerator and denominator of each fraction and reduce each fraction.

a. $\frac{875}{1000}$ **b.** $\frac{48}{400}$

c. A can of soup is an example of which geometric solid?

d. Sketch a triangular prism. Begin by drawing two triangles like these.



Problem set 66

1. (10) What is the sum of the first nine positive odd numbers?

2. (46) A fathom is about 6 feet. A nautical mile is 1000 fathoms. A nautical mile is about how many feet?

3. (42) Instead of dividing \$1.50 by \$0.05, Marcus made an equivalent division problem by mentally multiplying the dividend and divisor by 100. Then he performed the equivalent division problem. What is the equivalent division problem Marcus made and what is the quotient?

4. (3) 6 cm + l = 11 cm

5. (42) $8g = 9.6$

6. (18) The combined length of four sticks is 172 inches. What is the average length of each stick?

7. (5) 10,000 − (4675 + 968)

8. (58) $3\frac{1}{3} + 2\frac{3}{4}$

9. (42) $\frac{7}{10} - w = \frac{1}{2}$

10. (62) $4\frac{1}{4} - 2\frac{7}{8}$

11. (65) $2\frac{2}{3} \times 3$

12. (65) $1\frac{1}{3} \times 2\frac{1}{4}$

13. (41) $\frac{3}{5} = \frac{?}{100}$

14. (39) $(2 \times 0.3) - (0.2 \times 0.3)$

15. (44) 1.44 ÷ 60

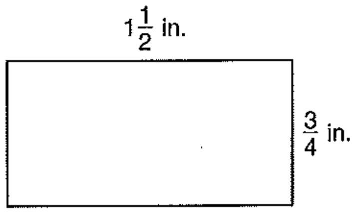
16. (48) \$6.00 ÷ \$0.15

17. (15) Five dollars was divided evenly among four people. How much money did each receive?

18. (63) The area of a regular quadrilateral is 100 square inches. What is its perimeter?

19. (66) Write the prime factorization of 625 and of 1000. Then reduce $\frac{625}{1000}$.

20. (31) What is the area of the rectangle shown below?



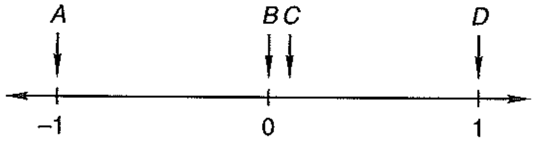
21. (29) Thirty-six of the eighty-eight piano keys are black. What fraction of the piano keys are black?

22. (66) Sketch a rectangular prism. Begin by drawing two congruent rectangles.

23. $1\frac{1}{2} \times \square = 1$
(30,60)

24. There are 1000 meters in a kilometer. How many meters are in 2.5 kilometers?
(15)

25. Which arrow could be pointing to 0.1 on the number line?
(49)

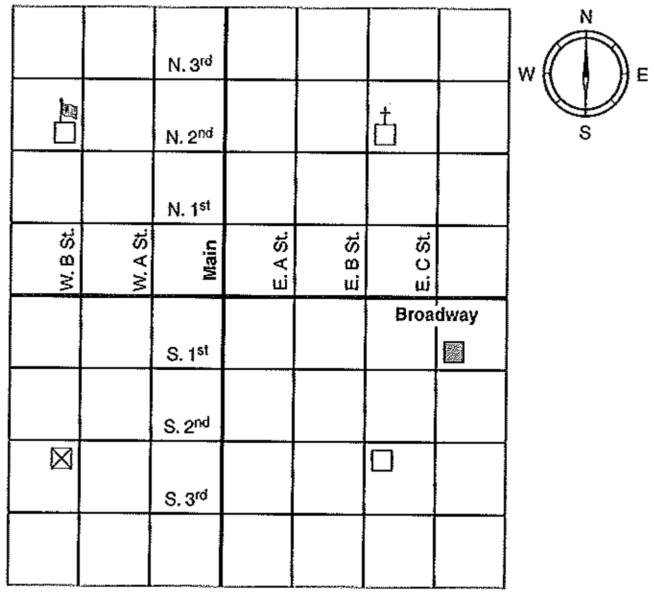


26. Which arrow in problem 25 is pointing to the number halfway between –1 and 1?
(49)

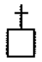
27. A basketball is an example of which geometric solid?
(66)

28. Write 51% as a fraction. Then write the fraction as a decimal number.
(33)

Refer to this map to answer questions 29 and 30.



29. Which building is on the corner of W. B St. and N. 2nd?
(N.R.)

A. 
B. 
C. ☐
D. ☒

30. The building ▨ is on the corner of which two streets?
(N.R.)

LESSON 67

Dividing Mixed Numbers

Facts Practice: 28 Fractions to Simplify (Test I in Test Masters)

Mental Math: Count down by 5's from 25 to negative 25.

a. 5×80 **b.** $275 + 1500$ **c.** 7×42
d. \$5.75 + 50¢ **e.** $\frac{1}{4}$ of \$48.00 **f.** $\frac{\$120}{10}$
g. $7 \times 8, - 1, \div 5, \times 2, - 1, \div 3, - 8$
h. Hold your hands one meter apart; then 90 cm apart.

Problem Solving: Which whole number greater than 90 but less than 100 is a prime number?

Recall the three steps to solving an arithmetic problem with fractions.

Step 1. Be sure the problem is in the correct shape.
Step 2. Perform the operation indicated.
Step 3. Simplify the answer if possible.

In Lesson 65 we noted that the correct shape for multiplying and dividing fractions is fraction form. In this lesson we will practice dividing mixed numbers. We first write any mixed numbers or whole numbers as improper fractions. Then we divide.

Example 1 $2\frac{2}{3} \div 4$

Solution We write the numbers in fraction form.

$$\frac{8}{3} \div \frac{4}{1}$$

We find the number of 4's in 1. Then we use the reciprocal of 4 to find the number of 4's in $\frac{8}{3}$.

$$\begin{tikzcd} 1 \div \frac{4}{1} = & \frac{1}{4} \arrow[dl] \\ \frac{8}{3} \times \frac{1}{4} & = \frac{8}{12} \end{tikzcd}$$

We simplify the answer.

$$\frac{8}{12} = \mathbf{\frac{2}{3}}$$

Notice that dividing a number by 4 is equivalent to finding $\frac{1}{4}$ of the number. Instead of dividing $2\frac{2}{3}$ by 4, we find $\frac{1}{4}$ of $2\frac{2}{3}$.

Example 2 $2\frac{2}{3} \div 1\frac{1}{2}$

Solution We write the mixed numbers as fractions.

$$\frac{8}{3} \div \frac{3}{2}$$

We find the number of $\frac{3}{2}$'s in 1. Then we use the reciprocal of $\frac{3}{2}$ to find the number of $\frac{3}{2}$'s in $\frac{8}{3}$.

$$1 \div \frac{3}{2} = \frac{2}{3}$$

$$\frac{8}{3} \times \frac{2}{3} = \frac{16}{9}$$

We simplify the answer.

$$\frac{16}{9} = \mathbf{1\frac{7}{9}}$$

Practice*

a. $\frac{1}{4}$ of $1\frac{3}{5}$

b. $1\frac{3}{5} \div 4$

c. $\frac{1}{3}$ of $2\frac{2}{5}$

d. $2\frac{2}{5} \div 3$

e. $1\frac{2}{3} \div 2\frac{1}{2}$

f. $2\frac{1}{2} \div 1\frac{2}{3}$

g. $1\frac{1}{2} \div 1\frac{1}{2}$

h. $7 \div 1\frac{3}{4}$

Problem set 67

1. (54) What is the difference between the sum of $\frac{1}{2}$ and $\frac{1}{4}$ and the product of $\frac{1}{2}$ and $\frac{1}{4}$?

2. (15) Bill ran a half mile in two minutes and fifty-five seconds. How many seconds is that?

3. (15) The gauge of a railroad—the distance between the two tracks—is usually 4 feet $8\frac{1}{2}$ inches. How many inches is that?

4. (67) $1\frac{1}{2} \div 2\frac{2}{3}$

5. (67) $1\frac{1}{3} \div 4$

6. (18) In six games Yvonne scored a total of 108 points. How many points per game did she average?

7. (66) Write the prime factorization of 24 and 200. Then reduce $\frac{24}{200}$.

8. (42) $m - 5\frac{3}{8} = 1\frac{3}{16}$

9. (42) $3\frac{3}{5} + 2\frac{7}{10} = n$

10. (62) $5\frac{1}{8} - 1\frac{1}{2}$

11. (65) $3\frac{1}{3} \times 1\frac{1}{2}$

12. (67) $3\frac{1}{3} \div 1\frac{1}{2}$

13. (31) What is the area of a rectangle that is 4 inches long and $1\frac{3}{4}$ inches wide?

14. (52) $(3.2 + 1) - (0.6 \times 7)$

15. (48) $12.5 \div 0.4$

16. (44) $0.375 \div 25$

17. (51) 3.2×10 equals which of the following?

A. $32 \div 10$ B. $320 \div 10$ C. $0.32 \div 10$

18. (16) Estimate the sum of 6416, 5734, and 4912 to the nearest thousand.

19. (42) Instead of dividing 880 by 24, Sam made an equivalent division problem with smaller numbers by dividing the dividend and divisor by 8. Then he quickly found the quotient of the equivalent problem. What was the equivalent problem Sam made, and what was the quotient? Write the quotient as a mixed number.

20. (63) The perimeter of a square is 2.4 meters. How long is each side of the square?

21. (63) What is the area of the square described in problem 20?

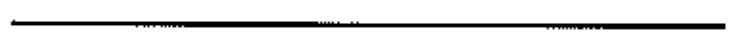
22. (29) What fraction of the months begin with the letter *M*?

23. (41) $\frac{3}{4} = \frac{?}{100}$

24. (65) Why is a circle not a polygon?

25. (67) Compare: $\frac{1}{3} \times 4\frac{1}{2}$ ○ $4\frac{1}{2} \div 3$

26. (17) Use your ruler to find the length of this segment to the nearest eighth of an inch.



27. (3) 36 mm + w = 63 mm

28. (33) Write 3% as a fraction. Then write the fraction as a decimal number.

29. (66) A shoe box is an example of which geometric solid?

30. (32) Sunrise occurred at 6:20 a.m. and sunset occurred at 5:45 p.m. How many hours and minutes were there from sunrise to sunset?

LESSON 68

Lines

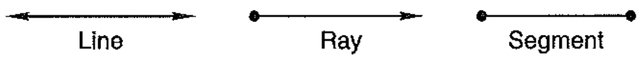
Facts Practice: Write 24 Mixed Numbers as Improper Fractions (Test J in Test Masters)

Mental Math: Count up and down by $\frac{1}{4}$'s between $\frac{1}{4}$ and 4.

a. 5×180 **b.** $530 - 50$ **c.** 6×44
d. \$6.00 − \$1.75 **e.** Double \$1.75 **f.** $\frac{\$120}{100}$
g. $6 \times 5, + 2, \div 4, \times 3, \div 4, - 2, \div 2, \div 2$
h. Hold your hands one inch apart; then one cm apart.

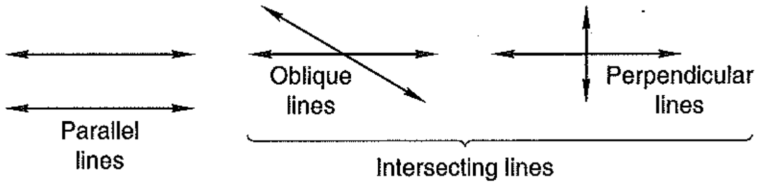
Problem Solving: Nathan used a one-foot length of string to make a rectangle that was twice as long as it was wide. What was the area that was enclosed by the string?

Here we show ways to illustrate a **line,** a **ray,** and a **segment.**



A **line** continues in two directions without end. A **ray** begins at one point and continues without end. A **segment** is a part of a line and has two endpoints.

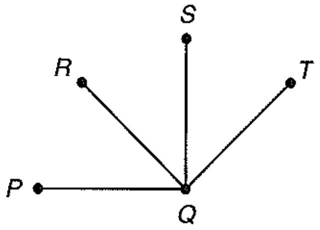
A **plane** in mathematics is a flat surface like a tabletop or a smooth sheet of paper. When two lines are drawn on the same plane, either they will cross at some point or they will not cross. When lines do not cross but stay the same distance apart, we say that the lines are **parallel.** When lines cross, we say they **intersect.** When they intersect and make square angles, we call the lines **perpendicular.** The square angles formed by perpendicular lines are called **right angles.** If lines intersect at a point but are not perpendicular, then the lines are **oblique.**



Letters are often used to designate points. We may use two points to identify a line, a ray, or a segment. Here we show a line that passes through points A and B. This line may be referred to as line AB or line BA. We may abbreviate line AB as $\overleftrightarrow{AB}$.

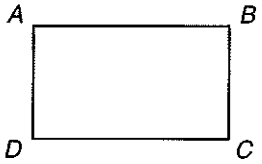
The ray that begins at point A and passes through point B is ray AB, which may be abbreviated $\overrightarrow{AB}$. The portion of line AB between and including points A and B is segment AB (or segment BA), which can be abbreviated $\overline{AB}$ (or $\overline{BA}$).

