Unit 6 Additional Practice

SECTION 1: Applications

Find the interest and the balance in the account for each given situation.

		Interest	Amount
1.	\$1,250 at 7% annually for 3 years	\$281.30	\$1531.30
2.	\$23,600 at 5% semi-annually for 10 years	\$15,071.35	\$38,671.35
3.	\$5,000 at 12% monthly for 5 years	\$4083.48	\$9083.48
4.	\$51,275 at 6.5% quarterly for 8.5 years	\$37,425.21	\$88,700.21
5.	\$7,250 at 18% annually for 15 years	\$79,559.67	\$86,809.67
6.	\$100,000 at 8% monthly for 25 years	\$634,017.60	\$734,017.60

Find the interest and the balance in the account for compounding continuously.

		Interest	Amount
7.	\$10,000 at 9% for 5 years	\$5,683.12	\$15,683.12
8.	\$240,000 at 7% for 25 years	\$1,141,164.64	\$1,381,164.64
9.	\$4,500 at 19% for 10 years	\$25,586.52	\$30,086.52
10.	\$500 at 5% for 2 years 6 months	\$66.57	\$566.57
11.	\$1,750 at 6.25% for 36 months	\$360.90	\$2110.90
12.	\$17,625 at 4.5% for 7.5 years	\$7075.37	\$24,700.37

Apply the appropriate formula to solve the following compound interest application problems.

13. Matt received a total of \$900 for his graduation.

a) If he invests in a local bank that pays 4.5% APR compounded quarterly, how much will he have in 4 years? (Assume he makes no withdrawals or deposits)

balance

$$A = 900\left(1 + \frac{.045}{4}\right)^{4*4} = \boxed{\$\,1076.41}$$

b) If another bank offers 4.5% APR compounded continuously, how much will he have in 4 years?

balance

$$A = 900e^{.045*4} = \boxed{\$1077.50}$$

14. A bank offers an APR of 8.5% and compounds interest semi-annually for savings accounts. If you were to deposit \$2250, what is the value of the account in 5 years?

$$A = 2250\left(1 + \frac{.085}{2}\right)^{2*5} = \boxed{\$\,3411.48}$$

Calculate an approximate value for the following expressions involving e to 3 decimal places.

15. $e^{2.3}$ 9.974

16. $e^{4.6}$ 99.484

17. $\sqrt{e}$ 1.649

18. $2\sqrt[3]{e^4}$ 7.587

19. $3\sqrt{e^3}$ = 13.445

20. e^{-2} 0.135

Unit 6 Additional Practice

Solve the following exponential application problems using growth/decay formula or the stated exponential model in the problem.

21. The function describing the number of a rare birds that are found in a specific region after ***t months*** is given by

$$P(t) = 150e^{.05t} \quad where \quad t \geq 0.$$

a) What is the initial population of rare birds? Is this a situation of growth or decay?

150 birds

growth

b) What is the population of rare birds after 7 months?

$P(7) = 150e^{(.05*7)} = 212.86 \rightarrow \boxed{213 \text{ birds}}$

c) What is the population of rare birds after 1 year?

$P(12) = 150e^{(.05*12)} = 273.32 \rightarrow \boxed{274 \text{ birds}}$

d) What is the population of rare birds after 1 decade? → 10 yrs → 120 months

$P(120) = 150e^{(.05*120)} = 60,514.32 \rightarrow \boxed{60,515 \text{ birds}}$

---

22. The population of a town is 50,000, and local authorities claim that the population is growing at an exponential rate of 4% per year.

a) Define the function that describes this situation: $\boxed{P(t) = 50,000(1.04)^t}$

b) Use your function to predict the population in 5 years?

$P(5) = 50,000(1.04)^5 \rightarrow 60,832.65 \rightarrow \boxed{60,833 \text{ people}}$

c) Use your function to predict the population in 10 years?

$P(10) = 50,000(1.04)^{10} \rightarrow 74,012.21 \rightarrow \boxed{74,013 \text{ people}}$

e) Use your function to predict the population in 25 years?

$P(25) = 50,000(1.04)^{25} \rightarrow 133,291.82 \rightarrow \boxed{133,292 \text{ people}}$

---

* 23. The number of people infected by the flu in a particular region after t hours is given by:

$$P(t) = 5e^{.03t} \quad where \quad t \geq 0.$$

a) Is this a growth or decay problem? What is the rate of growth/decay?

$\boxed{\text{growth}}$ $0.03 \rightarrow \boxed{3\%}$

b) What is the initial population of people infected by the flu?

$\boxed{5 \text{ people}}$

c) What is the population of people infected by the flu after 12 hours?

