

Section 1 → Multiplying : Dividing

$$1) \frac{x^2 - x - 12}{2x^2 + 8x} \div \frac{3x^3}{x - 4} = \frac{\cancel{(x-4)}(x+3)}{2\cancel{x}(x+4)} \cdot \frac{3\cancel{x}^3 x^2}{\cancel{(x-4)}} = \boxed{\frac{3x^2(x+3)}{x+4}}$$

$$2) \frac{x^2 - 8x + 15}{x^2 - 12x + 35} \div \frac{x^2 - 13x + 42}{x^2 - 8x + 12} = \frac{\cancel{(x-5)}(x-3)}{\cancel{(x-5)}(x-7)} \cdot \frac{\cancel{(x-6)}(x-7)}{\cancel{(x-6)}(x-2)} = \boxed{\frac{x-3}{x-2}}$$

$$3) \frac{8x}{4x^2 - 1} \div \frac{2x^2 + 5x + 2}{2} = \frac{\cancel{8x} \ 4x}{(\cancel{2x+1})(2x-1)} \cdot \frac{(\cancel{2x+1})(x+2)}{\cancel{2}} = \boxed{\frac{4x(x+2)}{2x-1}}$$

$$4) \frac{x^2 - 9}{x+5} \div \frac{x^2 - 6x + 9}{x^2 + 5x} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x+5)}} \cdot \frac{x(x+5)}{\cancel{(x-3)}(x-3)} = \boxed{\frac{x(x+3)}{x-3}}$$

$$5) \frac{4x+16}{x^2-16} \div \frac{3x+12}{4x^2+16x} = \frac{4\cancel{(x+4)}}{\cancel{(x+4)}(x-4)} \cdot \frac{4x\cancel{(x+4)}}{3\cancel{(x+4)}} = \boxed{\frac{16x}{3(x-4)}}$$

$$6) \frac{2x^2+7x+3}{x^2-6x+8} \div \frac{2x^2-5x-3}{x^2-12x+32} = \frac{\cancel{(2x+1)}(x+3)}{\cancel{(x-4)}(x-2)} \cdot \frac{\cancel{(x-4)}(x-8)}{\cancel{(2x+1)}(x-3)} = \boxed{\frac{(x+3)(x-8)}{(x-2)(x-3)}}$$

Section 2 → Adding/Subtracting Rational Expressions

$$1) \frac{4 \cdot 4}{3x \cdot 4} + \frac{7 \cdot 3}{4x \cdot 3} + \frac{1 \cdot 2}{6x \cdot 2} \rightarrow \frac{16+21+2}{12x} \rightarrow \boxed{\frac{39}{12x}}$$

LCD: 12x

$$2) \frac{4(x-5)}{(x-2)(x-5)} + \frac{3(x-2)}{(x-5)(x-2)} = \frac{4x-20+3x-6}{(x-5)(x-2)} = \boxed{\frac{7x-26}{(x-5)(x-2)}}$$

LCD: (x-2)(x-5)

$$3) \frac{3b}{8ab} + \frac{6a \cdot 2a}{4b \cdot 2a} = \frac{3b+12a^2}{8ab} = \boxed{\frac{3(b+4a^2)}{8ab}}$$

LCD: 8ab

Section 2 (Cont'd)

$$4) \frac{7}{x^2+8x+16} + \frac{3(x+4)}{(x+4)(x+4)} = \frac{7+3x+12}{(x+4)(x+4)} = \frac{3x+19}{(x+4)(x+4)} \quad \boxed{\frac{3x+19}{(x+4)^2}}$$

$$\text{LCD: } (x+4)(x+4)$$

$$5) \frac{6 \cdot 2}{x^2-2x-15} + \frac{8(x+3)}{2(x-5)(x+3)} = \frac{12+8x+24}{2(x-5)(x+3)} = \frac{8x+36}{2(x-5)(x+3)} = \frac{2(4(2x+9))}{2(x-5)(x+3)} \quad \boxed{\frac{2(2x+9)}{(x-5)(x+3)}}$$

$$\text{LCD: } 2(x-5)(x+3)$$

$$6) \frac{(5x+1)^{(5x+1)}}{1(5x+1)} - \frac{3x}{5x+1} = \frac{25x^2+10x+1-3x}{5x+1} = \frac{25x^2+7x+1}{(5x+1)} \quad \boxed{\frac{25x^2+7x+1}{(5x+1)}}$$

$$\text{LCD: } (5x+1)$$

$$7) \frac{x(x-6)}{x+6(x-6)} - \frac{6x}{x^2-36} = \frac{x^2-6x-6x}{(x+6)(x-6)} = \frac{x^2-12x}{(x+6)(x-6)} \quad \boxed{\frac{x(x-12)}{(x+6)(x-6)}}$$

$$\text{LCD: } (x-6)(x+6)$$

$$8) \frac{7 \cdot 4}{3x \cdot 4} - \frac{5 \cdot 3}{4x \cdot 3} + \frac{3 \cdot 2}{6x \cdot 2} = \frac{28-15+6}{12x} = \boxed{\frac{19}{12x}}$$