Example 1 Which segment appears to be perpendicular to $\overline{PQ}$ in this figure?



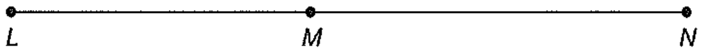
Solution The segment that appears to be perpendicular to segment PQ is **segment SQ** (or **segment QS**), which may be abbreviated **$\overline{SQ}$** or **$\overline{QS}$**. Together, $\overline{PQ}$ and $\overline{QS}$ appear to form a right angle. (It may be necessary to "mentally erase" or ignore the other segments in the figure to see the relationship of the segments.)

Example 2 In rectangle $ABCD$, which side is parallel to $\overline{AB}$?



Solution Points A, B, C, and D are the vertices of the rectangle. In rectangle $ABCD$, $\overline{BC}$ and $\overline{AD}$ are perpendicular to $\overline{AB}$. The side that is parallel to $\overline{AB}$ is **$\overline{DC}$**, which may also be named **$\overline{CD}$**.

Example 3 In this figure, the length of $\overline{LM}$ is 4 cm, and the length of $\overline{LN}$ is 9 cm. What is the length of $\overline{MN}$?



Solution The length of $\overline{LM}$ plus the length of $\overline{MN}$ equals the length of $\overline{LN}$. With the information in the problem we can make this equal. The letter l stands for the missing length.

$$4 \text{ cm} + l = 9 \text{ cm}$$

Since 4 cm plus 5 cm equals 9 cm, we find that the length of $\overline{MN}$ is **5 cm.**

Practice

a. What do we call a part of a line?

b. Is a beam of sunlight like a segment, a line, or a ray?

c. What do we call lines in the same plane that do not intersect?

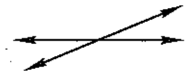
Name the following:

d.

e.

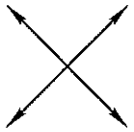
f.

g.

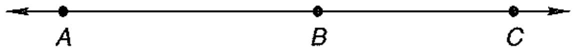


h.

i.



j. In this figure the length of $\overline{AC}$ is 60 mm, and the length of $\overline{BC}$ is 26 mm. Find the length of $\overline{AB}$.



k. A **horizontal** line is parallel with the horizon. A **vertical** line is perpendicular to the horizon. If a horizontal line and a vertical line intersect, what kind of angles are formed by the lines?

1. Draw a pair of parallel lines. Then draw a second pair of parallel lines that are perpendicular to the first pair. Trace over the quadrilateral that is formed by the intersecting pairs of lines. What kind of quadrilateral did you trace?

Problem set 68

1. (53) What is the quotient if the dividend is $\frac{1}{2}$ and the divisor is $\frac{1}{8}$?

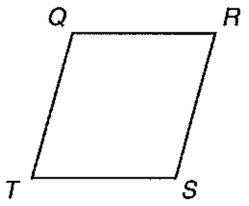
2. (14) The highest weather temperature recorded was 136°F in Africa. The lowest was –127°F in Antarctica. How many degrees difference is there between these temperatures?

3. (15) A dollar bill is about 6 inches long. Laid end to end, about how many **feet** would 1000 dollar bills reach?

4. (66) Write the prime factorization of the numerator and denominator of this fraction. Then reduce the fraction.

$$\frac{45}{72}$$

5. (68) In quadrilateral *QRST*, which segment appears to be parallel to $\overline{RS}$?



6. (18) In 10 days Carla saved $27.50. On the average, how much did she save each day?

7. (5) $\frac{1 \times 2 \times 3 \times 4 \times 5}{1 + 2 + 3 + 4 + 5}$

8. (30) $\frac{2}{5}w = 1$

9. (59) $3\frac{1}{2} + 2\frac{3}{4} + 1\frac{5}{8}$

10. (42) $\frac{3}{4} - f = \frac{1}{3}$

11. *(42)* $m + 1\frac{3}{4} = 5\frac{3}{8}$

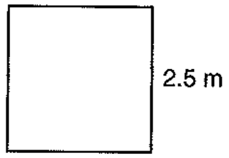
12. *(65)* $2\frac{2}{3} \times 1\frac{1}{5}$

13. *(67)* $1\frac{2}{3} \div 2$

14. *(48)* $\frac{2.4}{0.08}$

15. *(38)* What is the perimeter of this square?

16. *(38)* What is the area of this square?



17. *(64)* How do you decide if a counting number is a composite number?

18. *(64)* Make a factor tree to find the prime factorization of 250.

19. *(63)* A stop sign has the shape of an eight-sided polygon. What is the name of an eight-sided polygon?

20. *(29)* There were 15 boys and 12 girls in the class. What fraction of the class was made up of girls?

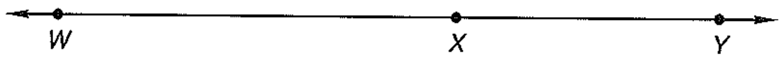
21. *(42)* Instead of dividing $4\frac{1}{2}$ by $1\frac{1}{2}$, Carla doubled both numbers before dividing mentally. What was Carla's mental division problem and its quotient?

22. *(30,60)* What is the reciprocal of $2\frac{1}{2}$?

23. *(15)* There are 1000 grams in a kilogram. How many grams is 2.25 kilograms?

24. *(7)* About how many **millimeters** long is the line?

25. *(68)* The length of $\overline{WX}$ is 53 mm. The length of $\overline{XY}$ is 35 mm. What is the length of $\overline{WY}$?



26. Sketch a cylinder.
(66)

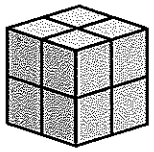
27. $\frac{8}{25} = \frac{?}{100}$
(41)

28. Arrange these numbers in order from least to greatest:
(49)

0.1, 1, −1, 0

29. Draw a circle and shade $\frac{1}{4}$ of it. What percent of the circle is shaded?
(33)

30. How many small cubes are in the big cube?
(66)



LESSON 69

Reducing Fractions Before Multiplying

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by 2's between negative 10 and 10.

a. 5×280

b. $476 + 99$

c. 3×54

d. $4.50 + $1.75

e. $\frac{1}{3}$ of $90.00

f. $\frac{\$250}{10}$

g. $5 \times 10, \div 2, + 5, \div 2, - 5, \div 10, - 1$

h. Hold your hands one yard apart; then two feet apart.

Problem Solving: Every even number greater than two is not prime but can be written as a product of primes. Chris thought that even numbers greater than two could also be written as a sum of primes ($4 = 2 + 2$, $6 = 3 + 3$, $8 = 5 + 3$, etc.). Show how the even numbers from 10 to 20 can be written as sums of prime numbers.

The terms of a fraction may be reduced before the fractions are multiplied, even though the terms to be reduced appear in different fractions. Notice that 3 appears as a numerator and as a denominator in these multiplied fractions.

$$\frac{\mathbf{3}}{5} \times \frac{2}{\mathbf{3}} = \frac{6}{15} \qquad \frac{6}{15} \text{ reduces to } \frac{2}{5}$$

We may reduce the common terms before we multiply. We reduce $\frac{3}{3}$ to $\frac{1}{1}$ by dividing both 3's by 3. Then we multiply the remaining terms.

$$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{2}{\underset{1}{\cancel{3}}} = \frac{2}{5}$$

Reducing before we multiply avoids the need to reduce after we multiply. Reducing before multiplying is also known as **canceling.**

Example 1 $\frac{5}{6} \times \frac{1}{5}$

Solution We will reduce before we multiply. Any numerator may be paired with any denominator to reduce multiplied fractions. Since 5 appears as a numerator and as a denominator, we will reduce $\frac{5}{5}$ to $\frac{1}{1}$ by dividing both 5's by 5. Then we multiply the remaining terms.

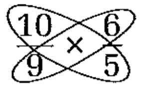
$$\frac{\overset{1}{\cancel{5}}}{6} \times \frac{1}{\underset{1}{\cancel{5}}} = \mathbf{\frac{1}{6}}$$

Example 2 $1\frac{1}{9} \times 1\frac{1}{5}$

Solution First we write the numbers in fraction form.

$$\frac{10}{9} \times \frac{6}{5}$$

We mentally pair 10 with 5 and 6 with 9.



We reduce $\frac{10}{5}$ to $\frac{2}{1}$ by dividing 10 and 5 by 5. We reduce $\frac{6}{9}$ to $\frac{2}{3}$ by dividing 6 and 9 by 3.

$$\frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}} = \frac{4}{3}$$

We multiply the remaining terms. Then we simplify the product.

$$\frac{4}{3} = \mathbf{1\frac{1}{3}}$$

Example 3 $\frac{5}{6} \div \frac{5}{2}$

Solution This is a division problem.

$$\frac{5}{6} \div \frac{5}{2}$$

First we find the number of $\frac{5}{2}$'s in 1. Then we use the reciprocal of $\frac{5}{2}$ to find the number of $\frac{5}{2}$'s in $\frac{5}{6}$.

$$1 \div \frac{5}{2} = \frac{2}{5}$$

$$\frac{5}{6} \times \frac{2}{5}$$

Since this is now a multiplication problem, we may reduce before we multiply.

$$\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{5}}} = \mathbf{\frac{1}{3}}$$

Note: We only cancel the terms of multiplied fractions. We may cancel the terms of divided fractions only when the problem has been rewritten as a multiplication problem. We do not cancel the terms of added or subtracted fractions.

Practice

Reduce before multiplying:

a. $\frac{3}{4} \cdot \frac{4}{5}$ **b.** $\frac{2}{3} \cdot \frac{3}{4}$ **c.** $\frac{8}{9} \cdot \frac{9}{10}$

Write in fraction form. Then reduce before multiplying.

d. $2\frac{1}{4} \times 4$ **e.** $1\frac{1}{2} \times 2\frac{2}{3}$ **f.** $3\frac{1}{3} \times 2\frac{1}{4}$

Rewrite each division problem as a multiplication problem. Then reduce before multiplying.

g. $\frac{2}{5} \div \frac{2}{3}$ **h.** $\frac{8}{9} \div \frac{2}{3}$ **i.** $\frac{9}{10} \div 1\frac{1}{5}$

Problem set 69

1. Alaska was purchased from Russia in 1867 for seven million, two hundred thousand dollars. Write that amount.
(12)

2. How many eighth notes equal a half note?
(53)

3. (42) Instead of dividing $12\frac{1}{2}$ by $2\frac{1}{2}$, Shannon doubled both numbers and then divided. Write the division problem Shannon formed and its quotient.

In problems 4, 5, and 6, reduce before multiplying:

4. (69) $\frac{5}{6} \cdot \frac{4}{5}$

5. (69) $\frac{5}{6} \div \frac{5}{2}$

6. (69) $\frac{9}{10} \cdot \frac{5}{6}$

7. (17) What number is halfway between $\frac{1}{2}$ and 1 on the number line?

8. (42) $f - \frac{3}{4} = \frac{5}{6}$

9. (58) $3\frac{2}{3} + 4\frac{5}{6}$

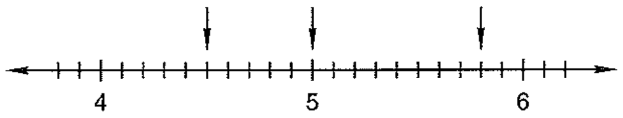
10. (62) $7\frac{1}{8} - 2\frac{1}{2}$

11. (38) $4.37 + 12.8 + 6$

12. (44) $0.46 \div 5$

13. (48) $60 \div 0.8$

14. (18) What is the average of the three numbers marked by the arrows on this decimal number line? (First estimate whether the average will be more than 5 or less than 5.)



15. (48) $1.5 \div 0.06$ is equivalent to which of the following?

A. $15 \div 6$ B. $150 \div 6$ C. $150 \div 60$

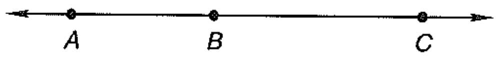
16. (39) There are 1000 milliliters in a liter. How many milliliters are in 3.8 liters?

17. (42) $\frac{2}{3} + n = 1$

18. (30) $\frac{2}{3}m = 1$

19. (64) Use division by primes to find the prime factorization of 150.

20. (68) Segment AC is 47 mm. Segment AB is 19 mm. How long is segment BC?

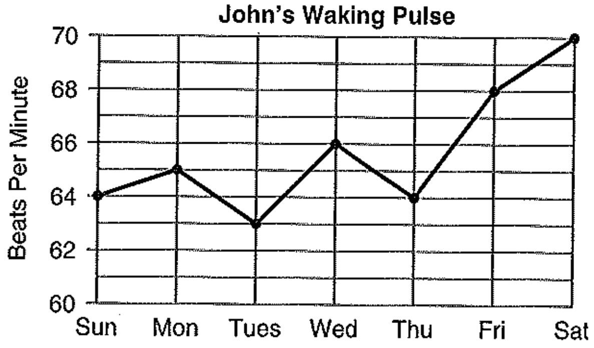


Write problems 21 and 22 in fraction form; then reduce before multiplying.