$P(12) = 5e^{.03(12)} = 7.17 \rightarrow \boxed{8 \text{ people}}$

d) What is the population of people infected by the flu after 1 day? $t = 24$

$P(24) = 5e^{.03(24)} = 10.27 \rightarrow \boxed{11 \text{ people}}$

e) What is the population of people infected by the flu after 1 week? $t = 168$

$P(168) = 5e^{.03*168} = 772.35 \rightarrow \boxed{773 \text{ people}}$

Unit 6 Additional Practice

*

24. The population of mosquitoes after t days is given by:

$$P(t) = 500e^{-.055t} \quad where \;\; t \geq 0.$$

a) Is this a growth or decay problem? What is the rate of growth/decay?

$\boxed{\text{Decay}}$ $-.055 \rightarrow \boxed{5.5\%}$

b) What is the initial population of mosquitoes?

$\boxed{\text{500 mosquitoes}}$

c) What is the population of mosquitoes after 1 day?

$P(1) = 500e^{-.055*1} \rightarrow 473.24 \rightarrow \boxed{\text{474 mosquitoes}}$

d) What is the population of mosquitoes after 72 hours? $t=3$

$P(3) = 500e^{-.055*3} \rightarrow 423.95 \rightarrow \boxed{\text{424 mosquitoes}}$

e) What is the population of mosquitoes after 2 weeks?

$t = 14$

$P(14) = 500e^{-0.055*14} \rightarrow 231.5 \rightarrow \boxed{\text{232 mosquitoes}}$

---

25. The number of computers infected by a certain virus that is rapidly spreading is determined by the following model: $V(t) = 25e^{.6t}$ where t is represented in hours.
Determine the number of computers infected after:

a) **1 hour?** $V(1) = 25e^{.6*1} = 45.6 \rightarrow \boxed{\text{46 computers}}$

b) **10 hours?** $V(10) = 25e^{.6*10} = 10{,}085.7 \rightarrow \boxed{\text{10,086 computers}}$

c) **1 day?** 24 hrs. $V(24) = 25e^{.6*24} = 44{,}851{,}869.32 \rightarrow \boxed{\text{44,851,870 computers}}$

---

26 . If the annual rate of inflation averages 4% over the next ten years, then the cost of goods or services in that decade will be determined according to the following cost model: $C(t) = P(1.04)^t$. Estimate the cost of the following goods/services in
2 years, 5 years and **10 years.**

The following represent the price of items/services at the present time (P).

	2 years	*5 years*	*10 years*
a) Gallon of Milk costs $3.00	$3.24	$3.65	$4.44
b) A ticket to the Movies costs $7.00	$7.57	$8.52	$10.36
c) Oil change costs $19.99	$21.62	$24.32	$29.59

Unit 6 Additional Practice

SECTION 2: Evaluating Exponential and Logarithmic Functions

Use the definition of a logarithm to write the given equation in logarithmic form.

1.	$5^3 = 125$	$\log_5 125 = 3$	6.	$81^{1/4} = 3$	$\log_{81} 3 = \frac{1}{4}$
2.	$6^{-2} = \frac{1}{36}$	$\log_6 \frac{1}{36} = -2$	7.	$10^{-3} = 0.001$	$\log 0.001 = -3$
3.	$e^3 = 20.085$	$\ln 20.085 = 3$	8.	$e^0 = 1$	$\ln 1 = 0$
4.	$e^x = 4$	$\ln 4 = x$	9.	$u^v = w$	$\log_u w = v$
5.	$8^2 = 64$	$\log_8 64 = 2$	10.	$9^{3/2} = 27$	$\log_9 27 = 3/2$

Use the definition of a logarithm to write the given equation in exponential form.

11.	$\log_2 8 = x$	$2^x = 8$	16.	$\ln 143 = x$	$e^x = 143$
12.	$\log_5 625 = 4$	$5^4 = 625$	17.	$\log 1000 = 3$	$10^3 = 1000$
13.	$\log_x 13 = 5$	$x^5 = 13$	18.	$\ln x = 14$	$e^{14} = x$
14.	$\log_2 \frac{1}{8} = -3$	$2^{-3} = 1/8$	19.	$\log \frac{1}{100} = -2$	$10^{-2} = \frac{1}{100}$
15.	$\log_4 64 = 3$	$4^3 = 64$	20.	$\ln 18 = x$	$e^x = 18$

Use your calculator to evaluate the following. Round to four decimal places.

21.	log 68	1.8325	26.	ln 9548	9.1641
22.	log 100	2	27.	log 0.0001	−4
23.	ln 9	2.1972	28.	log 17	1.2304
24.	log 10	1	29.	ln 125	4.8283
25.	ln 216	5.3753	30.	log 6158	3.7894

Use the change of base formula to evaluate. Round to four decimal places.

