

**Vocabulary:** Fundamental counting principle, permutations, combinations, factorial, independent event, dependent event, mutually exclusive, compound probability, Venn Diagram, odds in favor, odds against, term, arithmetic sequence, arithmetic series, geometric sequence, geometric series

**Skill**

1 - Determine the number of outcomes using the FCP, Permutations or Combinations.

2 - Calculate basic and compound probability.

3 - Determine the odds of a situation.

4 - Write a formula to represent a given arithmetic or geometric sequence.

5 - Determine the sum of an arithmetic or geometric series.

6 - Determine the sum of a series using summation notation.

**Skill 1 - Determine the number of outcomes using the FCP, Permutations or Combinations.**

1) The letters A, B, C, and D, are used to form four letter passwords for entering a computer file. How many passwords are possible if letters can be repeated any number of times?

$$4 \cdot 4 \cdot 4 \cdot 4$$

$$\boxed{256}$$

2) How many ways can the first five letters of the alphabet be arranged if each is used only once?

$$5P_5 \text{ or } 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$\boxed{120}$$

3) A restaurant serves 5 main dishes, 3 salads and 4 desserts. How many different meals could be ordered if each has a main dish, salad and dessert?

$$5 \cdot 3 \cdot 4$$

$$\boxed{60}$$

4) How many different ways can 4 different books be arranged on the shelf?

$$4P_4 \text{ or } 4 \cdot 3 \cdot 2 \cdot 1$$

$$\boxed{24}$$

5) How many 5-digit even numbers can be formed using the digits 4, 6, 7, 2, 8 if the digits can be repeated any number of times?

$$5 \cdot 5 \cdot 5 \cdot 5 \cdot 4$$

$$\boxed{2,500}$$

6) How many 4-digit positive even integers are there?

$$9 \cdot 10 \cdot 10 \cdot 5$$

$$\boxed{4,500}$$

7) How many license plate numbers consisting of three letters followed by three numbers are possible when repetition is allowed?

$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$$

$$\boxed{1,757,600}$$

8) How many combinations are possible using the information in problem 15 if no repetition is allowed?

$$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8$$

$$11,232,000$$



Skill 1, Continued - Determine the number of outcomes using the FCP, Permutations or Combinations.

9) How many ways can the letters in MONDAY be arranged?

$$6P_6$$

\*no repeats in MONDAY

$$720$$

10) How many ways can the letters in CENTRAL BUCKS be arranged?

$$\frac{12!}{2!} = 239,500,800$$

11) How many ways can the letters in NITTANY LIONS be arranged?

$$\frac{12!}{3!2!2!} = 199,584,000$$

12) How many ways can 8 members of a family be seated side-by-side in a movie theater?

$$8 \ 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1$$

$$40,320$$

13) How many ways can 8 member of a family be seated side-by-side in a movie theater if the father is seated on the aisle?

$$1 \ 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1$$

$$5040$$

14) How many ways can 3 different books be placed on a shelf if chosen from a selection of 7 books?

$$7P_3$$

$$210$$

15) How many 5-digit numbers can be made using the digits from 19,323?

$$\frac{5!}{2!} = 60$$

16) Eight playing cards are to be arranged in a row, no two cards are exactly alike, how many different arrangements of 8 cards are possible?

$$8P_8$$

$$40,320$$

17) There are 15 different books. How many groups of 6 books can be selected?

$$15C_6 = 5005$$

18) From a group of 10 men and 12 women how many committees of 5 men and 6 women can be formed?

$$10C_5 * 12C_6$$

$$232,848$$

19) From a standard deck of 52 cards how many ways can 5 cards be dealt?

$$52P_5 = 31,875,200$$

20) How many tennis teams of 6 players can be formed from 14 players without regard to position?

$$14C_6 = 3003$$



Skill 2 - Calculate basic and compound probability. Express your answer as a fraction in simplest form.

1) A card is drawn from a standard deck of 52 cards. Find the probability of selecting a Jack or a Ten. \* no overlap

$$\frac{8}{52} = \boxed{\frac{2}{13}}$$

2) A card is drawn from a standard deck of 52 cards. Find the probability of selecting a Queen or a Spade. overlap!

$$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \boxed{\frac{4}{13}}$$

3) A card is drawn from a standard deck of 52 cards. Find the probability of selecting Black or a Face Card. \* overlap

$$\frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \boxed{\frac{8}{13}}$$

4) Each time a card is drawn from a standard deck of cards, it is not replaced. Find the probability of selecting an Ace then a 6.

$$\frac{4}{52} \times \frac{4}{51} = \boxed{\frac{4}{663}}$$

5) Each time a card is drawn from a standard deck of cards, it is not replaced. Find the probability of selecting two Aces in a row.

$$\frac{4}{52} \times \frac{3}{51} = \boxed{\frac{1}{221}}$$

6) Each time a card is drawn from a standard deck of cards, it is not replaced. Find the probability of selecting two Clubs in a row.

$$\frac{13}{52} \times \frac{12}{51} = \boxed{\frac{1}{17}}$$

7) A pair of dice are thrown. Find the probability of rolling doubles (the numbers matching).

$$\frac{6}{36} = \boxed{\frac{1}{6}}$$

8) A pair of dice are thrown. Find the probability of getting a sum of exactly 10.

$$\frac{3}{36} = \boxed{\frac{1}{12}}$$

Skill 3 - Determine the odds of a situation. Express your answer as a ratio in simplest form.