21. (69) $1\frac{2}{3} \times 1\frac{1}{5}$

22. (69) $\frac{8}{9} \div 2\frac{2}{3}$

Use the graph to answer questions 23, 24, and 25:



23. (15) When John woke on Saturday, his pulse was how many beats per minute more than it was on Tuesday?

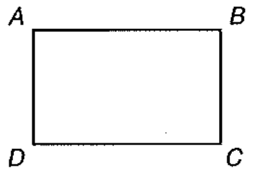
24. (18) On Monday, John took his pulse for 3 minutes before marking the graph. How many times did his heart beat in those 3 minutes?

25. (18) Write a question that refers to this graph and answer the question.

26. (66) Write the prime factorization of the numerator and denominator of this fraction. Then reduce the fraction.

$$\frac{72}{300}$$

In rectangle *ABCD*, the length of $\overline{AB}$ is 2.5 cm, and the length of $\overline{BC}$ is 1.5 cm. Use this information and the figure to answer problems 27 through 30.



27. (68) What is the perimeter of the rectangle?

28. (31) What is the area of the rectangle?

29. (68) Name two segments perpendicular to $\overline{DC}$.

30. (68) If segment *BD* were drawn on the figure dividing the rectangle into two equal parts, what would be the area of each part?

LESSON 70

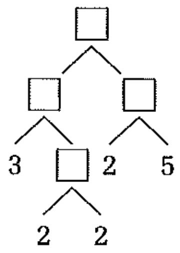
Rectangular Coordinates

Facts Practice: 64 Multiplication Facts (Test D in Test Masters)

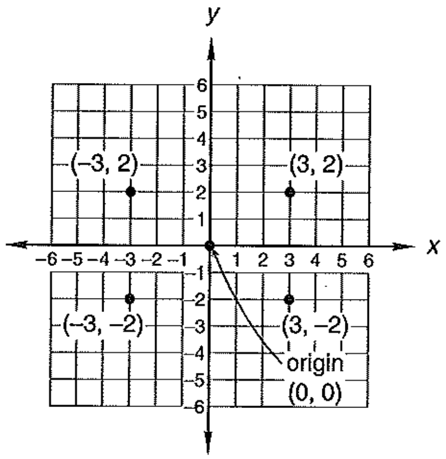
Mental Math: Count by 12's from 12 to 144.

a. 5 × 480 **b.** 367 − 99 **c.** 8 × 43
d. \$10.00 − \$8.75 **e.** Double \$2.25 **f.** $\frac{\$250}{100}$
g. 8 × 9, + 3, ÷ 3, × 2, − 10, ÷ 5, + 3, ÷ 11
h. Hold your hands one yard apart; then one meter apart.

Problem Solving: Copy this factor tree and fill in the missing numbers.



By drawing two number lines perpendicular to each other and by extending the unit marks, we can create a grid or graph called a **coordinate plane.**

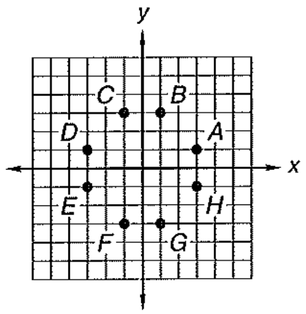


The point at which the number lines intersect is called the **origin.** The horizontal number line is called the ***x*-axis,** and the vertical number line is called the ***y*-axis.** We **graph** a point when we make a dot at the location of the point. We can name the location of any point on this coordinate plane with two numbers. The numbers that tell the location of the point are called the **coordinates** of the point.

The coordinates are written as a pair of numbers in parentheses, like (3, –2). The first number is the *x*-coordinate and shows the horizontal (⟷) direction and distance from the origin. The second number, the *y*-coordinate, shows the vertical (↕) direction and distance from the origin. The sign of the coordinate shows direction. Positive coordinates are to the right or up, and negative coordinates are to the left or down.

To graph (3, –2), we begin at the origin and move to the right along the *x*-axis three units to 3 on the number line. From there we move down two units and make a dot. We may label the point we graphed (3, –2). We have graphed and marked the coordinates of three additional points on the coordinate plane. Notice that each pair of coordinates is different and designates a different point.

Example Refer to this coordinate plane to answer questions (a) and (b).



(a) What are the coordinates of point *A*?

(b) Which point has the coordinates (–1, 3)?

Solution (a) We see that point *A* aligns with 3 on the *x*-axis and 1 on the *y*-axis. We write the *x*-coordinate first. So the coordinates of point *A* are **(3, 1).**

(b) We find the point that aligns with –1 on the *x*-axis and 3 on the *y*-axis. This is point **C.** Some people think of starting at the origin and moving to the left one unit and then up three units.

Practice* Refer to the coordinate plane in the example to answer questions (a)–(f). Name the points that have the following coordinates:

a. (3, –1) **b.** (1, 3) **c.** (–3, 1)

Identify the coordinates of the following points:

d. *G* **e.** *E* **f.** *F*

> **Teacher Note:** Beginning in Lesson 74, students will be drawing their own graphs. Having graph paper available for Lesson 74 and for subsequent problem sets will be helpful to students. "Activity Master 6" in the Test Masters may also be copied for student use.

Problem set 70

1. What is the least common multiple of 6 and 10?
(30)

2. The highest point on land is Mt. Everest, which is 29,028 feet above sea level. The lowest point on land is the Dead Sea, which is 1229 feet below sea level. What is the difference in elevation between these two points?
(13)

3. The movie lasted 105 minutes. If it started at 1:15 p.m., at what time did it end?
(32)

In problems 4 through 7, reduce the fractions, if possible, before multiplying.

4. $\frac{2}{3} \cdot \frac{3}{8}$ (69)

5. $1\frac{1}{4} \cdot 2\frac{2}{3}$ (69)

6. $\frac{3}{4} \div \frac{3}{8}$ (69)

7. $4\frac{1}{2} \div 6$ (69)

8. $6 + 3\frac{3}{4} + 2\frac{1}{2}$ (58)

9. $5 - 3\frac{1}{8}$ (62)

10. $5\frac{1}{4} - 1\frac{7}{8}$ (62)

11. (437)(86) (2)

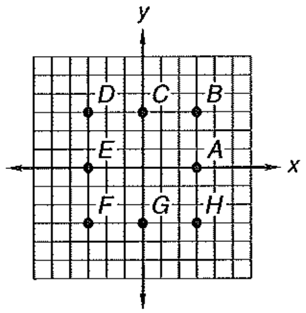
12. $\frac{5472}{18}$ (2)

13. $15\overline{)\$75.00}$ (2)

14. (1) \$100 − \$10.87

15. (48) $(1 + 0.6) \div (1 - 0.6)$

Refer to this coordinate plane to answer questions 16 and 17:



16. (70) Name the points that have the following coordinates:
(a) (−3, 3) (b) (0, −3)

17. (70) Identify the coordinates of the following points:
(a) *H* (b) *E*

18. (42) $1.2f = 120$

19. (42) $\frac{120}{f} = 1.2$

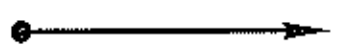
20. (66) Write the prime factorization of the numerator and the denominator of this fraction. Then reduce the fraction.

$$\frac{64}{224}$$

21. (63) The perimeter of a square is 6.4 meters. What is its area?

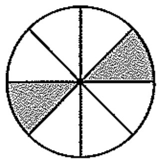
22. (68) Which diagram illustrates a ray?

A. 

B. 

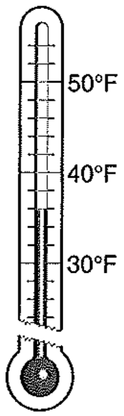
C. 

23. (21) What fraction of the circle is **not** shaded?



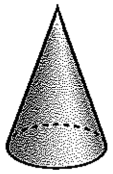
24. (7) A centimeter is about this long ——. About how many centimeters long is your little finger?

25. Water freezes at 32° Fahrenheit. The temperature shown on the thermometer is how many degrees Fahrenheit above the freezing point of water?
(10)

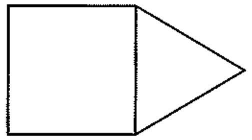


26. Ray found that 20% of an hour of TV that he watched was commercial time. Write 20% as a reduced fraction. Then find the number of minutes of commercials there were in the hour.
(29,33)

27. Name this geometric solid.
(66)



28. This square and regular triangle share a common side. The perimeter of the square is 24 cm. What is the perimeter of the triangle?
(63)



29. $\frac{7}{20} = \frac{?}{100}$
(41)

30. What is the name of the point on the coordinate plane that has the coordinates (0, 0)?
(70)

LESSON 71

Fractions Chart • Multiplying Three Fractions

Facts Practice: 72 Mixed Multiplication and Division (Test H in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 2.

a. 3 × 125
b. 275 + 50
c. 3 × \$0.99 *(3 × \$1.00 − 3 × 1¢)*
d. \$20.00 − \$9.99
e. $\frac{1}{3}$ of \$6.60
f. \$2.50 × 10
g. 2 × 2, × 2, × 2, × 2, − 2, ÷ 2
h. Hold your hands one foot apart; then six inches apart.

Problem Solving: Nelson was thinking of two numbers whose average was 24. If one of the numbers was half of 24, then what was the other number?

Fractions chart

We have studied the three steps to take when performing pencil-and-paper arithmetic with fractions and mixed numbers.

Step 1. Write the problem in the correct shape.
Step 2. Perform the operation.
Step 3. Simplify the answer.

The letters S.O.S. may help us remember the steps as "shape," "operate," and "simplify." We will assemble the rules we have learned in a fractions chart. Below the + − signs, we will list the steps we take when adding and subtracting fractions. Below the × ÷ signs, we will list the steps we take when multiplying and dividing fractions.

Fractions Chart

<table>
<tr><th></th><th>+ −</th><th colspan="2">× ÷</th></tr>
<tr><td>Shape</td><td>Write fractions with common denominators.</td><td colspan="2">Write numbers in fraction form.</td></tr>
<tr><td rowspan="3">Operate</td><td rowspan="3">Add or subtract the numerators.</td><td>×</td><td>÷</td></tr>
<tr><td>cancel. ←</td><td rowspan="2">Find reciprocal of divisor; then</td></tr>
<tr><td>$\frac{n \times n}{d \times d}$</td></tr>
<tr><td>Simplify</td><td colspan="3">Reduce fractions. Convert improper fractions.</td></tr>
</table>

At the "operate" step we separate multiplication and division. When multiplying fractions we may reduce (cancel) before we multiply. Then we multiply the numerators to find the numerator of the product, and we multiply the denominators to find the denominator of the product.

When dividing fractions we first find the reciprocal of the divisor. Then we treat the division problem like a multiplication problem as we multiply by the reciprocal of the divisor.

The "simplify" step is the same for all four operations. We reduce answers when possible and convert answers that are improper fractions to mixed numbers.

Multiplying three fractions

To multiply three or more fractions, we follow the same steps we take when multiplying two fractions:

1. We write the numbers in fraction form.
2. We may reduce before we multiply (cancel) by reducing any numerator-denominator pair of terms. Then we multiply the remaining terms.
3. We simplify if possible.

Example $\frac{2}{3} \times 1\frac{3}{5} \times \frac{3}{4}$

Solution First we write $1\frac{3}{5}$ as the improper fraction $\frac{8}{5}$. Then we reduce where possible before multiplying. Multiplying the remaining terms, we find the product.

$$\frac{2}{\underset{1}{\cancel{3}}} \times \frac{\overset{2}{\cancel{8}}}{5} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} = \mathbf{\frac{4}{5}}$$

Practice **a.** Draw the fractions chart from this lesson.

b. $\frac{2}{3} \cdot \frac{4}{5} \cdot \frac{3}{8}$

c. $2\frac{1}{2} \times 1\frac{1}{10} \times 4$

Problem set 71

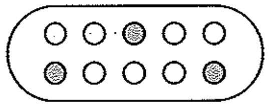
1. *(18)* What is the average of 4.2, 2.61, and 3.6?

2. *(53)* Four tablespoons equal $\frac{1}{4}$ of a cup. How many tablespoons would equal a full cup?

3. *(14)* The temperature on the moon ranges from a high of 134°C to a low of about −170°C. This is a difference of how many degrees?

4. *(21)* (a) What fraction of this group is shaded?

(b) What fraction of this group is not shaded?



5. *(29)* What fraction of a meter is a centimeter?

6. *(29)* What fraction of a dollar is a nickel?

7. *(71)* $\frac{1}{2} \cdot \frac{5}{6} \cdot \frac{3}{5}$

8. *(71)* $3 \times 1\frac{1}{2} \times 2\frac{2}{3}$

9. *(53)* $\frac{3}{4} \div 2$

10. *(67)* $1\frac{1}{2} \div 1\frac{2}{3}$

11. *(42)* $n - \frac{1}{2} = \frac{3}{5}$

12. *(42)* $1 - w = \frac{7}{12}$

13. *(42)* $w + 2\frac{1}{2} = 3\frac{1}{3}$

14. *(38)* $(1 + 2.3) - 0.45$

15. *(48)* $(0.12)(0.24)$

16. *(39)* $0.6 \div 0.25$

17. *(45)* Write the standard decimal number for the following:

$$(6 \times 10) + \left(4 \times \frac{1}{10}\right) + \left(3 \times \frac{1}{100}\right)$$

18. *(49)* Which is closest to 1?