31.	$\log_3 7 =$	1.7712	36.	$\log_{0.5} 4 =$	−2
32.	$\log_9 0.4 =$	−.4170	37.	$\log_{15} 1250 =$	2.6332
33.	$\log_7 4 =$	0.7124	38.	$\log_4 0.55 =$	−0.4312
34.	$\log_{20} 125 =$	1.6117	39.	$\log_{1/3} 0.015 =$	3.8227
35.	$\log_6 95 =$	2.5416	40.	$\log_{17} 2 =$	0.2447

Unit 6 Additional Practice

SECTION 3: Solving Exponential and Logarithmic Equations

Solve the following exponential equations. (Round answers to three decimal places)

1. $10^x = 42$

$\log 42 = x$

$\boxed{x \approx 1.623}$

2. $\frac{1}{3}\left(10^{2x}\right) = 12$

$10^{2x} = 36$

$\log 36 = 2x$

$\boxed{x \approx 0.778}$

3. $3\left(10^{x-1}\right) = 2$

$10^{x-1} = \frac{2}{3}$

$\log \frac{2}{3} = x-1$

$\boxed{x \approx .824}$

4. $e^x = 10$

$\ln 10 = x$

$\boxed{x \approx 2.303}$

5. $2^{3x} = 565$

$\log_2 565 = 3x$

$\boxed{x \approx 3.047}$

6. $1000e^{-4x} = 75$

$e^{-4x} = .075$

$\ln .075 = -4x$

$\boxed{x \approx 0.648}$

7. $25e^{2x+1} = 962$

$e^{2x+1} = 38.48$

$\ln 38.48 = 2x+1$

$\boxed{x \approx 1.325}$

8. $\frac{1250}{1.04^x} = \frac{500}{1}$

$1250 = 500 * 1.04^x$

$2.5 = 1.04^x$

$\log_{1.04} 2.5 = x$

$\boxed{x \approx 23.362}$

9. $e^{0.09x} = 3$

$\ln 3 = 0.09x$

$\boxed{x \approx 12.207}$

10. $\frac{1000}{1+19e^{-x/5}} = \frac{2000}{1}$

$1000 = 2000(1+19e^{-x/5})$

$.5 = 1+19e^{-x/5}$

$-.5 = 19e^{-x/5}$

$-.5/19 = e^{-x/5}$

$\ln\left(-\frac{.5}{19}\right) = \frac{-x}{5}$

$x = -5 * \ln\left(-\frac{.5}{19}\right)$

$\boxed{\varnothing}$

Unit 6 Additional Practice

Solve the following logarithmic equations. (Round answers to three decimal places)

11. $\ln x = 5$ ✓

$e^5 = x$

$\boxed{x \approx 148.413}$

12. $2\ln x = 7$ ✓

$\ln x = 3.5$

$e^{3.5} = x$

$\boxed{x \approx 33.115}$

13. $2\ln 4x = 0$ ✓

$\ln 4x = 0$

$e^0 = 4x$

$\boxed{x = \frac{1}{4}}$

14. $\log(x-3) = 2$ ✓

$10^2 = x - 3$

$\boxed{x = 103}$

15. $6\ln(x+1) = 2$ ✓

$\ln(x+1) = 1/3$

$e^{1/3} = x + 1$

$\boxed{x \approx 0.396}$

16. $\log 2 + \log x = 3$ ✓

$\log 2x = 3$

$10^3 = 2x$

$\boxed{x = 500}$

✓Challenge!

*17. $\ln x + \ln(x-2) = 1$ * CTS

$\ln(x(x-2)) = 1$

$e^1 = x(x-2)$

$1 + e = x^2 - 2x + 1$

$\sqrt{e+1} = \sqrt{(x-1)^2}$

$x = 1 \pm \sqrt{e+1}$

$\boxed{x \approx 2.928}$

~~$x \approx -.928$~~

18. $\log x - \log(2x-1) = 0$ ✓

$\log\left(\frac{x}{2x-1}\right) = 0$

$10^0 = \frac{x}{2x-1}$

$1 = \frac{x}{2x-1}$

$2x - 1 = x$

$\boxed{x = 1}$

$1/2 - 0$

19. $\log_2(x+5) - \log_2(x-2) = 3$ ✓

$\log_2 \frac{x+5}{x-2} = 3$

$2^3 = \frac{x+5}{x-2}$

$8x - 16 = x + 5$

$7x = 21$

$\boxed{x = 3}$

20. $\log_4 x - \log_4(x-1) = \frac{1}{2}$ ✓

$\log_4 \frac{x}{x-1} = \frac{1}{2}$

$4^{\frac{1}{2}} = \frac{x}{x-1}$

$2 = \frac{x}{x-1}$

$2x - 2 = x$

$\boxed{x = 2}$