CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

✓ ✓

rate out ⬅

| t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

✓ ✓ ✓ ✓

I.C.

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

(b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

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a) $R'(2) = \dfrac{950 - 1190}{3 - 1} = -120 \ L/hr^2$

b) $1(1340) + 2(1190) + 3(950) + 2(740) = 8050$ Liters.
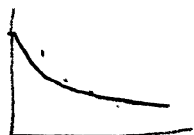
Since $R(t)$ is decreasing, LRAM gives an <u>overestimate</u> of the total amt of H_2O removed.

c) $I.C + IN - OUT$

$50,000 + \displaystyle\int_0^8 2000e^{-t^2/20} \, dt - 8050 \approx 49,786 \ L$

7836.195

d) $W(0) = 2000$
$W(8) = 81.524$

Since $W(t)$ and $R(t)$ are continuous and differentiable, by IVT at some time b/w $t=0$ & $t=8$, $W(t)$ takes on all values b/w 2000 & 81.524, so $W(t) = R(t)$ since $R(t) \ge 700$,

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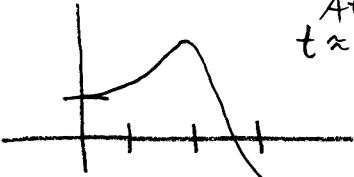
2. For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by

$v(t) = 1 + 2\sin\left(\dfrac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$. ~~I.C.~~

✓ (a) At time $t = 4$, is the particle speeding up or slowing down? $v(t) = 0$

✓ (b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

✓ (c) Find the position of the particle at time $t = 0$.

✓ (d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.
Split ∫

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a) $v(4) = 1 + 2\sin(8) \approx 2.979$ Since $v(4) + a(4)$ have opposite

$a(4) = -1.164$ signs, the particle slows down at $t=4$.

b) $0 = 1 + 2\sin\left(\dfrac{t^2}{2}\right)$

At $t \approx 2.707$, the $v(t)$ changes from > 0 to < 0, so the particle changes from moving right to left +
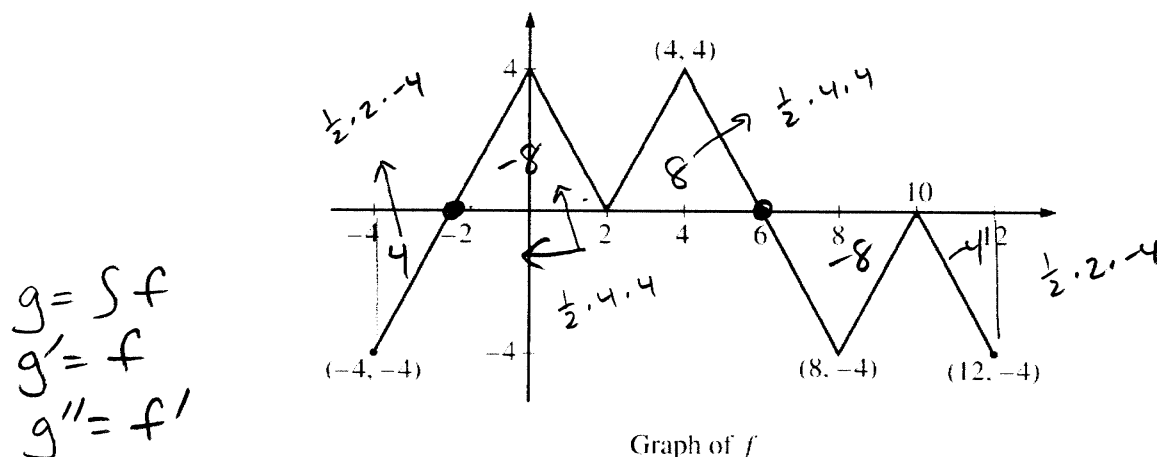
END OF PART A OF SECTION II

c) $\displaystyle\int_0^4 v(t)\,dt \approx 5.815$ $I.P + 5.815 = 2$

$I.P = -3.815$

d) $\displaystyle\int_0^3 |v(t)|\,dt \approx 5.301$ or $\displaystyle\int_0^{2.707} v(t)\,dt + \left|\int_{2.707}^3 v(t)\,dt\right|$

5.13695 $.164247$

CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.

Graph of f

$g = \int f$
$g' = f$
$g'' = f'$

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_{2}^{x} f(t)\, dt.$

   (a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

   (b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

   (c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

   (d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

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a) Neither. $g' = f$, and f does not switch signs at $x = 10$,

b) Yes, because $g'' = f'$ and f' switches from positive to negative when $x = 4$,

c) Max/mins occur when $g' = f$ switches signs, which is at $x = -2, 6$, or at the endpts, $x = -4, 12$

$\int_{2}^{-2} f(t)\, dt = -8$ $\int_{2}^{6} f(t)\, dt = 8$ $\int_{2}^{-4} f(t)\, dt = -4$ $\int_{2}^{12} f(t)\, dt = 4$

     abs min   abs max

d) $[-4, 2] \cup [10, 12]$

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4. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

$\dfrac{4}{-1}$

$\dfrac{4}{1}$

$\dfrac{1}{-1}$

$\dfrac{1}{1}$

BAm!
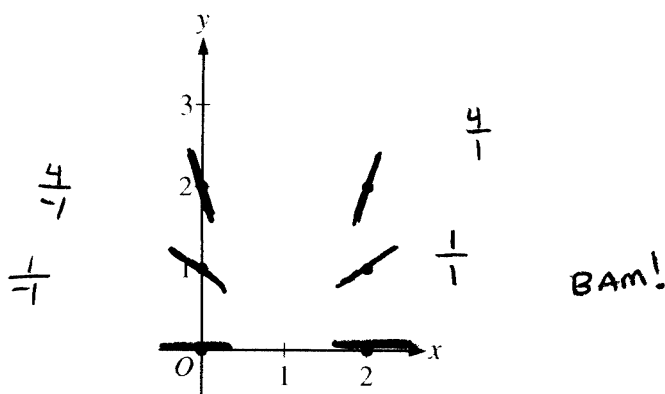
(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.