A. −1 B. 0.1 C. 10

19. *(61)* What is the largest prime number that is less than 100?

20. $6w = 300$
(4)

21. $a + 47 = 300$
(3)

22. A loop of string two feet around is formed to make a square.
(63)

(a) How many inches long is each side of the square?

(b) What is the area of the square?

Refer to this menu and the following information to answer questions 23, 24, and 25.

Menu

Grilled Chicken Sandwich	$3.49	Drinks:	Small	$0.89
Taco Salad	$3.29		Medium	$1.09
Pasta Salad	$2.89		Large	$1.29

From this menu the Johnsons ordered two Grilled Chicken Sandwiches, one Taco Salad, a small drink, and two medium drinks.

23. What was the total price of the items the Johnsons ordered?
(1)

24. If 93¢ tax is added to the bill, and if the Johnsons pay for the food with a $20 bill, then how much money should they get back?
(12)

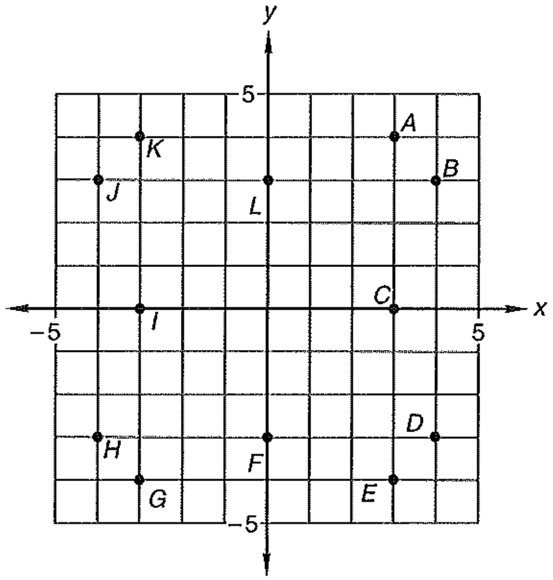
25. Make up an order from the menu. Then calculate the bill, not including tax.
(10)

26. If l equals 2.5 and w equals 0.4, then what does lw equal?
(45)

27. Write the prime factorization of the numerator and denominator of this fraction and then reduce the fraction.
(66)

$$\frac{72}{120}$$

Refer to this coordinate plane to answer questions 28 and 29.



28. Identify the coordinates of the following points:
(70)
(a) *K* (b) *F*

29. Name the points that have the following coordinates:
(70)
(a) (3, −4) (b) (−3, 0)

30. Draw a pair of parallel lines. Then draw a second pair of parallel lines perpendicular to the first pair of lines and about the same distance apart. Trace over the quadrilateral that is formed by the intersecting lines. Is the quadrilateral a rectangle?
(68)

LESSON 72

Exponents • Writing Decimal Numbers as Fractions, Part 2

Facts Practice: Write 24 Mixed Numbers as Improper Fractions (Test J in Test Masters)

Mental Math: Count up and down by 25's between 25 and 300.

a. 4 × 112 **b.** 475 − 150 **c.** 4 × \$0.99
d. \$2.99 + \$1.99 **e.** Double \$3.50 **f.** \$3.50 ÷ 10
g. 3 × 3, × 3, + 3, ÷ 3, − 3, × 3
h. Hold your hands one yard apart; then one meter apart.

Problem Solving: Describe the steps from the fractions chart that would be used to find the answer to the following problem: $\frac{3}{4} + \frac{2}{3}$

Exponents

Exponents are used to indicate repeated multiplication. The 2 in the following expression is an exponent:

$$5^2$$

Notice that the exponent is elevated and written to the right of the 5. The exponent shows how many times the other number, the **base,** is to be used as a factor. In this case, 5 is to be used as a factor twice.

$$5^2 \text{ means } 5 \cdot 5$$

Since $5 \cdot 5$ equals 25, the expression equals 25.

$$5^2 = 25$$

We read numbers with exponents as **powers.** Note that when the exponent is 2 we usually say "squared" and when the exponent is 3 we usually say "cubed."

> We read 5^2 as "five to the second power" or "five squared."
>
> We read 10^3 as "ten to the third power" or "ten cubed."
>
> We read 3^4 as "three to the fourth power."
>
> We read 2^5 as "two to the fifth power."

Example 1 Compare: $3^4 \bigcirc 4^3$

Solution We find the value of each expression.

3^4 means $3 \cdot 3 \cdot 3 \cdot 3$, which equals 81.

4^3 means $4 \cdot 4 \cdot 4$, which equals 64.

Since 81 is greater than 64, we find that 3^4 is greater than 4^3.

$$\mathbf{3^4 > 4^3}$$

Example 2 Write the prime factorization of 1000 using exponents to group factors.

Solution Using a factor tree or division by primes, we find the prime factorization of 1000.

$$1000 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$$

We group the three 2's and the three 5's with exponents.

$$\mathbf{1000 = 2^3 \cdot 5^3}$$

Example 3 $100 - 10^2$

Solution We perform operations with exponents before we add, subtract, multiply, or divide. Ten squared is 100. So when we subtract 10^2 from 100 the difference is zero.

$$100 - 10^2$$

$$\mathbf{100 - 100 = 0}$$

Writing decimal numbers as fractions, part 2

We will review changing a decimal number to a fraction or to a mixed number. Recall that the denominator of a decimal number (10 or 100 or 1000 ...) is indicated by the number of decimal places in the decimal number. The digits to the right of the decimal point make up the numerator of the fraction.

Example 4 Write 0.5 as a common fraction.

Solution We read 0.5 as "five tenths," which also names the fraction $\frac{5}{10}$. We reduce the fraction.

$$\frac{5}{10} = \mathbf{\frac{1}{2}}$$

Example 5 Write 3.75 as a mixed number.

Solution The whole number part of 3.75 is 3. The fraction part is 0.75, which has two decimal places.

$$3.75 = 3\frac{75}{100}$$

We reduce the fraction.

$$3\frac{75}{100} = 3\frac{3}{4}$$

Practice Find the value of each expression:

a. 10^4 **b.** $2^3 + 2^4$ **c.** $2^2 \cdot 5^2$

d. Write the prime factorization 72 using exponents.

Write each decimal number as a fraction or mixed number:

e. 12.5 **f.** 1.25 **g.** 0.125

h. 0.05 **i.** 0.24 **j.** 10.2

Problem set 72

1. (38) Mark's temperature was 102°F. Normal body temperature is 98.6°F. How many degrees above normal was Mark's temperature?

2. (11) Jill has read 42 pages of a 180-page book. How many pages are left to read?

3. (18) If Jill wants to finish the book in the next three days, then she should read an average of how many pages?

4. (72) Write 2.5 as a reduced mixed number.

5. (72) Write 0.35 as a reduced fraction.

6. (33) Write 7% as a fraction. Then write the fraction as a decimal number.

7. (71) $\frac{3}{4} \times 2 \times 1\frac{1}{3}$

8. (72) $(100 - 10^2) \div 25$

9. (59) $3 + 2\frac{1}{3} + 1\frac{3}{4}$

10. (62) $5\frac{1}{6} - 3\frac{1}{2}$

11. (67) $\frac{3}{4} \div 1\frac{1}{2}$

12. (72) $7 \div 0.4$

13. (72) Compare: $5^2 \bigcirc 2^5$

14. (43) Compare: $0.3 \bigcirc 0.125$

15. (39) (6.3)(0.48)

16. (44) $0.175 \div 25$

17. (12) Which digit is in the ten-thousands' place in 123,456.78?

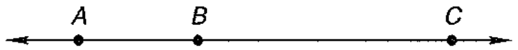
18. (57) Arrange in order from least to greatest:

$$1, \frac{1}{2}, \frac{1}{10}, \frac{1}{4}, 0$$

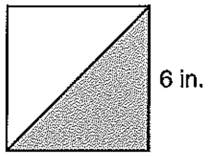
19. (72) Write the prime factorization of 200 using exponents.

20. (42) $1.2 + y + 4.25 = 7$

21. (68) The length of $\overline{AB}$ is 16 mm. The length of $\overline{AC}$ is 50 mm. What is the length of $\overline{BC}$?



22. (38) One half of the area of the square is shaded. What is the area of the shaded region?

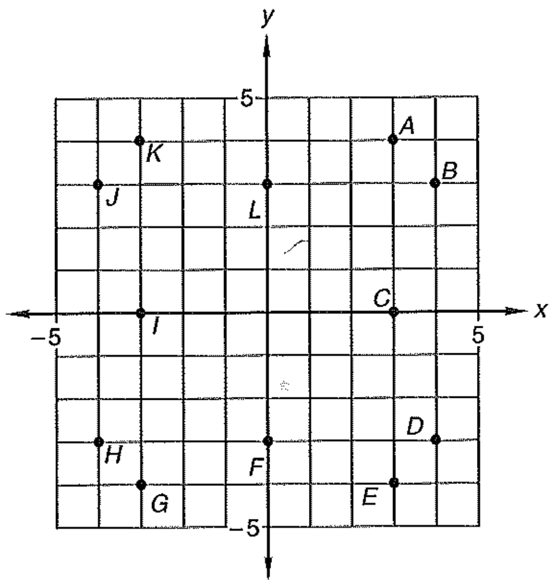


23. (63) Is every square a rectangle?

24. (72) $\frac{2^2 + 2^3}{2}$

25. (71) We read on the fractions chart that the proper "shape" for multiplying fractions is "fraction form." What does that mean?

Refer to this coordinate plane to answer questions 26 and 27:



26. Identify the coordinates of the following points:
(70)
(a) *H* (b) *L*

27. Name the points that have the following coordinates:
(70)
(a) (−4, 3) (b) (3, 0)

28. If *s* equals 9, then what does s^2 equal?
(72)

29. Sketch a cylinder.
(66)

30. Draw a pair of parallel lines. Then draw a second pair of parallel lines intersecting but not perpendicular to the first pair of lines. Trace over the quadrilateral that is formed by the intersecting lines. Is the quadrilateral a rectangle?
(68)

LESSON 73

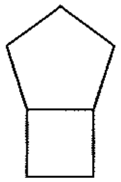
Writing Fractions as Decimal Numbers

Facts Practice: 28 Fractions to Simplify (Test I in Test Masters)

Mental Math: Count up and down by 5's between negative 25 and 25.

a. 3 × 230
b. 430 + 270
c. 5 × \$0.99
d. \$5.00 − \$1.98
e. $\frac{1}{4}$ of \$2.40
f. \$1.25 × 10
g. 5 × 5, − 5, × 5, ÷ 2, + 5, ÷ 5
h. Hold your hands one meter apart; then 100 centimeters apart.

Problem Solving: In this figure a square and a regular pentagon share a common side. The area of the square is 25 square centimeters. What is the perimeter of the pentagon?



We learned earlier that a fraction bar indicates division. So the fraction $\frac{1}{2}$ also means 1 divided by 2, which we can write as $2\overline{)1}$. By attaching a decimal point and zeros, we can perform the division and write the quotient as a decimal number.

$$\begin{array}{r} 0.5 \\ \frac{1}{2} \longrightarrow 2\overline{)1.0} \\ \underline{1\ 0} \\ 0 \end{array}$$

We find that $\frac{1}{2}$ is equivalent to the decimal number 0.5. We divide the numerator by the denominator to find the decimal number that is equivalent to the fraction.

Example 1 Convert $\frac{1}{4}$ to a decimal number.

Solution The fraction $\frac{1}{4}$ means 1 divided by 4, which is $4\overline{)1}$. By attaching a decimal point and zeros, we may complete the division.

$$\begin{array}{r} \mathbf{0.25} \\ 4\overline{)1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Example 2 Use a calculator to convert $\frac{15}{16}$ to a decimal number.

Solution We begin by clearing the calculator. Then we enter the fraction with these key strokes.

$$\boxed{1}\ \boxed{5}\ \boxed{\div}\ \boxed{1}\ \boxed{6}\ \boxed{=}$$

After striking the equals sign, the display shows the decimal equivalent of $\frac{15}{16}$.

0.9375

The answer is reasonable because both $\frac{15}{16}$ and 0.9375 are less than but close to 1.

Example 3 Write $7\frac{2}{5}$ as a decimal number.

Solution The whole number part of $7\frac{2}{5}$ is written to the left of the decimal point. We convert $\frac{2}{5}$ to a decimal by dividing 2 by 5.

$$\frac{2}{5} \longrightarrow \begin{array}{r} 0.4 \\ 5\overline{)2.0} \end{array}$$

Since $\frac{2}{5}$ equals 0.4, the mixed number $7\frac{2}{5}$ equals **7.4.**

Practice* Convert each fraction or mixed number to decimal form:

a. $\frac{3}{4}$

b. $4\frac{1}{5}$

c. $\frac{1}{8}$

d. $\frac{7}{20}$

e. $3\frac{3}{10}$

f. $\frac{7}{25}$

You may use a calculator to convert these fractions to decimals:

g. $\frac{11}{16}$

h. $\frac{31}{32}$

i. $3\frac{24}{64}$

Problem set 73

1. (72) What is the difference when five squared is subtracted from four cubed?

