Answer Key

Section 1

1.)

$$\begin{array}{r} 2x+4 \\ x+3\,\overline{)\,2x^2+10x+12} \\ -(2x^2+6x) \\ \hline 4x+12 \\ -(4x+12) \\ \hline 0 \end{array}$$

$$\boxed{2x+4}$$

2)

$$\begin{array}{r} x^3+3x^2+0-1 \\ x+2\,\overline{)\,x^4+5x^3+6x^2-x-2} \\ -(x^4+2x^3) \\ \hline 3x^3+6x^2 \\ -(3x^3+6x^2) \\ \hline 0-x \\ -(0+0) \\ \hline -x-2 \\ -(-x-2) \\ \hline 0 \end{array}$$

$$\boxed{x^3+3x^2-1}$$

Section 2

1)

$$\begin{array}{r|rrrr} 5 & 3 & -17 & 15 & -25 \\ & & 15 & -10 & 25 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$

$$\boxed{3x^2-2x+5}$$

2)

$$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 8 \\ & & 20 & 56 & 224 \\ \hline & 5 & 14 & 56 & 232 \end{array}$$

$$\boxed{5x^2+14x+56+\frac{232}{x-4}}$$

Section 5

1) As $x \to -\infty$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to -\infty$

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2) As $x \to -\infty$, $f(x) \to -\infty$
As $x \to \infty$, $f(x) \to \infty$

3) As $x \to -\infty$, $f(x) \to \infty$
As $x \to \infty$, $f(x) \to -\infty$

Section 4

1)
$$\begin{array}{r|rrrr} -2 & 1 & -5 & -8 & 12 \\ & & -2 & 14 & -12 \\ \hline & 1 & -7 & 6 & 0 \end{array}$$

YES!

$(x+2)(x^2-7x+6)$

$\boxed{(x+2)(x-6)(x-1)}$ Factors

$\boxed{\{-2, 6, 1\}}$ Zeros

2)
$$\begin{array}{r|rrrrr} 2 & 2 & 7 & -4 & -27 & -18 \\ & & 4 & 22 & 36 & 18 \\ \hline & 2 & 11 & 18 & 9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 2 & 11 & 18 & 9 \\ & & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array} \nearrow$$

$(x-2)(x+3)(2x^2+5x+3)$

$\underbrace{2x^2+2x}\underbrace{+3x+3}$

$2x(x+1)+3(x+1)$

$\boxed{(x-2)(x+3)(2x+3)(x+1)}$

$\boxed{\left\{2, -3, -\frac{3}{2}, -1\right\}}$

Section 3

1)
$$\begin{array}{r|rrrr} -2 & 3 & 8 & 5 & -7 \\ & & -6 & -4 & -2 \\ \hline & 3 & 2 & 1 & \boxed{-9} \end{array}$$

Or

$3(-2)^3+8(-2)^2+5(-2)-7$
$=\boxed{-9}$

2)
$$\begin{array}{r|rrrr} 3 & 4 & 10 & -3 & -8 \\ & & 12 & 66 & 189 \\ \hline & 4 & 22 & 63 & \boxed{181} \end{array}$$