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b) $\dfrac{dy}{dx} = \dfrac{9}{1} = 9$ $y - 3 = 9(x-2)$

$y = 9x - 15$

$y = 9(2.1) - 15$

$= 18.9 - 15 = 3.9$

c) $\displaystyle\int \dfrac{1}{y^2}\, dy = \int \dfrac{1}{x-1}\, dx$

$-\dfrac{1}{y} = \ln|x-1| + C$

$\dfrac{1}{y} = -\ln|x-1| + C$

$y = \dfrac{-1}{\ln|x-1| + C}$

$3 = \dfrac{-1}{\ln 1 + C}$

$3C = -1$

$c = -\dfrac{1}{3}$

$y = \dfrac{-1}{\ln|x-1| - \frac{1}{3}} = \dfrac{-1}{\frac{3\ln|x-1| - 1}{3}} = \dfrac{-3}{3\ln|x-1| - 1}$

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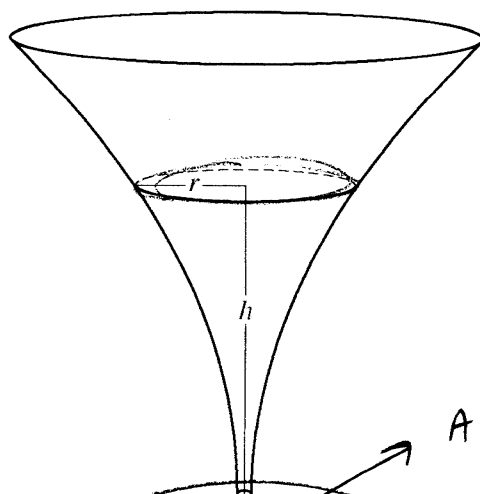
$A = \pi r^2$

5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \dfrac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

$\dfrac{dr}{dt} = \dfrac{-1}{5}$ $h = 3$

✓ (a) Find the average value of the radius of the funnel.

✓ (b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\dfrac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

$\dfrac{dh}{dt} = ?$

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a) $\dfrac{1}{10}\displaystyle\int_0^{10} \left(\dfrac{1}{20}(3+h^2)\right) dh = \dfrac{1}{200}\int_0^{10} 3+h^2 \, dh = \dfrac{1}{200}\left(3h + \dfrac{1}{3}h^3\right)\Big|_0^{10}$

$\dfrac{1}{200}\left(30 + \dfrac{1000}{3}\right)$

$\dfrac{1}{200}\left(\dfrac{1090}{3}\right) = \dfrac{1090}{600}$ in.

b) $\pi \displaystyle\int_0^{10} \left(\dfrac{1}{20}(3+h^2)^2\right) dh$

$\dfrac{\pi}{400}\displaystyle\int_0^{10} h^4 + 6h^2 + 9 \, dh$

$\dfrac{\pi}{400}\left(\dfrac{1}{5}h^5 + 2h^3 + 9h \Big|_0^{10}\right)$

$\dfrac{\pi}{400}\left(\dfrac{100000}{5} + 2000 + 90\right)$

$\dfrac{\pi}{400}\left(22,090\right)$

$\dfrac{22090\pi}{400}$

$\dfrac{2209\pi}{40}$ in

c) $r = \dfrac{3}{20} + \dfrac{h^2}{20}$

$\dfrac{dr}{dt} = \dfrac{h}{10}\dfrac{dh}{dt}$

$\dfrac{-1}{5} = \dfrac{3}{10}\dfrac{dh}{dt}$

$\dfrac{dh}{dt} = \dfrac{-10}{15} = -\dfrac{2}{3}$ in/sec.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	−6	3	2	8
2	2	−2	−3	0
3	8	7	6	2
6	4	5	3	−1

6. The functions f and g have <u>continuous second derivatives</u>. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the <u>line tangent</u> to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\, dx$.

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a) $\quad f(g(3)) = f(6) = 4 \qquad (3,4) \qquad\qquad y-4 = 10(x-3)$

$\quad k'(x) = f'(g(x)) \cdot g'(x) \qquad\qquad\qquad y = 10x - 26$

$\qquad = f'(6) \cdot g'(3) = 5 \cdot 2 = 10$

STOP

END OF EXAM

b) $\quad h'(x) = \dfrac{f(x) g'(x) - g(x) f'(x)}{(f(x))^2} = \dfrac{-6 \cdot 8 - 2 \cdot 3}{36} = \dfrac{-48-6}{36} = \dfrac{-54}{36} = \dfrac{-3}{2}$

$\quad x=1$

c) $\quad \int_1^3 f''(2x)\, dx \qquad u = 2x$

$\qquad\qquad\qquad\qquad du = 2\, dx$

$\quad \frac{1}{2}\left(f'(u) \Big|_2^6 \right) = \frac{1}{2}\left(5 - -2 \right) = \frac{7}{2}$

$\qquad\qquad\qquad\qquad\qquad\qquad$ Trump!
$\qquad\qquad\qquad\qquad\qquad\qquad$ 2016.

$\qquad$ This problem was HUUUUUGELY EASY!