2. (15) On a certain map, 1 inch represents a distance of 10 miles. How many miles apart are two towns that are 3 inches apart on the map?

3. (15) Steve hit the baseball 400 feet. Tom hit the golf ball 300 yards. How many feet farther than the baseball did the golf ball travel?

4. (73) Convert $2\frac{3}{4}$ to a decimal number.

5. (73) To what decimal number is $\frac{4}{5}$ equal?

6. (72) Write 0.24 as a reduced fraction.

7. (45) If b equals 12 and h equals 8, then what does bh equal?

8. (72) Compare: $3^2 \bigcirc 3 + 3$

9. (59) $\frac{1}{2} + \frac{2}{3} + \frac{1}{6}$

10. (62) $3\frac{1}{4} - 1\frac{7}{8}$

11. (71) $\frac{5}{8} \cdot \frac{3}{5} \cdot \frac{4}{5}$

12. (65) $3\frac{1}{3} \times 3$

13. (67) $\frac{3}{4} \div 1\frac{1}{2}$

14. (38) $(4 + 3.2) - 0.01$

15. (66) Sketch a triangular prism.

16. (15) Nancy bought a dozen golf balls for $10.44. What was the cost for each golf ball?

17. (16) Estimate the product of 81 and 38.

18. (18) In four days Jill read 42 pages, 46 pages, 35 pages, and 57 pages. What was the average number of pages she read each day?

19. (30) What is the least common multiple of 6, 8, and 12?

20. (3) $24 + c + 96 = 150$

21. (66) Write the prime factorization of the numerator and of the denominator of this fraction. Then reduce the fraction.

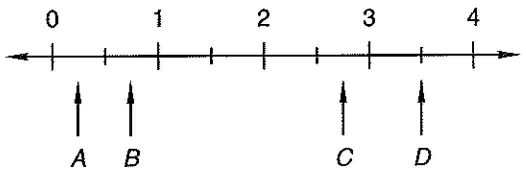
$$\frac{40}{96}$$

22. What number is $\frac{1}{2}$ of 360?
(22)

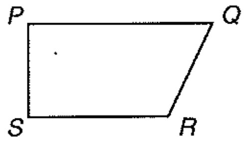
23. Twenty-four of the three dozen bicyclists rode mountain bikes. What fraction of the bikers rode mountain bikes?
(29)

24. Why are some rectangles not squares?
(63)

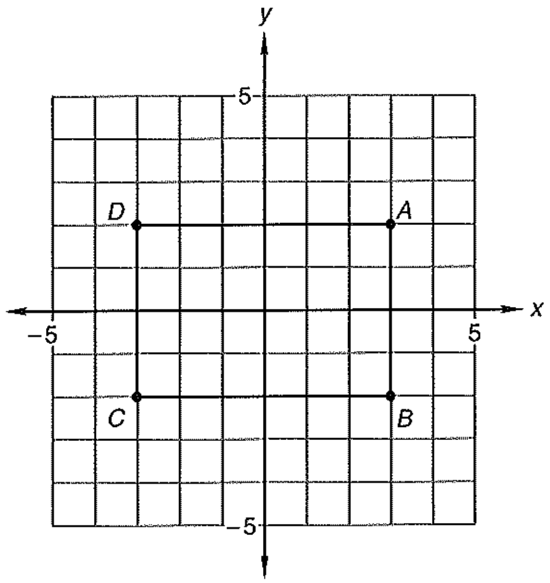
25. Which arrow could be pointing to $\frac{3}{4}$?
(17)



26. In quadrilateral $PQRS$, which segment appears to be
(68)
(a) parallel to $\overline{PQ}$?
(b) perpendicular to $\overline{PQ}$?



Refer to this coordinate plane to answer questions 27–29:



27. Identify the coordinates of the following points:
(70)
(a) C (b) Origin

28. (70) Name the points that have the following coordinates:

(a) (–3, 2) (b) (3, –2)

29. (68) One pair of parallel segments in rectangle $ABCD$ is $\overline{AB}$ and $\overline{DC}$. Name a second pair of parallel segments.

30. (29,33) Farmer John planted corn on 60% of his 300 acres. Write 60% as a reduced fraction. Then find the number of acres planted in corn.

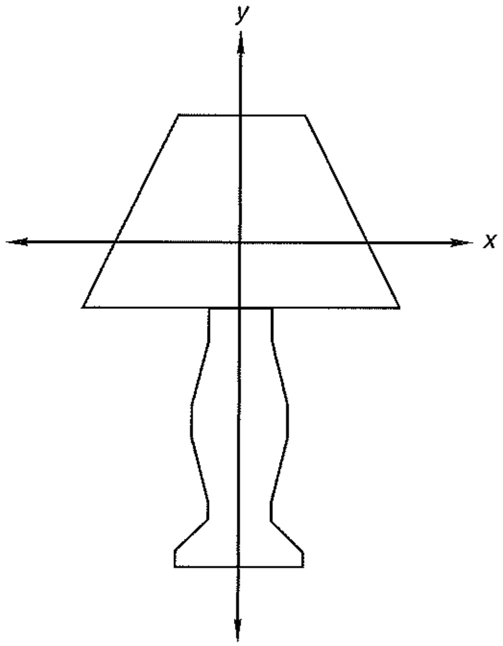
INVESTIGATION 2

Drawing on the Coordinate Plane

Materials needed for investigation:

- Three sheets of grid paper per student

Christy made the following drawing on a coordinate plane. Then Christy wrote directions for making the drawing. These directions are listed on the next page.



Draw segments to connect the following points in order:

1. $(-1, -2)$
2. $(-1, -3)$
3. $(-1\frac{1}{2}, -5)$
4. $(-1\frac{1}{2}, -6)$
5. $(-1, -8)$
6. $(-1, -8\frac{1}{2})$
7. $(-2, -9\frac{1}{2})$
8. $(-2, -10)$
9. $(2, -10)$
10. $(2, -9\frac{1}{2})$
11. $(1, -8\frac{1}{2})$
12. $(1, -8)$
13. $(1\frac{1}{2}, -6)$
14. $(1\frac{1}{2}, -5)$
15. $(1, -3)$
16. $(1, -2)$

Lift your pencil and restart:

1. $(-2\frac{1}{2}, 4)$
2. $(2\frac{1}{2}, 4)$
3. $(5, -2)$
4. $(-5, -2)$
5. $(-2\frac{1}{2}, 4)$

Refer to the example above to complete the following instructions:

a. The coordinates of the vertices are listed in order, as in a dot-to-dot drawing. Follow Christy's directions to make a similar drawing on your own grid paper.

b. Jenny wrote the following directions for a drawing. Follow her directions to make the drawing on your own grid paper:

1. $(-9, 0)$
2. $(6, -1)$
3. $(8, 0)$
4. $(7, 1)$
5. $(6, \frac{1}{2})$
6. $(6, -1)$
7. $(9, -2\frac{1}{2})$
8. $(10, -2)$
9. $(7, 1)$
10. $(6, 1\frac{1}{2})$
11. $(-10\frac{1}{2}, 3)$
12. $(-11, 2)$
13. $(-10\frac{1}{2}, 0)$
14. $(-10, -1\frac{1}{2})$
15. $(9, -2\frac{1}{2})$
16. $(-3, -3\frac{1}{2})$
17. $(-7, -8)$
18. $(-10, -8)$
19. $(-9, -1\frac{1}{2})$

Lift your pencil and restart:

1. $(-10\frac{1}{2}, 0)$
2. $(-11, -\frac{1}{2})$
3. $(-12, \frac{1}{2})$
4. $(-11\frac{1}{2}, 1)$
5. $(-12, 1\frac{1}{2})$
6. $(-11\frac{1}{2}, 2)$

7. $(-12, 2\frac{1}{2})$
8. $(-11, 3\frac{1}{2})$
9. $(-10\frac{1}{2}, 3)$
10. $(-11\frac{1}{2}, 8)$
11. $(-9\frac{1}{2}, 8)$
12. $(-7, 3)$
13. $(-6, 2\frac{1}{2})$
14. $(-7, 3)$
15. $(-6, 5)$
16. $(-4, 5)$
17. $(-1, 2)$

c. On a coordinate plane make a straight segment drawing. Then write directions for the drawing by listing the coordinates of the vertices of the drawing in "dot-to-dot" order.

Selected drawings may be used to provide periodic practice of graphing skills.

LESSON 74

Coordinate Geometry

Facts Practice: Linear Measure Facts (Test K in Test Masters)

Mental Math: Count by 12's from 12 to 144.

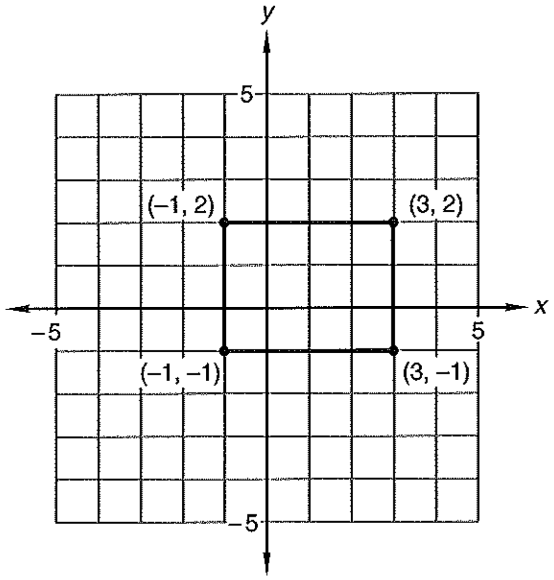
a. 504 × 6
b. 625 − 250
c. 3 × $1.99
d. $2.50 + $1.99
e. Double $1.60
f. $12.50 ÷ 10
g. 6 × 6, − 6, ÷ 6, − 5, × 2, + 1
h. Hold your hands one yard apart; then 18 inches apart.

Problem Solving: Describe the steps from the fractions chart that would be used to find the answer to the following problem: $3\frac{1}{3} \div 2\frac{1}{2}$

In this lesson we will use graph paper to create a coordinate plane (or use "Activity Master 6" in the Test Masters). Then we will graph points on the coordinate plane that are vertices of a rectangle. We will use the figure that we draw to help us answer questions about the rectangle.

Example The vertices of a rectangle are located at points (–1, –1), (3, –1), (3, 2), and (–1, 2). Graph the rectangle and find its perimeter and its area.

Solution Using graph paper we trace over two perpendicular lines on the graph paper to make the *x*-axis and the *y*-axis. Then we graph and label the points. The problem states that these points are the vertices of a rectangle. So we draw horizontal and vertical segments to connect the points and draw the rectangle.



We see that the rectangle is four units long and three units wide. Adding, we find the perimeter is **14 units.** To find the area we may count the unit squares within the rectangle. There are three rows of four squares, so the area of the rectangle is 3 times 4, which is **12 square units.**

Practice Use graph paper to create a coordinate plane. Then graph the points and use the figure to help you answer the following questions:

The vertices of a rectangle are located at (–2, –1), (2, –1), (2, 3), and (–2, 3).

a. Graph the rectangle. What do we call this special type of rectangle?

b. What is the perimeter of the rectangle?

c. What is the area of the rectangle?

Problem set 74

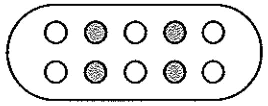
1. *(30,60)* What is the reciprocal of two and three fifths?

2. *(32)* What time is one hour and thirty-five minutes after 2:30 p.m.?

3. *(15)* A 1-pound box of candy cost $4.00. What was the cost per ounce? (1 pound = 16 ounces)

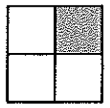
4. *(73)* (a) What fraction of the group is shaded?

(b) Write the fraction in part (a) as a decimal number.

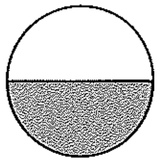


5. *(73)* (a) What fraction of the square is shaded?

(b) Write the fraction in part (a) as a decimal number.



6. *(23)* What percent of the circle is shaded?



7. *(73)* Write $3\frac{1}{8}$ as a decimal number.

8. *(72)* Write 1.8 as a mixed number.

9. *(33)* Write 12% as a reduced fraction. Then write the fraction as a decimal number.

10. *(56)* $\left(\frac{1}{2} + \frac{1}{3}\right) - \frac{1}{6}$

11. *(42)* $5 - m = 3\frac{1}{8}$

12. *(71)* $3\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{2}$

13. *(42)* $m + 1\frac{2}{3} = 3\frac{1}{6}$

14. $\$4 + \$6.37 + 94¢$ (1)

15. $1 - 0.95$ (38)

16. $(0.43)(2.6)$ (39)

17. $0.26 \div 5$ (44)

18. Which digit in 4.87 has the same place value as the 9 in 0.195? (34)

19. $\frac{7}{10} = \frac{?}{100}$ (41)

20. $\frac{3^3}{0.3}$ (72)

21. Write the prime factorization of the numerator and denominator of $\frac{18}{30}$. Then reduce the fraction. (66)

22. What is the greatest common factor of 18 and 30? (20)

23. If the product of two numbers is 1, then the two numbers are which of the following? (30)

A. Equal B. Reciprocals C. Opposites D. Prime

24. Why is every rectangle a quadrilateral? (63)

25. If b equals 8 and h equals 6, then what does $\frac{bh}{2}$ equal? (45)

26. Find the prime factorization of 400 using a factor tree. Then write the prime factorization of 400 using exponents. (64,72)

27. Sketch a coordinate plane on graph paper. Then draw a rectangle with vertices located at (3, 1), (3, –1), (–1, 1), and (–1, –1). (74)

Refer to the rectangle drawn in problem 27 to answer questions 28 and 29.