Or

$4(3)^3+10(3)^2-3(3)-8$
$=\boxed{181}$

Section 6

1) $(x-4)(x+3)(x-1)$

$x^2+3x-4x-12$

$(x^2-x-12)(x-1)$

$x^3-x^2-12x-x^2+x+12 \doteq \boxed{x^3-2x^2-11x+12}$

2) $(x-2)(x+5)(x^2+16)$

$x^2+5x-2x-10$

$(x^2+3x-10)(x^2+16)$

$x^4+3x^3-10x^2+16x^2+48x-160$

$= \boxed{x^4+3x^3+6x^2+48x-160}$

3) $(x+3)(x-6)(x^2-3)$

$x^2+3x-6x-18$

$(x^2-3x-18)(x^2-3)$

$x^4-3x^3-18x^2-3x^2+9x+54$

$\boxed{x^4-3x^3-21x^2+9x+54}$

Section 7

1) $\underbrace{x^3+2x^2}+\underbrace{2x+4}$

$x^2(x+2)+2(x+2)$

$(x^2+2)(x+2)$

$\boxed{x=-2,\ i\sqrt{2},\ -i\sqrt{2}}$

$x^2+2=0$ $\quad$ $x+2=0$

$x^2=-2$ $\quad$ $x=-2$

$x=\pm\sqrt{-2}$

2) $x^3+27=0$

$a=x$ $\quad$ $b=3$

$(x+3)(x^2-3x+9)$

$x=-3$ $\quad$ $\dfrac{3\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)} = \dfrac{3\pm\sqrt{9-36}}{2} = \dfrac{3\pm\sqrt{-27}}{2} = \dfrac{3\pm 3i\sqrt{3}}{2} = \dfrac{3\pm3i\sqrt{3}}{2}$

$\boxed{x=-3,\ \dfrac{3\pm 3i\sqrt{3}}{2}}$

3) $x^4+x^2-20=0$

-20

$5\ \ -4 \mid 1$

$(x^2+5)(x^2-4)$

$x^2+5=0$ $\quad$ $x^2-4=0$

$x^2=-5$ $\quad$ $x^2=4$

$x=\pm\sqrt{-5}$ $\quad$ $x=\pm\sqrt{4}$

$\boxed{x=\pm i\sqrt{5} \quad x=\pm 2}$

Section 7 continued

4) $x^3 + 4x^2 - x = 0$

$x(x^2 + 4x - 1) = 0$

-1
can't | 4

$\boxed{x=0}$ $x^2 + 4x - 1 = 0$

$$\frac{-4 \pm \sqrt{16+4}}{2}$$

$$\frac{-4 \pm \sqrt{20}}{2}$$

$$\frac{-4 \pm 2\sqrt{5}}{2}$$

$$\boxed{-2 \pm \sqrt{5}}$$

$$\boxed{x = 0, -2 \pm \sqrt{5}}$$

5) $2x^4 - 22x^3 + 36x^2 = 0$

$2x^2(x^2 - 11x + 18) = 0$

18
-9 -2 | -11

$2x^2(x-9)(x-2) = 0$

$2x^2 = 0$ $x-9=0$ $x-2=0$

$x=9$ $x=2$

$x=0$

$\boxed{0, 9, 2}$

Algebra 2/Trigonometry
Test Review Graphing

Name: ______________________
Date: ______________________ Block: ________

Section 8- Write f(x) as a product of linear factors and list all of its zeros. You must verify all rational zeros (using synthetic division). Use our calculator to find the relative maxima and minima (to the nearest hundredth). Then sketch a graph.

↓↑

1.) $x^3 - x^2 - 13x - 3$ 3 real

$$\begin{array}{r|rrrr} -3 & 1 & -1 & -13 & -3 \\ & & -3 & 12 & 3 \\ \hline & 1 & -4 & -1 & 0 \end{array}$$ YES

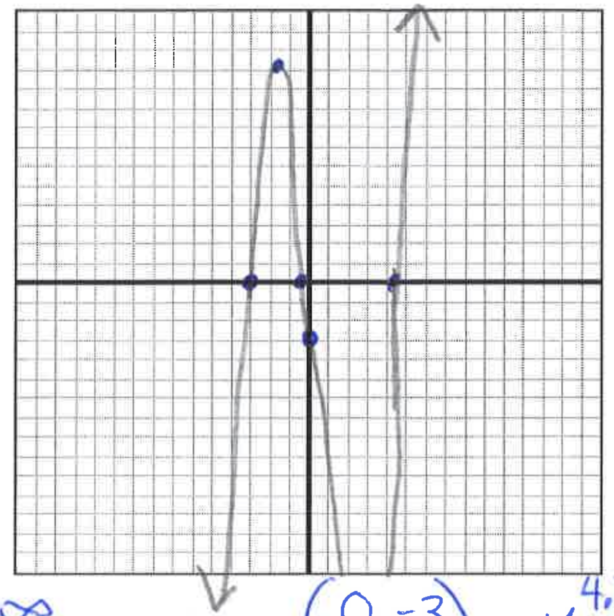
$(x+3)(x^2-4x-1)=0$

Use quadratic!

