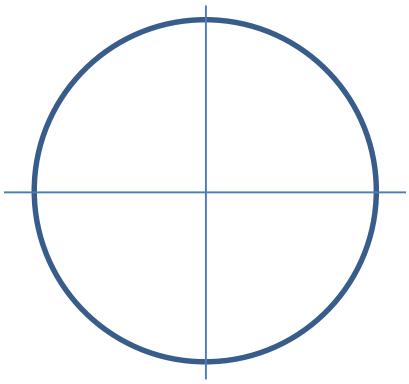
4-4 Trigonometric Functions of Any Angle

Objective:

a. Evaluate trigonometric functions of any angle;
b. Use reference angles to evaluate trigonometric functions;
c. Evaluate trigonometric functions of real numbers.

For any angle Θ, we can draw the angle in a circle of radius r with endpoint of the terminal side located at (x, y).



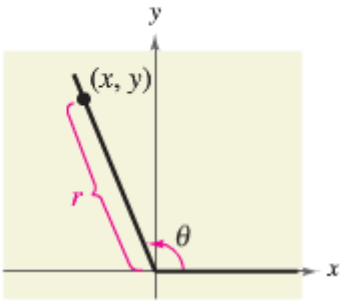
Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}, \quad x \neq 0 \qquad \cot\theta = \frac{x}{y}, \quad y \neq 0$$

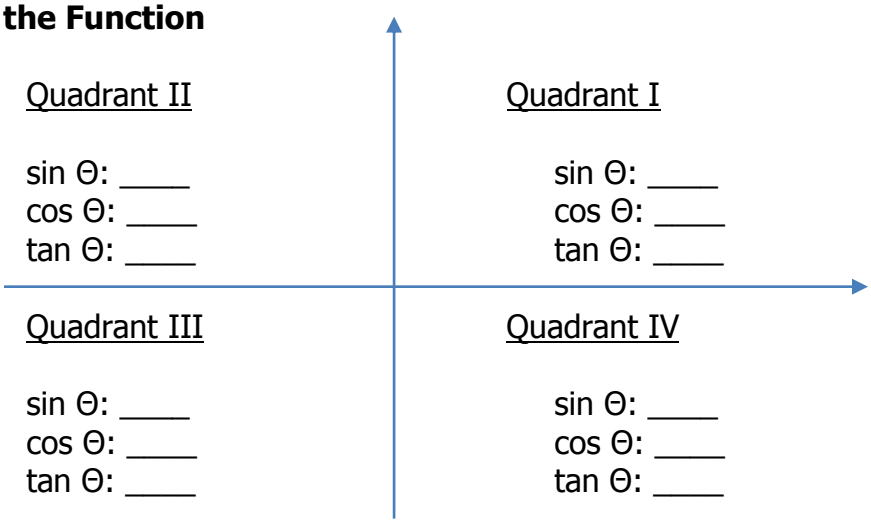
$$\sec\theta = \frac{r}{x}, \quad x \neq 0 \qquad \csc\theta = \frac{r}{y}, \quad y \neq 0$$



Example:

Let (-3, 4) be a point on the terminal side of Θ. Find the sine, cosine, and tangent of Θ.

Sign of the Function



Quadrant II

sin Θ: _____
cos Θ: _____
tan Θ: _____

Quadrant I

sin Θ: _____
cos Θ: _____
tan Θ: _____

Quadrant III

sin Θ: _____
cos Θ: _____
tan Θ: _____

Quadrant IV

sin Θ: _____
cos Θ: _____
tan Θ: _____

Example 1:

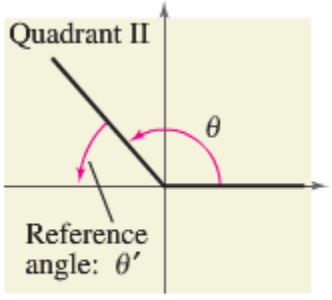
Given tan Θ = $-\frac{5}{4}$ and cos Θ > 0, find sin Θ and sec Θ.

Example 2:

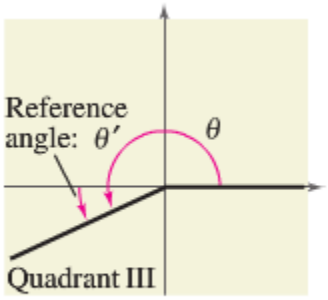
Evaluate the cosine and tangent functions at the four quadrant angles 0, $\frac{\pi}{2}, \pi, \text{ and } \frac{3\pi}{2}$.

Recall Reference Angles

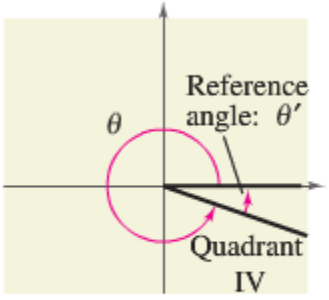
As a way to simplify trigonometric calculations, every angle can be converted to a corresponding acute angle called the _____. Every angle that is already in Quadrant I does not need to be converted.



$\theta' = \pi - \theta$ (radians)
$\theta' = 180° - \theta$ (degrees)



$\theta' = \theta - \pi$ (radians)
$\theta' = \theta - 180°$ (degrees)



$\theta' = 2\pi - \theta$ (radians)
$\theta' = 360° - \theta$ (degrees)

PRACTICE

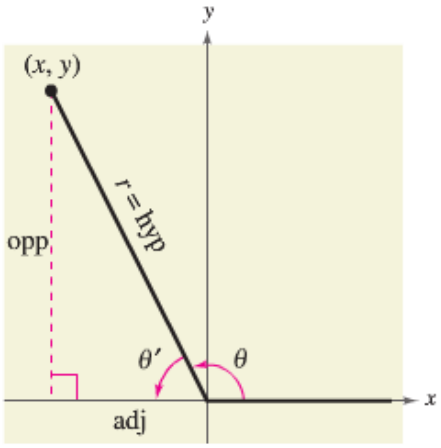
1) $\theta = 80°$; $\theta' =$ _____

2) $\theta = \frac{5\pi}{3}$; $\theta' =$ _____

3) $\theta =$ -260°; $\theta' =$ _____

4) $\theta = -\frac{7\pi}{6}$; $\theta' =$ _____

Trigonometric Functions of Real Numbers



opp = $|y|$, adj = $|x|$

Evaluating Trigonometric Functions of Any Angle

To find the value of a trigonometric function of any angle θ:

1. Determine the function value for the associated reference angle θ'.
2. Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

Examples: Find each trigonometric function.

1) $cos\frac{4\pi}{3} =$

2) $\tan(-210°) =$

3) $csc\frac{11\pi}{4} =$

Example: Let Θ be an angle in Quadrant II such that sin Θ = 1/3. Find (a) cos Θ and (b) tan Θ by using trigonometric identities.