28. What is the perimeter of the rectangle? (8)

29. What is the area of the rectangle? (31)

30. Draw a pair of parallel segments of different lengths. Then form a quadrilateral by drawing two segments that connect the endpoints of the parallel segments. Is the quadrilateral a rectangle? (68)

LESSON 75

Comparing Fractions by Converting to Decimal Form

Facts Practice: 30 Fractions to Reduce (Test G in Test Masters)

Mental Math: Count up and down by 2's between negative 12 and 12.

a. 4×208 **b.** $380 + 155$ **c.** $4 \times \$1.99$
d. $\$10.00 - \$4.99$ **e.** $\frac{1}{5}$ of \$4.50 **f.** $\$0.95 \times 100$
g. $8 \times 8,\ - 4,\ \div 2,\ + 2,\ \div 4,\ \times 3,\ + 1,\ \div 5$
h. Hold your hands one foot apart; then 24 inches apart.

Problem Solving: Celina used division by primes to find the prime factorization of a number. Copy her work and fill in the missing numbers.

$$\begin{array}{r} 1 \\ 7\overline{)\square} \\ 5\overline{)\square} \\ 3\overline{)\square} \\ 2\overline{)\square} \\ 2\overline{)\square} \end{array}$$

We have compared fractions by sketching pictures of fractions and by writing fractions with common denominators. Another way to compare fractions is to convert the fractions to decimal form.

Example 1 To compare these fractions, first convert each fraction to decimal form:

$$\frac{3}{5} \bigcirc \frac{5}{8}$$

Solution We convert each fraction to a decimal number by dividing the numerator by the denominator.

$$\frac{3}{5} \longrightarrow \begin{array}{r} 0.6 \\ 5\overline{)3.0} \end{array} \qquad \frac{5}{8} \longrightarrow \begin{array}{r} 0.625 \\ 8\overline{)5.000} \end{array}$$

We will write both decimal numbers with the same number of decimal places and compare the decimal numbers.

$$0.600 < 0.625$$

Since 0.6 is less than 0.625, we know that $\frac{3}{5}$ is less than $\frac{5}{8}$.

$$\mathbf{\frac{3}{5} < \frac{5}{8}}$$

Example 2 Compare: $\frac{3}{4} \bigcirc 0.7$

Solution First we write the fraction as a decimal.

$$\frac{3}{4} \longrightarrow \begin{array}{r} 0.75 \\ 4\overline{)3.00} \end{array}$$

Then we compare the decimal numbers.

$$0.75 > 0.70$$

Since 0.75 is greater than 0.7, we know that $\frac{3}{4}$ is greater than 0.7.

$$\mathbf{\frac{3}{4} > 0.7}$$

Practice Change the fractions to decimal numbers to compare these numbers:

a. $\frac{3}{20} \bigcirc \frac{1}{8}$ **b.** $\frac{3}{8} \bigcirc \frac{2}{5}$ **c.** $\frac{15}{25} \bigcirc \frac{3}{5}$

d. $0.7 \bigcirc \frac{4}{5}$ **e.** $\frac{2}{5} \bigcirc 0.5$ **f.** $\frac{3}{8} \bigcirc 0.325$

Problem set 75

1. *(72)* What is the product of ten squared and two cubed?

2. *(49)* What number is halfway between 4.5 and 6.7?

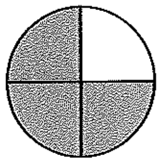
3. *(15)* It is said that each year of a dog's life is equivalent to 7 years of a human's life. In that case, a dog that is 13 years old is the equivalent age of a human that is how many years old?

4. *(75)* To compare these fractions, first convert each fraction to decimal form:

$$\frac{2}{5} \bigcirc \frac{1}{4}$$

5. (73) (a) What fraction of the circle is shaded?

(b) Convert the answer to part (a) to a decimal number.



6. (73) Convert $2\frac{1}{2}$ to a decimal number.

7. (72) Write 3.45 as a reduced mixed number.

8. (72) Write 0.04 as a reduced fraction.

9. (42) Instead of dividing 200 by 18, Sam found half of each number and then divided. Show Sam's division problem and write the quotient as a mixed number.

10. (59) $6\frac{1}{3} + 3\frac{1}{4} + 2\frac{1}{2}$

11. (41) $\frac{4}{5} = \frac{?}{100}$

12. (71) $\left(2\frac{1}{2}\right)\left(3\frac{1}{3}\right)\left(1\frac{1}{5}\right)$

13. (67) $5 \div 2\frac{1}{2}$

14. (42) $6.7 + 0.48 + n = 8$

15. (42) $12 - d = 4.75$

16. (39) 0.35×0.45

17. (51) $4.3 \div 100$

18. (43) Arrange these numbers in order from least to greatest:

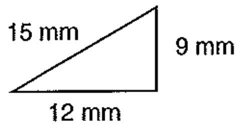
0.3, 0.25, 0.313

19. (16) Estimate the sum of 3926 and 5184 to the nearest thousand.

20. (61) List all the prime numbers between 40 and 50.

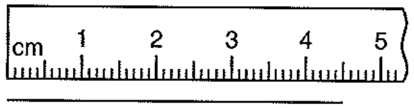
21. (42) $47.6 - w = 28.4$

22. (8) What is the perimeter of the triangle?

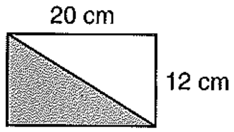


23. (63) Draw a quadrilateral that is not a rectangle.

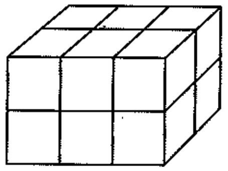
24. *(7)* About how many **millimeters** long is the line segment?



25. *(31)* One half of the area of the rectangle is shaded. What is the area of the shaded region?



26. *(66)* How many small cubes were used to make this rectangular prism?



27. *(74)* Sketch a coordinate plane on graph paper. Graph point *A* (1, 2), point *B* (–3, –2), and point *C* (1, –2). Then draw segments to connect each point. What type of polygon is figure *ABC*?

28. *(68)* In the figure drawn in problem 27, which segment is perpendicular to segment *AC*?

29. *(45)* If b equals 12 and h equals 9, then what does $\frac{bh}{2}$ equal?

30. *(68)* Draw a pair of parallel lines. Draw a third line perpendicular to the parallel lines. Complete a quadrilateral with a fourth line that intersects but is not perpendicular to the pair of parallel lines. Trace over the quadrilateral that is formed. Is the quadrilateral a rectangle?

LESSON 76

Finding Unstated Information in Fractional-Part Problems

Facts Practice: Linear Measure Facts (Test K in Test Masters)

Mental Math: Count up and down by $\frac{1}{8}$'s between $\frac{1}{8}$ and 2.

a. 311×5
b. $565 - 250$
c. $5 \times \$1.99$
d. $\$7.50 + \$1.99$
e. Double 80¢
f. $6.5 \div 100$
g. $10 \times 10, \times 10, - 1, \div 9, - 11, \div 10$
h. Hold your hands one meter apart; then one yard apart.

Problem Solving: Jill read an average of 45 pages a day for four days. If she read a total of 123 pages during the first three days, then how many pages did she read on the fourth day?

Often fractional-parts statements contain more information than is directly stated. Consider this fractional-part statement.

> *Three fourths of the 28 students in the class are boys.*

This sentence directly states information about the number of boys in the class. It also *indirectly* states information about the number of girls in the class. In this lesson we will practice finding several pieces of information from fractional-part statements.

Example Three fourths of the 28 students in the class are boys. Make a sketch that illustrates this statement; then answer the following questions.

(a) Into how many parts is the class divided?

(b) How many students are in each part?

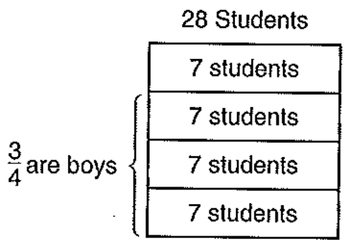
(c) How many parts are boys?

(d) How many boys are in the class?

(e) How many parts are girls?

(f) How many girls are in the class?

Solution We sketch a rectangle to represent the whole class. Since the problem describes $\frac{3}{4}$ of the class, we divide the rectangle into four parts. Dividing the total number of students by four, we find there are seven students in each fourth. We identify three of the four parts as boys. Now we will answer the questions.



(a) The denominator of the fraction indicates that the class is divided into **four parts** for the purpose of this statement. It is important to distinguish between the number of *parts* (as indicated by the denominator) and the number of *categories*. There are two categories of students implied by the statement—boys and girls.

(b) In each of the four parts there are **seven students.**

(c) **Three parts are boys.**

(d) Since three parts are boys, and since there are seven students in each part, we find that there are **21 boys** in the class.

(e) Three of the four parts are boys, so only **one part is girls.**

(f) There are seven students in each part, so there are **seven girls.**

Practice* Make a sketch to illustrate the following statement, and then answer the questions.

Three eighths of the 40 little engines could climb the hill.

a. Into how many parts was the group divided?

b. How many engines are in each part?

c. How many parts could climb the hill?

d. How many engines could climb the hill?

e. How many parts could not climb the hill?

f. How many engines could not climb the hill?

Teacher Note: A tagboard display of a parallelogram is suggested in Lesson 79. You may want to start preparing this early. Please refer to Lesson 79 for instructions.

Problem set 76

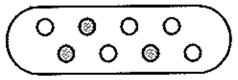
1. (29) The weight of an object on the moon is $\frac{1}{6}$ of its weight on earth. A person weighing 114 pounds on earth would weigh how much on the moon?

2. (22) Use the information in problem 1 to calculate what your weight would be on the moon.

3. (29) Cupid shot 24 arrows and hit 6 targets. What fraction of his shots hit the target?

4. (76) There are 30 students in the class. Three fifths of them are boys. Make a sketch to illustrate this statement. Then use this information to answer questions (a)–(d).
   (a) Into how many parts was the class divided?
   (b) How many students are in each part?
   (c) How many boys are in the class?
   (d) How many girls are in the class?

5. (73) (a) Find the fraction of the group that is shaded.
   (b) Convert the fraction in part (a) to a decimal number.



6. (72) Write the decimal number 3.6 as a mixed number.

7. (72) $3^2 - 2^3$

8. (30) $\frac{2}{5}x = 1$

9. (22) Three fifths of a dollar is how many cents?

10. (28) Three fifths of a circle is what percent of a circle?

11. (14) A temperature of –3°F is how many degrees below the temperature at which water freezes?

12. (75) Compare: 0.35 ○ $\frac{7}{20}$

13. (56) $\frac{1}{2} + \frac{2}{3}$

14. (62) $3\frac{1}{5} - 1\frac{3}{5}$

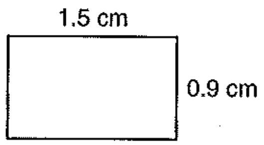
15. (59) $\frac{1}{2} + \frac{3}{4} + \frac{7}{8}$

16. (65) $3 \times 1\frac{1}{3}$

17. (67) $3 \div 1\frac{1}{3}$

18. (67) $1\frac{1}{3} \div 3$

19. (8) What is the perimeter of the rectangle?



20. (31) What is the area of the rectangle?

21. (12) Which digit in 6734.2198 is in the ones' place?

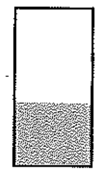
22. (42) $3.6 + a = 4.15$

23. (50) Round $357.64 to the nearest dollar.

24. (63) Is every quadrilateral a polygon?

25. (32) What time is one hour and fourteen minutes before noon?

26. (28) What percent of the rectangle appears to be shaded?



A. 20%　　B. 40%

C. 60%　　D. 80%

27. Sketch a coordinate plane on graph paper. Graph point W (2, 3), point X (1, 0), point Y (–3, 0), and point Z (–2, 3). Then draw $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, and $\overline{ZW}$.
(74)

28. (a) Which segment in problem 27 is parallel to $\overline{WX}$?
(68)
(b) Which segment in problem 27 is parallel to $\overline{XY}$?

29. Write the prime factorization of the numerator and the denominator of this fraction. Then reduce the fraction.
(66)

$$\frac{210}{350}$$

30. The moon has the shape of what geometric solid?
(66)

LESSON 77

Liquid Measure

Facts Practice: Write 24 Mixed Numbers as Improper Fractions (Test J in Test Masters)

Mental Math: Count up and down by 3's between negative 15 and 15.

a. 4 × 325
b. 1500 + 275
c. 3 × $2.99
d. $20.00 − $2.99
e. $\frac{1}{3}$ of $2.40
f. 1.75 × 100
g. 9 × 11, + 1, ÷ 2, − 1, ÷ 7, − 2, × 5
h. Hold your hands one foot apart; then 18 inches apart.