-1 / \ 1 -4 Not Factorable

$\frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2}$

$x = -3,\ 2 \pm \sqrt{5}$



End Behavior: As x→∞, f(x) → ∞, As x→-∞, f(x) → -∞   y-intercept: (0,-3)   4.24, -.24

Product of ~~Linear~~ Factors: $(x+3)(x^2-4x-1)$ Zeros: $-3, 2+\sqrt{5}, 2-\sqrt{5}$

Relative Minima/Maxima: (-1.77,11.33) Max (2.44,-26.14) Min.

↓↑

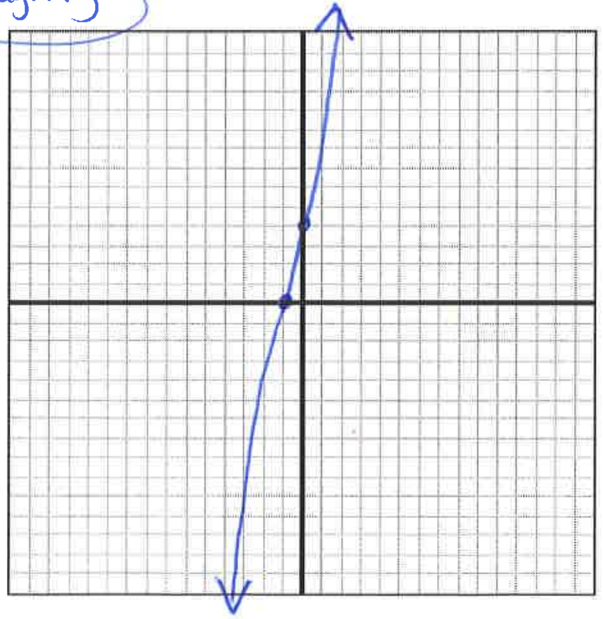
2.) $x^3 + x^2 + 4x + 4$ 1 real 2 imaginary

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 4 & 4 \\ & & -1 & 0 & -4 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$(x+1)(x^2+4)=0$

$x+1=0$
$x=-1$

$x^2+4=0$
$x^2=-4$
$x=\pm\sqrt{-4}$
$x=\pm 2i$



End Behavior: As x→∞, f(x) → ∞, As x→-∞, f(x) → -∞   y-intercept: (0,4)

Product of ~~Linear~~ Factors: $(x+1)(x^2+4)$ Zeros: $-1, 2i, -2i$

Relative Minima/Maxima: None!

Algebra 2/Trigonometry
Test Review Graphing

Name: ______________________
Date: ____________________ Block: ______

Section 8 - Write f(x) as a product of linear factors and list all of its zeros. You must verify all rational zeros (using synthetic division). Use our calculator to find the relative maxima and minima (to the nearest hundredth). Then sketch a graph.

↓↑

3.) $x^3 - 4x$ 3 real

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -4 & 0 \\ & & -2 & 4 & 0 \\ \hline & 1 & -2 & 0 & 0 \end{array}$$ YES!

or $x(x^2-4)$

$x(x+2)(x-2)$

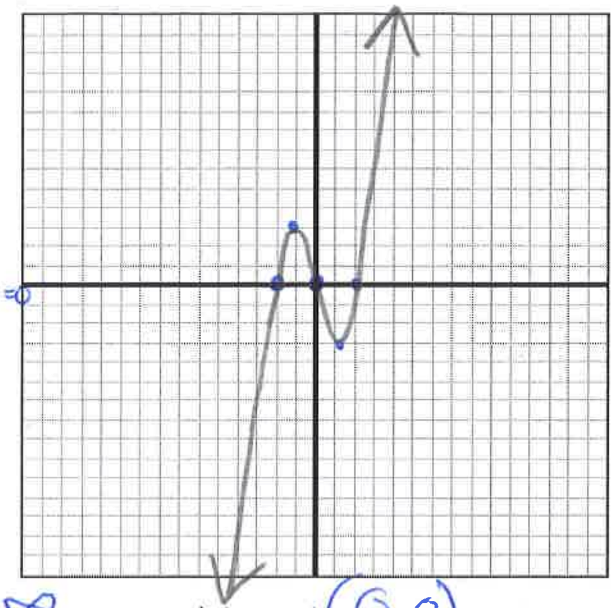
$x=0 \quad x+2=0 \quad x-2=0$

$x=0,-2,2$

$(x+2)(x^2-2x)$

$(x+2)\,x\,(x-2)$

$x = -2 \quad 0 \quad 2$



End Behavior: As x→∞, f(x) → ∞, As x→-∞, f(x) → -∞   y-intercept: (0,0)