1) The odds of an event occurring are 1:3. Find the probability of that event.

$$\frac{1}{4}$$

2) The odds of an event occurring are 5:2. Find the probability of that event NOT occurring.

$$\frac{2}{7}$$

3) The probability of an event is 10/13. Find the odds of the event occurring.

$$\frac{10}{3}$$

4) Determine the odds of selecting a King from a standard deck of cards. 4/52 < prob.

$$4:48$$
  

$$\boxed{1:12}$$

5) Determine the odds of selecting a black card from a standard deck of cards. 26/52 < prob

$$26:26$$
  

$$\boxed{1:1}$$

6) Determine the odds of selecting either an 8 or a Heart from a standard deck of cards. 4+13-1 → 16/52 < prob

$$16:36$$
  

$$\boxed{4:9}$$



Skill 4 - Write a formula to represent each arithmetic or geometric sequence. Then find  $a_{10}$ .

1)  $3, 7, 11, 15, \dots$   $+4$   
 $a_1 = 3$   $d = 4$   
 $a_n = 3 + (n-1)4$   
 $a_{10} = -1 + 40$   
 $a_n = -1 + 4n$   
 $a_{10} = 39$

2)  $3, 9, 27, 81, \dots$   $\times 3$   
 $a_1 = 3$   
 $a_n = 3 \cdot (3)^{n-1}$   
 $a_{10} = 3 \cdot 3^{10-1}$   
 $a_{10} = 59049$   
 $\ast$  technically this is  $a_n = 3^n$

3)  $-6, -11, -16, -21, \dots$   $d = -5$   
 $a_n = -6 + (n-1)(-5)$   
 $-6 + -5n + 5$   
 $a_n = -1 - 5n$   
 $a_{10} = -1 - 5(10)$   
 $a_{10} = -1 - 50$   
 $a_{10} = -51$

4)  $80, -40, 20, -10, 5, \dots$   $r = -\frac{1}{2}$   
 $a_n = 80 \cdot \left(-\frac{1}{2}\right)^{n-1}$   
 $a_{10} = 80 \cdot \left(-\frac{1}{2}\right)^{10-1}$   
 $a_{10} = 80 \cdot \left(-\frac{1}{2}\right)^9$   
 $a_{10} = -0.15625 \text{ or } -\frac{5}{32}$   
 $a_{10} = -0.15625 \text{ or } -5/32$

Skill 5 - Determine the sum of the first 15 terms for each arithmetic or geometric series.

1)  $3 + 6 + 9 + 12 + \dots$   $n = 15$   
 $S_{15} = 15 \left( \frac{3 + 45}{2} \right)$   
 $a_{15} = 3 + (15-1)(3)$   
 $a_{15} = 45$   
 $S_{15} = 360$

2)  $19 + 17 + 15 + 13 + \dots$   $n = 15$   
 $S_{15} = 15 \left( \frac{19 + (-9)}{2} \right)$   
 $a_{15} = 19 + (15-1)(-2)$   
 $a_{15} = -9$   
 $S_{15} = 75$

3)  $2 + 4 + 8 + 16 + 32 + \dots$   $\times 2$   
 $S_{15} = 2 \left( \frac{1 - 2^{15}}{1 - 2} \right)$   
 $S_{15} = 65534$

4)  $20 + 10 + 5 + 2.5 + 1.25 + \dots$   $\times \frac{1}{2}$   
 $S_{15} = 20 \left( \frac{1 - \left(\frac{1}{2}\right)^{15}}{1 - \left(\frac{1}{2}\right)} \right)$   
 $S_{15} = 39.999$

Skill 6 - Determine the sum of a series using summation notation.

1)  $\sum_{n=1}^5 4n - 5$   $= 35$   
 $4(1) - 5 + 4(2) - 5 + 4(3) - 5 + 4(4) - 5 + 4(5) - 5$   
 $-1 + 3 + 7 + 11 + 15$

2)  $\sum_{n=1}^4 n^2 + 3$   $= 42$   
 $(1)^2 + 3 + (2)^2 + 3 + (3)^2 + 3 + (4)^2 + 3$   
 $4 + 7 + 12 + 19$