Name

# **3-3 Properties of Logarithms**

## **Objective:**

- a. Use change-of-base formula to rewrite and evaluate logarithmic expressions;
- b. Use properties of logarithms to evaluate or rewrite logarithmic expressions;
- c. Use properties of logarithms to expand or condense logarithmic expressions.

## Change-of-Base Formula

Previously, we discussed using the calculator to evaluate both \_\_\_\_\_\_ logarithms (base 10) and \_\_\_\_\_\_ logarithms (base *e*).

In order to evaluate logarithms with other bases, you need to use the **change-of-base formula.** 

Change-of-Base Formula

Let a, b, and x be positive real numbers such that  $a \neq 1$  and  $b \neq 1$ . Then  $\log_a x$  can be converted to a different base as follows.

Base b	Base 10	Base e
$\log_a x = \frac{\log_b x}{\log_b a}$	$\log_a x = \frac{\log x}{\log a}$	$\log_a x = \frac{\ln x}{\ln a}$

Examples: Evaluate the following logarithms.

## PRACTICE

Rewrite the logarithm as a ratio of (a) common logarithms and (b) natural logarithms

1)  $\log_3 x$  2)  $\log_{1/3} x$  3)  $\log_x \frac{3}{4}$ 

Evaluate the logarithm.

**4)** log<sub>7</sub> 4

### **Properties of Logarithms**

Logarithmic expressions can be simplified when they have the same base (b) by applying one or a combination of logarithmic properties associated with the properties of multiplication, division and exponents.

What is log 10?	If u = 10 and v = 100, then uv = 1000.
What is log 100?	therefore
What is log 1000?	

# **Properties of Logarithms**

Let *a* be a positive number such that  $a \neq 1$ , and let *n* be a real number. If *u* and *v* are positive real numbers, the following properties are true.

		Logarithm with Base a	Natural Logarithm
1.	Product Property:	$\log_a(uv) = \log_a u + \log_a v$	$\ln(uv) = \ln u + \ln v$
2.	Quotient Property:	$\log_a \frac{u}{v} = \log_a u - \log_a v$	$\ln\frac{u}{v} = \ln u - \ln v$
3.	Power Property:	$\log_a u^n = n \log_a u$	$\ln u^n = n \ln u$

## **Using Properties of Logarithms.**

Examples: Write each logarithm in terms of In 2 and In 3

a) ln 6 b) ln  $\frac{2}{3}$  c) ln  $\frac{9}{16}$ 

## PRACTICE

Find the exact value of each expression without using a calculator.

1)  $\log_5 \sqrt[3]{5} =$ \_\_\_\_\_

2)  $ln e^6 - ln e^2 =$  \_\_\_\_\_

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## Expanding/Condensing a Logarithmic Expression

Use the quotient property, product property and power property of logarithms to expand or condense the following.

1) Expand  $\log_4 5x^3y$ 

- Product property
- Power property

2) Expand  $\ln \frac{\sqrt{3x-5}}{7}$ 

- Rewrite using rational exponent
- Quotient property
- Power property

3) Expand  $\ln \frac{xz^2}{y}$  (you try)

- 4) Condense  $\frac{1}{2} \log x + 3 \log(x+1)$ 
  - Power property
  - Product property
- 5) Condense  $2\ln(x+2) \ln x$ 
  - Power property
  - Quotient property
- 6) Condense  $\frac{1}{3}[\log_2 x + \log_2(x+1)]$  (you try)