Product of Factors: $x(x+2)(x-2)$ Zeros: 0, 2, -2

Relative Minima/Maxima: Max (-1.15, 3.08)   Min (1.15, -3.08)

↑↓

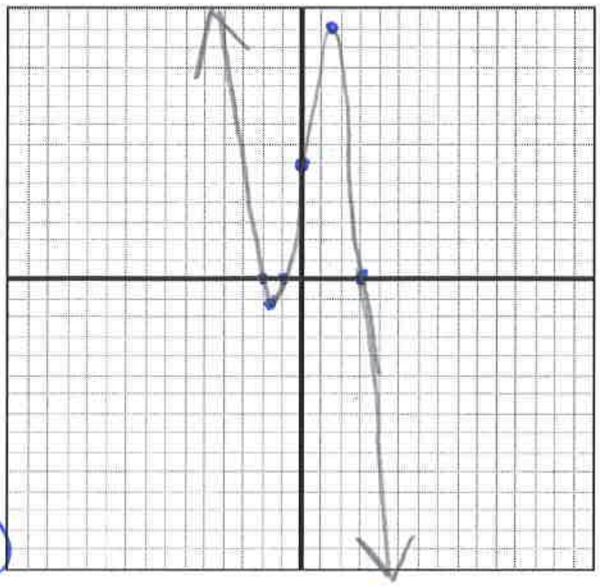
4.) $-x^3 + 7x + 6$ 3 real

$$\begin{array}{r|rrrr} 3 & -1 & 0 & 7 & 6 \\ & & -3 & -9 & -6 \\ \hline & -1 & -3 & -2 & 0 \end{array}$$ YES!

$(x-3)(-x^2-3x-2)$ 2, -2 -1 | -3

$-x^2 - x - 2x - 2$

$-x(x+1) - 2(x+1) \Rightarrow (x-3)(-x-2)(x+1)$



End Behavior: As x→∞, f(x) → -∞, As x→-∞, f(x) → ∞   y-intercept: (0,6)

Product of Factors: $(x-3)(-x-2)(x+1)$ Zeros: 3, -2, -1

Relative Minima/Maxima: (-1.53, -1.13) Min   (1.53, 13.13) Max

Algebra 2/Trigonometry
Test Review Graphing

Name: ____________________
Date: ________________ Block: ______

Section 8- Write f(x) as a product of linear factors and list all of its zeros. You must verify all rational zeros (using synthetic division). Use our calculator to find the relative maxima and minima (to the nearest hundredth). Then sketch a graph.

#

↓↑

5.) $x^3 - 3x^2 + 4x - 2$

1 real

2 imaginary

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 4 & -2 \\ & & 1 & -2 & 2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$ YES!