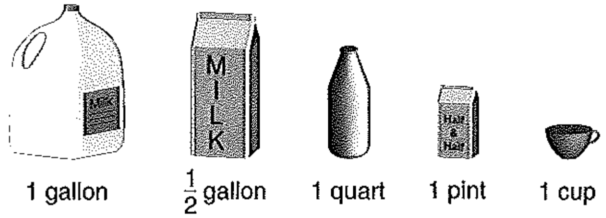
Problem Solving: Jim was thinking of a prime number between 75 and 100 which did **not** have a 9 as one of its digits. Of what number was he thinking?

To measure quantities of liquid we use units like gallons (gal), quarts (qt), pints (pt), and ounces (oz) in the U.S. Customary System, and we use liters (L) and milliliters (mL) in the metric system. The relationships between the units within each system are shown in the following table.

Equivalence Table for Units of Liquid Measure

U.S. Customary System	Metric System
1 gallon = 4 quarts	1 liter = 1000 milliliters
1 quart = 2 pints	
1 pint = 2 cups	
1 pint = 16 ounces	
1 cup = 8 ounces	

Commonly used container sizes in the U.S. Customary system are illustrated below. Notice that the next smaller container size is half the capacity of the larger container. Also, notice that a quart is a "quarter" of a gallon.



Food and beverage containers often have both U.S. Customary and metric capacities printed on the containers. Relating the two systems of measure, we find that one liter is a little more than one quart.

Example 1 A half-gallon of milk is how many pints of milk?

Solution Two pints equal a quart, and two quarts equal a half-gallon. So a half-gallon of milk is **4 pints.**

Example 2 Compare: 12 oz pop can ○ 1 pint container

Solution A pint equals 16 ounces. **So a pint container is larger than a 12-ounce pop can.**

Practice

a. What fraction of a gallon is a quart?

b. A 2-liter pop bottle has a capacity of how many milliliters?

c. A half-gallon of orange juice will fill how many 8-ounce cups?

Problem set 77

1. *(54)* What is the difference when the product of $\frac{1}{2}$ and $\frac{1}{2}$ is subtracted from the sum of $\frac{1}{2}$ and $\frac{1}{2}$?

2. *(7)* The claws of a Siberian tiger are 10 centimeters long. How many millimeters long is that?

3. *(26)* Sue was thinking of a number between 40 and 50 that is a multiple of 3 and 4. Of what number was she thinking?

4. *(76)* Make a sketch to illustrate the following statement and use the information to answer questions (a) through (d).

Four fifths of the 60 lights were on.

(a) Into how many parts have the 60 lights been divided?
(b) How many lights are in each part?
(c) How many lights are "on"?
(d) How many lights are "off"?

5. *(64)* Which counting number is neither a prime number nor a composite number?

6. *(30)* $\frac{4}{5}m = 1$

7. *(42)* $\frac{4}{5} + w = 1$

8. *(42)* $\frac{4}{5} \div x = 1$

9. *(42)* $y - \frac{4}{5} = 1$

10. *(73)* (a) What fraction of the rectangle is shaded?

(b) Write the answer to part (a) as a decimal number.

11. *(72)* Convert the decimal number 1.15 to a mixed number.

12. *(75)* Compare: $\frac{3}{5}$ ○ 0.35

13. *(56)* $\frac{5}{6} - \frac{1}{2}$

14. *(41)* $\frac{3}{4} = \frac{?}{100}$

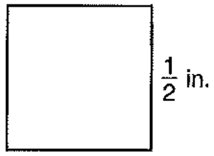
15. *(59)* $\frac{1}{2} + \frac{2}{3} + \frac{5}{6}$

16. *(71)* $1\frac{1}{2} \times 2\frac{2}{3}$

17. *(67)* $1\frac{1}{2} \div 2\frac{2}{3}$

18. *(67)* $2\frac{2}{3} \div 1\frac{1}{2}$

19. *(38)* What is the perimeter of the square?



20. *(38)* What is the area of the square?

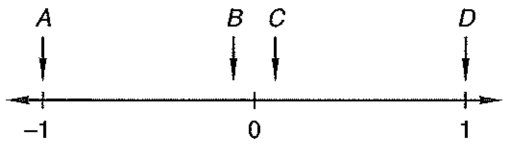
21. *(63)* The opposite sides of a rectangle are parallel. True or false?

22. *(72)* What is the average of 3^3 and 5^2?

23. *(50)* Round 1.3579 to the hundredths' place.

24. *(65)* How many inches is $2\frac{1}{2}$ feet?

25. *(49)* Which arrow could be pointing to 0.1?



26. *(63)* Draw a polygon that is not a quadrilateral.

27. *(64,72)* Find the prime factorization of 900 by using a factor tree. Then write the prime factorization using exponents.

28. *(74)* Three vertices of a rectangle have the coordinates (5, 3), (5, –1), and (–1, –1). What are the coordinates of the fourth vertex of the rectangle?

Refer to this table to answer questions 29 and 30.

3 teaspoons = 1 tablespoon
16 tablespoons = 1 cup
2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

29. *(77)* A teaspoon of soup is what fraction of a tablespoon of soup?

30. *(77)* How many cups of milk is a gallon of milk?

LESSON
78

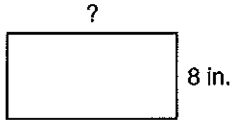
Classifying Quadrilaterals

Facts Practice: Linear Measure Facts (Test K in Test Masters)

Mental Math: Count up and down by 12's between 12 and 144.

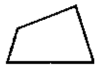
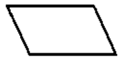
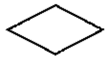
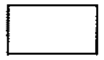
a. 307 × 6 **b.** 1000 − 420 **c.** 4 × $2.99
d. $5.75 + $2.99 **e.** Double $24 **f.** 0.125 × 100
g. 2 × 2, × 2, × 2, − 1, × 2, + 2, ÷ 2, ÷ 2
h. Hold your hands one inch apart; then one centimeter apart.

Problem Solving: The perimeter of the rectangle is 48 inches. The width is 8 inches. What is the length?



Quadrilaterals are polygons with four sides. Quadrilaterals are classified in the following way:

Quadrilaterals

SHAPE	CHARACTERISTIC	NAME
	No sides parallel	Trapezium
	One pair of parallel sides	Trapezoid
	Two pairs of parallel sides	Parallelogram
	Parallelogram with equal sides	Rhombus
	Parallelogram with right angles	Rectangle
	Rectangle with equal sides	Square

Notice that rhombuses, rectangles, and squares are all parallelograms. Also notice that a square is a special kind of rectangle, which is a special kind of parallelogram, which is a special kind of quadrilateral, which is a special kind of polygon. A square is also a special kind of rhombus.

Example Is the following statement true or false?

"All parallelograms are rectangles."

Solution We are asked to decide if every parallelogram is a rectangle. Since a rectangle is a special kind of parallelogram, some parallelograms are rectangles. However, some parallelograms are not rectangles. So the statement is **false.**

Practice State whether each statement is true or false:

a. All quadrilaterals are four-sided polygons.

b. Some parallelograms are trapezoids.

c. Every square is a rhombus.

d. Every rhombus is a square.

e. Some rectangles are squares.

Problem set 78

1. *(8)* If you know the perimeter of a rectangle and the length of the rectangle, how can you figure out the width of the rectangle?

2. *(77)* A 2-liter beverage bottle contained 2 qt, 3.6 oz of beverage. Use this information to compare a liter and a quart.

Compare: 1 liter ○ 1 quart

3. *(11)* Uncle Bill was 38 when he started his job. He worked for 33 years. How old was he when he retired?

4. *(78)* Is the following statement true or false?

"Every rectangle is a square."

5. *(78)* "Every rectangle is a parallelogram." True or false?

6. *(28)* Ninety percent of the 30 students were right-handed. What percent of the students were left-handed?

7. (76) If $\frac{3}{4}$ of the 24 runners finished the race, then how many runners did not finish the race?

8. (72) $10^3 \div 10^2$

9. (38) $6.42 + 12.7 + 8$

10. (42) $10 - q = 9.87$

11. (39) 1.2×0.12

12. (44) $0.288 \div 24$

13. (48) $64 \div 0.08$

14. (58) $3\frac{1}{3} + 2\frac{3}{4}$

15. (42) $w + \frac{1}{4} = \frac{5}{6}$

16. (71) $3\frac{1}{3} \times \frac{1}{5} \times \frac{3}{4}$

17. (67) $2\frac{1}{2} \div 3$

18. (38) The perimeter of a square is 80 cm. What is its area?

19. (45) Write the decimal number for the following:

$$(9 \times 10) + (6 \times 1) + \left(3 \times \frac{1}{100}\right)$$

20. (45) If b equals 6 and h equals 8, then what does $\frac{1}{2}bh$ equal?

21. (49) Which of these numbers is closest to zero?

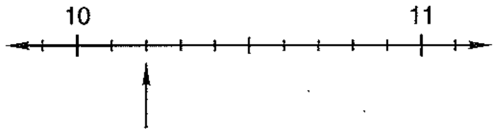
$$-2,\ 0.2,\ 1,\ \frac{1}{2}$$

22. (50) Estimate the product of 6.7 and 7.3 by rounding each number to the nearest whole number before multiplying.

23. (72) The fourth power of 2 (which is 2^4) equals 16. What number does the fourth power of 3 equal?

24. (18) What number is halfway between 0.2 and 0.3?

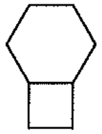
25. (49) To what decimal number is the arrow pointing?



26. (78) Which quadrilateral has one pair of parallel sides but not two pairs of parallel sides?

27. (74) The coordinates of the vertices of a quadrilateral are (–5, 5), (1, 5), (3, 1), and (–3, 1). What is the name for this kind of quadrilateral?

In this figure a square and a regular hexagon share a common side. The area of the square is 100 sq. cm. Use this information to answer problems 28 and 29.



28. (63) (a) What is the length of each side of the square?

(b) What is the perimeter of the square?

29. (63) (a) What is the length of each side of the hexagon?

(b) What is the perimeter of the hexagon?

30. (66) Write the prime factorization of the numerator and the denominator of this fraction. Then reduce the fraction.

$$\frac{32}{48}$$

LESSON
79

Area of a Parallelogram

Facts Practice: 28 Fractions to Simplify (Test I in Test Masters)

Mental Math: Count up and down by 7's between negative 35 and 35.

a. 4 × 315
b. 380 + 170
c. 5 × $2.99
d. $10.00 − $7.99
e. $\frac{1}{4}$ of $4.80
f. 37.5 ÷ 100
g. 5 × 5, × 5, − 25, ÷ 4, ÷ 5, − 5
h. Hold your hands a meter apart; then four feet apart.

Problem Solving: A seven-digit phone number consists of a three-digit prefix followed by four digits. How many different phone numbers are possible for a particular prefix?

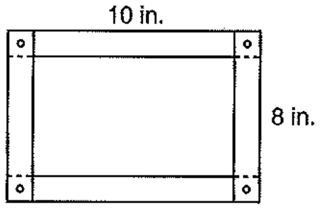
A flexible model of a parallelogram is useful when discussing the area of a parallelogram. A model can be constructed of stiff tagboard or cardboard and brads.

Materials needed:

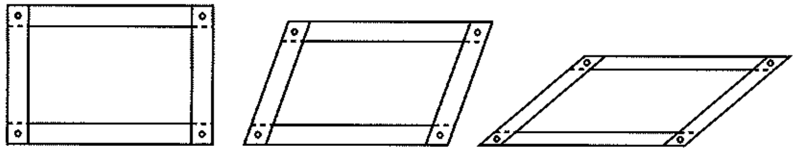
- Two strips of tagboard 1 in. × 10 in.
- Two strips of tagboard 1 in. × 8 in.
- Hole punch
- 4 brads

Assembly:

Lay the two 8 in. strips over the two parallel 10 in. strips as shown. Punch a hole at the center of the overlapping ends. Insert and open brads to hold the strips together.

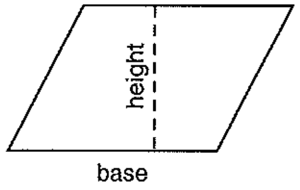


With this model we can demonstrate that the area of the parallelogram changes as the angles between the adjacent sides change. We hold the model with two hands and slide the opposite sides in opposite directions.



Although the area changes as the angles change, the opposite sides remain parallel, and the perimeter does not change.

The flexible model shows that the two parallelograms may have sides that are equal in length but areas that are different. To find the area of a parallelogram, we multiply two **perpendicular** measurements. We multiply the **base** and the **height** of the parallelogram.



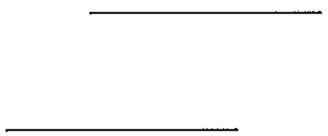
The base of a parallelogram is the length of one of the parallel sides. The height of a parallelogram is the perpendicular distance between the parallel sides. The following activity will illustrate why the area of a parallelogram equals the base times the height.

Activity: Area of a Parallelogram

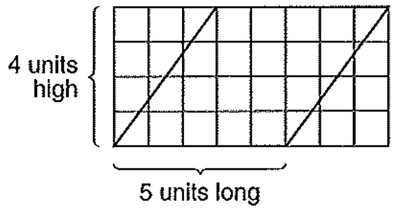
Materials needed:

- Graph paper
- Ruler
- Pencil
- Scissors

Step 1. Tracing over the lines on the graph paper, draw two parallel segments the same number of units long but shifted slightly as shown.