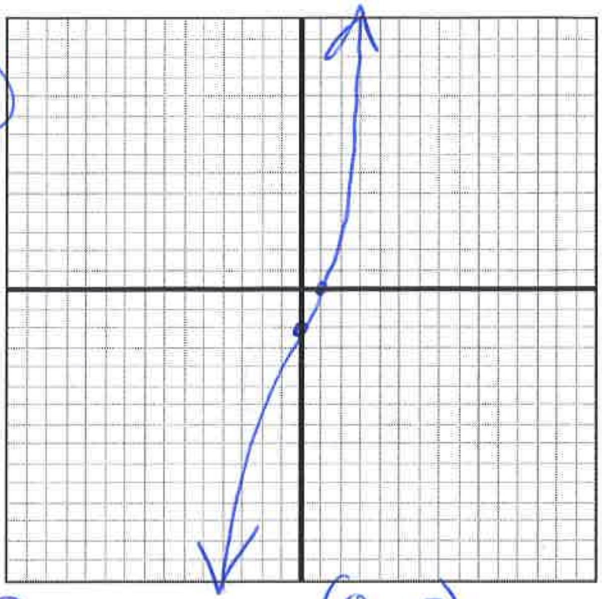
$(x-1)(x^2-2x+2)$

$\downarrow$

1

$$\frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2}$$

$1 \pm i$



End Behavior: As x→∞, f(x) → ∞, As x→-∞, f(x) → $-\infty$ y-intercept: (0,-2)

Product of Factors: $(x-1)(x^2-2x+2)$ Zeros: $\{1, 1 \pm i\}$

Relative Minima/Maxima: None!

6.) $2x^3 + 3x^2 - 1$

Relative Min/Max:
(0,-1)

p: 1
q: 2 1
Possible Roots: $\pm 1, \frac{1}{2}$

$$\begin{array}{r|rrrr} 1 & 2 & 3 & 0 & -1 \\ & & 2 & 5 & 5 \\ \hline & 2 & 5 & 5 & 4 \end{array}$$ No!

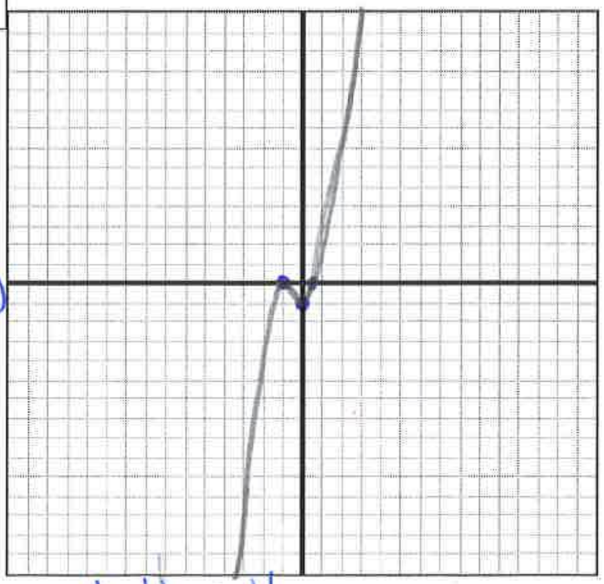
$$\begin{array}{r|rrrr} -1 & 2 & 3 & 0 & -1 \\ & & -2 & -1 & 1 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$ YES!

$(x+1)(2x^2+x-1)$

$(x+1)(\underbrace{2x^2+2x}\underbrace{-x-1})$

$(x+1)\,2x(x+1) - 1(x+1)$

$(x+1)(2x-1)(x+1)$



doubleroot!
↓

End Behavior: As x→∞, f(x) → ∞, As x→-∞, f(x) → $-\infty$ y-intercept: (0,-1)

Product of Factors: $(x+1)(2x-1)(x+1)$ Zeros: $-1, \frac{1}{2}$

Relative Minima/Maxima: Min at (0,-1) Max at (-1,0)

B/C It's a double root

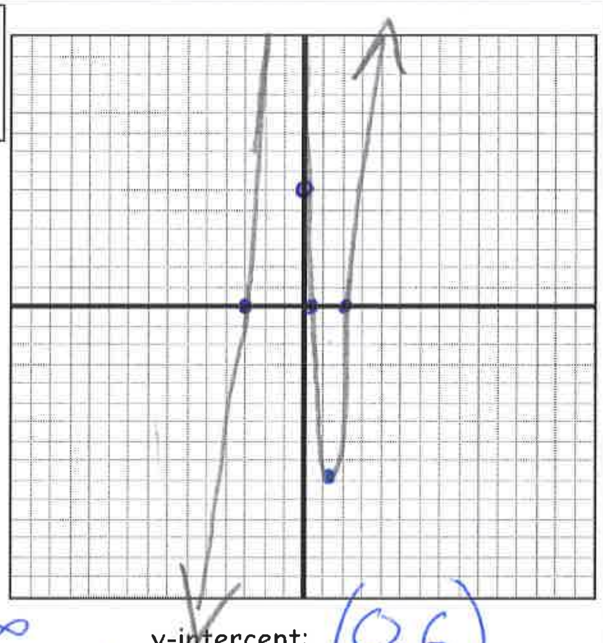
Algebra 2/Trigonometry
Test Review Graphing

Name: ______________________
Date: ________________ Block: ______

Section 8- Write f(x) as a product of linear factors and list all of its zeros. You must verify all rational zeros (using synthetic division). Then sketch a graph.

7.) $3x^3+2x^2-19x+6$

Relative Min/Max:
(-1.69,29.34)
(1.25, -8.77)



P: 1, 2, 3, 6
q: 1, 3
Possible Roots: $\pm 1, 2, 3, 6, \frac{1}{3}, \frac{2}{3},$

1| 3 2 -19 6
3 5 -14
3 5 -14 -8 NO!

-1| 3 2 -19 6
-3 1 18 NO!
3 -1 -18 24

2| 3 2 -19 6
6 16 -6 YES!
3 8 -3 0

$(x-2)(3x^2+8x-3)$

$(x-2)(3x-1)(x+3)$

End Behavior: As x→∞, f(x) → ∞, As x→-∞, f(x) → $-\infty$

y-intercept: (0, 6)

Product of Factors: $(x-2)(3x-1)(x+3)$

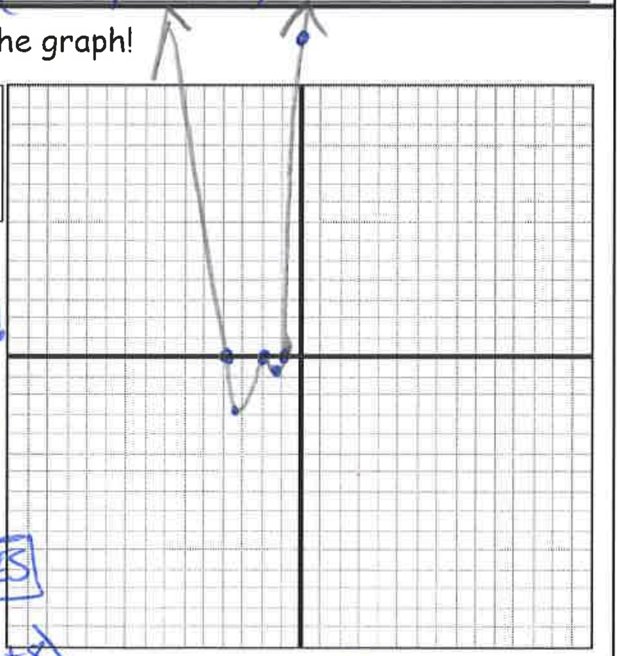
Zeros: $\{2, \frac{1}{3}, -3\}$

Relative Minima/Maxima: (-1.69, 29.34) Max (1.25, -8.77) Min

8.) * Hint: Don't forget how a double root impacts the graph!

↑↑ $x^4+9x^3+28x^2+36x+16$

Relative Min/Max:
(-3.44, -2.83)
(-1.31,-0.40)



P: 1, 2, 4, 8, 16
q: 1
Possible Roots: $\pm 1, 2, 4, 8, 16$

1| 1 9 28 36 16
1 10 38 74
1 10 38 74 90 NO!

-1| 1 9 28 36 16
-1 -8 -20 -16
1 8 20 16 0 YES

2| 1 8 20 16
2 20 80 NO!
1 10 40 96

-2| 1 8 20 16
-2 -12 -16
1 6 8 0 YES

$(x+1)(x+2)(x^2+6x+8)$
$(x+4)(x+2)$

End Behavior: As x→∞, f(x) →________, As x→-∞, f(x) →________

y-intercept: (0, 16)

Product of Factors: $(x+1)(x+2)(x+4)(x+2)$

Zeros: $\{-4, -1, -2 \text{ (double)}\}$!

Relative Minima/Maxima: (-3.44, -2.83) Min (-1.31, -0.40) Min (-2, 0) Max