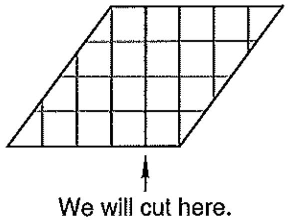


Then draw segments between the endpoints of the pair of parallel segments to complete the parallelogram.

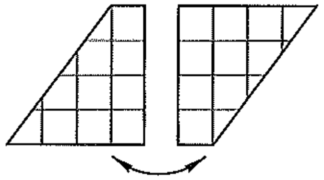


The parallelogram we drew is 5 units long and 4 units high. Your parallelogram may be different. How many units long and high is your parallelogram? Can you easily count the number of square units in the area of your parallelogram?

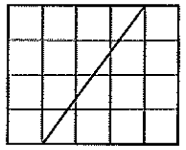
Step 2. Use your scissors to cut out your parallelogram.



Then select a line on the graph paper that is perpendicular to one of the parallel sides of the parallelogram and cut the parallelogram into two pieces.



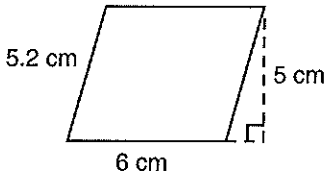
Step 3. Rearrange the two pieces of the parallelogram to make a rectangle. What is the length and width of the rectangle? How many square units is the area of the rectangle?



Our rectangle is 5 units long and 4 units wide. The area of the rectangle is 20 square units. So the area of the parallelogram is also 20 square units.

By making a perpendicular cut across the parallelogram and rearranging the pieces, we formed a rectangle with the same area as the parallelogram. The length and width of the rectangle equaled the base and height of the parallelogram. By multiplying the base and height of a parallelogram, we find the area of a parallelogram.

Example Find the area of this parallelogram.



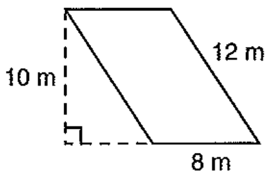
Solution We multiply two perpendicular measurements, the base and the height. The height is often shown as a dashed line segment. The base is 6 cm. The height is 5 cm.

$$6 \text{ cm} \times 5 \text{ cm} = 30 \text{ sq. cm}$$

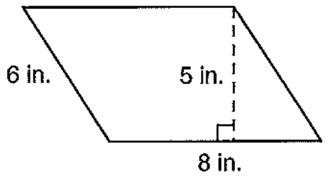
The area of the parallelogram is **30 sq. cm.**

Practice Find the perimeter and area of these parallelograms:

a.



b.



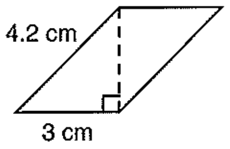
Problem set 79

1. *(18)* What is the average of 96, 49, 68, and 75?

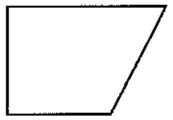
2. *(15)* The average depth of the ocean beyond the edges of the continents is $2\frac{1}{2}$ miles. How many feet is that? (1 mile = 5280 ft)

3. *(15)* The 168 girls who signed up for soccer were divided into 12 teams. How many players were on each team?

4. *(78)* What is the perimeter of the parallelogram?



5. *(78)* This quadrilateral has one pair of parallel sides. What is the name of this kind of quadrilateral?



6. *(78)* "All squares are rectangles." True or false?

7. *(76)* If four fifths of the 30 students in the class were present, then how many students were absent?

8. *(29)* If $\frac{1}{6}$ of the one-hour show was taken up with commercials, then how many minutes did the commercials last?

9. Compare: 0.5 ○ $\frac{3}{4}$
(75)

10. Write 4.4 as a reduced mixed number.
(72)

11. Write $\frac{1}{8}$ as a decimal number.
(73)

12. (6.3)(0.36)
(39)

13. $0.36 \div 5$
(44)

14. $63 \div 0.9$
(48)

15. $\frac{5}{6} + \frac{1}{2}$
(56)

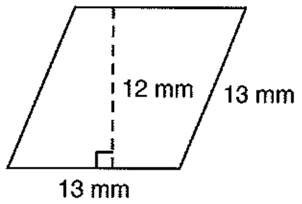
16. $\frac{5}{8} - \frac{1}{4}$
(56)

17. $2\frac{1}{2} \times 1\frac{1}{3} \times \frac{3}{5}$
(71)

18. $\frac{17}{20} = \frac{?}{100}$
(41)

19. $1\frac{1}{2} \div 3$
(67)

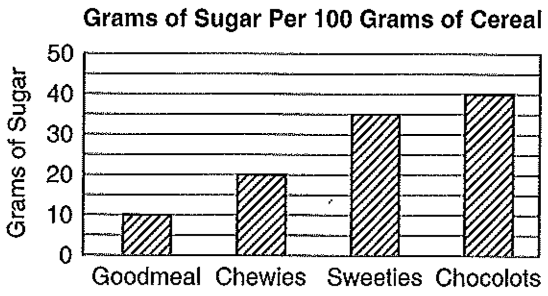
20. What is the area of the parallelogram?
(79)



21. Round 0.4287 to the hundredths' place.
(50)

22. $4 - a = 2.6$
(42)

Use the graph of sugar in breakfast cereals to answer questions 23, 24, and 25.



23. *Sweeties* contains about how many grams of sugar per 100 grams of cereal?
(16)

24. Fifty grams of *Chocolots* would contain about how (16) many grams of sugar?

25. Write a "larger-smaller-difference" problem that refers (16) to the bar graph and answer the question.

26. There was a quart of milk in the bottle. Oscar poured (77) one cup of milk on his cereal. How many cups of milk were left in the bottle?

27. Three vertices of a square are (3, 0), (3, 3), and (0, 3). (74) What are the coordinates of the fourth vertex of the square?

Refer to the square in problem 27 to answer problems 28 and 29.

28. What is the perimeter of the square? (38)

29. What is the area of the square? (38)

30. Draw a pair of parallel segments that are the same (78) length. Make a quadrilateral by drawing two more segments between the endpoints of the parallel segments. Is the quadrilateral a parallelogram?

LESSON 80

Arithmetic with Units of Measure

Facts Practice: Linear Measure Facts (Test K in Test Masters)

Mental Math: Count up and down by 25's between negative 150 and 150.

a. 311 × 7 **b.** 2000 − 1250 **c.** 4 × $9.99

d. $2.50 + $9.99 **e.** Double $5.50 **f.** 0.075 × 100

g. 8 × 8, + 6, ÷ 2, + 1, ÷ 6, × 3, ÷ 2

h. Hold your hands six inches apart; then 6 cm apart.

Problem Solving: Copy this problem and fill in the missing digits.

Recall that the operations of arithmetic are addition, subtraction, multiplication, and division. In this lesson we will practice adding, subtracting, multiplying, and dividing units of measure.

We may add or subtract measurements that have the same units. If the units are not the same, we first convert one or more measurements so that the units are the same. Then we add or subtract.

Example 1 2 ft + 12 in.

Solution The units are not the same. Before we add we either convert 2 feet to 24 inches, or we convert 12 inches to 1 foot.

CONVERT TO INCHES

2 ft + 12 in.

24 in. + 12 in. = **36 in.**

CONVERT TO FEET

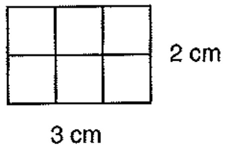
2 ft + 12 in.

2 ft + 1 ft = **3 ft**

Either answer is correct because 3 ft equals 36 in.

Notice that the units of the sum in Example 1 are the same as the units of the addends. The units do not change when we add or subtract measurements. However, the units *do* change when we multiply or divide measurements.

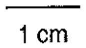
When we find the area of a figure we multiply the lengths. Notice how the units change when we multiply.



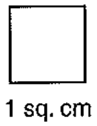
To find the area of this rectangle we multiply 2 cm and 3 cm. The product has a different unit of measure than the factors.

$$2\ \textbf{cm} \times 3\ \textbf{cm} = 6\ \textbf{sq. cm}$$

A centimeter and a square centimeter are two different kinds of units. A centimeter is a line segment used to measure length.



A square centimeter is a square used to measure area.



The unit of the product is a different unit because we multiplied the units of the factors. When we multiply 2 cm and 3 cm, we multiply the 2 and the 3 and we also multiply the cm and cm.

$$2\text{ cm} \times 3\text{ cm} = \underbrace{2 \cdot 3}_{6}\ \underbrace{\text{cm} \cdot \text{cm}}_{\text{sq. cm}}$$

Instead of writing sq. cm, we may use exponents to write cm · cm as cm^2. We read cm^2 as "square centimeters."

$$2 \text{ cm} \times 3 \text{ cm} = \underbrace{2 \cdot 3}_{6}\ \underbrace{\text{cm} \cdot \text{cm}}_{\text{cm}^2}$$

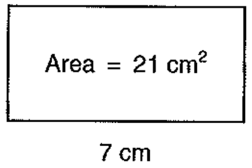
Example 2 6 ft × 4 ft

Solution We multiply the number of units, and we also multiply the units.

$$6 \text{ ft} \times 4 \text{ ft} = \underbrace{6 \cdot 4}_{24}\ \underbrace{\text{ft} \cdot \text{ft}}_{\text{ft}^2}$$

The product is **24 ft^2,** which is also 24 sq. ft.

Units also change when we divide measurements. For example, if we know the area of a rectangle and the length of the rectangle, we can find the width of the rectangle by dividing.



To find the width of this rectangle, we divide 21 cm^2 by 7 cm.

$$\frac{21 \text{ cm}^2}{7 \text{ cm}} = \frac{\overset{3}{\cancel{21}}}{\underset{1}{\cancel{7}}}\ \frac{\cancel{\text{cm}} \cdot \text{cm}}{\cancel{\text{cm}}}$$

We divide the numbers and reduce the units. The quotient is 3 cm, which is the width of the rectangle.

Example 3 $\dfrac{25 \text{ mi}^2}{5 \text{ mi}}$

Solution To divide the units, we write mi² as mi · mi and reduce.

$$\frac{\overset{5}{\cancel{25}}}{\underset{1}{\cancel{5}}}\ \frac{\cancel{\text{mi}} \cdot \text{mi}}{\cancel{\text{mi}}}$$

The quotient is **5 mi.**

Sometimes when we divide measurements the units will not reduce. When units will not reduce, we leave the units in division form. For example, if a car travels 300 miles in 6 hours, we can find the average speed of the car by dividing.

$$\frac{300\text{ mi}}{6\text{ hr}} = \frac{\overset{50}{\cancel{300}}}{\underset{1}{\cancel{6}}}\ \frac{\text{mi}}{\text{hr}}$$

The quotient is $50\frac{\text{mi}}{\text{hr}}$, which is 50 miles per hour (50 mph).

The word "per" means "each" and is used in place of the division sign. Notice that speed is a quotient of distance divided by time.

Example 4 $\frac{300\text{ miles}}{10\text{ gallons}}$

Solution We divide the numbers. The units do not reduce.

$$\frac{300\text{ mi}}{10\text{ gal}} = \frac{\overset{30}{\cancel{300}}}{\underset{1}{\cancel{10}}}\ \frac{\text{mi}}{\text{gal}}$$

The quotient is $\mathbf{30\frac{mi}{gal}}$, or 30 miles per gallon.

Practice

a. 2 ft − 12 in. (Write the difference in inches.)

b. 2 ft × 4 ft

c. $\frac{12\text{ cm}^2}{3\text{ cm}}$

d. $\frac{300\text{ mi}}{5\text{ hr}}$

Problem set 80

1. *(77)* The Jones family had two gallons of milk before breakfast. The family used two quarts of milk during breakfast. How many quarts of milk did the Jones family have after breakfast?

2. *(77)* One quart of milk is about 945 milliliters of milk. Use this information to help you with this comparison:

Compare: 1 quart ○ 1 liter

3. *(65)* Carol cut $2\frac{1}{2}$ inches off her hair three times last year. How much longer would her hair have been if she had not cut it?

4. *(80)* The plane flew 1200 miles in 3 hours. Divide the distance by the time to find the average speed of the plane.

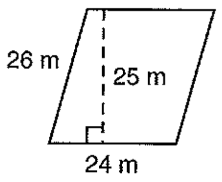
5. *(66)* Write the prime factorization of the numerator and denominator of this fraction. Then reduce the fraction.

$$\frac{54}{135}$$

6. *(29,33)* The basketball team scored 60% of its 80 points in the second half. Write 60% as a reduced fraction. Then find the number of points the team scored in the second half.

7. *(79)* What is the area of the parallelogram?

8. *(78)* What is the perimeter of the parallelogram?



9. *(78)* "Some rectangles are trapezoids." True or false?

10. *(45)* If b equals 12 and h equals 16, then what does $\frac{1}{2}bh$ equal?

11. *(75)* Arrange these numbers in order from least to greatest:

$$\frac{1}{2}, \frac{1}{5}, 0.4$$