$$\text{LCD: } 12x$$

$$9) \frac{x}{x^2+4x+4} - \frac{x+5(x+2)}{(x+2)(x+2)} - \frac{2x(x+2)}{(x+2)(x+2)} = \frac{x - (x^2+7x+10) - (2x^2+4x)}{(x+2)(x+2)} \rightarrow \frac{x - x^2 - 7x - 10 - 2x^2 - 4x}{(x+2)(x+2)}$$

$$\text{LCD: } (x+2)(x+2)$$

$$\frac{x - x^2 - 7x - 10 - 2x^2 - 4x}{(x+2)(x+2)} = \frac{-3x^2 - 10x - 10}{(x+2)(x+2)} = \frac{-1(3x^2+10x+10)}{(x+2)^2}$$

$$\boxed{\frac{-(3x^2+10x+10)}{(x+2)^2}}$$

Section 3 → Solving Rational Equations. Check & watch for Extraneous solutions.

$$1) \frac{y+1}{3} + \frac{y-1}{3} = \frac{8}{3} \rightarrow \frac{2y}{3} \times \frac{8}{3} \quad 6y = 24 \quad \boxed{y=4}$$

✓: $\frac{5}{3} + \frac{3}{3} = \frac{8}{3}$
 $\frac{8}{3} = \frac{8}{3}$ ✓

$$2) \left(\frac{2y-5}{6} - \frac{y-5}{4} = \frac{3}{4} \right) \cdot 12 \quad 2(2y-5) - 3(y-5) = 3 \cdot 3$$

$$4y - 10 - 3y + 15 = 9$$

$$y + 5 = 9$$

$$\boxed{y=4}$$

✓: $\frac{8-5}{6} - \frac{-1}{4} = \frac{3}{4}$
 $\frac{3}{6} - \frac{-1}{4} = \frac{3}{4}$
 $\frac{3}{6} = \frac{2}{4}$
 ✓ $\frac{1}{2} = \frac{1}{2}$

$$3) \frac{5x}{x+3} - \frac{5}{6} = 2$$

$x \neq -3$

$$\frac{5x}{x+3} \times \frac{17}{6} \quad 17(x+3) = 30x$$

$$17x + 51 = 30x$$

$$51 = 13x$$

$$\boxed{x = 51/13}$$

ok ✓

✓ $\frac{255}{90/13} - \frac{5}{6} = 2 \rightarrow \frac{17}{6} - \frac{5}{6} = 2 \rightarrow \frac{12}{6} = 2 \rightarrow 2 = 2$ ✓

$$4) \left(8 - \frac{2-5x}{4} = \frac{4x+9}{3} \right) \cdot 12 = 96 - 3(2-5x) = 4(4x+9)$$

$$96 - 6 + 15x = 16x + 36$$

$$90 = x + 36$$

$$\boxed{x = 54}$$

✓: $8 - \frac{2-5(54)}{4} = \frac{4(54)+9}{3} \rightarrow 8 - \frac{268}{4} = \frac{225}{3} \quad 8 + 67 = 75$
 $75 = 75$ ✓

$$5) \left(1 + \frac{2}{x} = \frac{3}{x^2} \right) \cdot x^2$$

$x \neq 0$

$$x^2 + 2x = 3 \rightarrow x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$\boxed{x = -3, 1}$$

ok

✓: $1 + \frac{2}{-3} = \frac{3}{(-3)^2}$
 $1 - \frac{2}{3} = \frac{3}{9}$
 $\frac{1}{3} = \frac{1}{3}$ ✓

$1 + \frac{2}{1} = \frac{3}{(1)^2}$
 $1 + 2 = \frac{3}{1}$
 $3 = 3$ ✓

$$6) \left(\frac{x^2 - x}{x^2 - 9} + \frac{2}{(x+3)} = \frac{-4}{(x-3)} \right) (x-3)(x+3)$$

$x \neq -3 \quad x \neq 3$

$$x^2 - x + 2(x-3) = -4(x+3)$$

$$x^2 - x + 2x - 6 = -4x - 12$$

$$x^2 + x - 6 = -4x - 12$$

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

$$\boxed{x = -2} \quad \text{and} \quad \boxed{x = -3}$$

$$\sqrt{\frac{(-2)^2 - (-2)}{(-2)^2 - 9}} + \frac{2}{-2+3} = \frac{-4}{-2-3}$$

$$\frac{4+2}{-5} + \frac{2}{1} = \frac{-4}{-5}$$

$$\frac{6}{-5} + \frac{2}{1} = \frac{-4}{-5}$$

$$2 = \frac{-10}{-5}$$

$$2 = 2 \quad \checkmark$$

7) It takes Bill 3 days to cultivate a garden. It takes Fred 11 days to cultivate the same garden. If they work together, how long does it take to cultivate the garden?

$$\left(\frac{1}{3}t + \frac{1}{11}t = 1 \right) \quad 33$$

$$11t + 3t = 33$$

$$14t = 33$$

$$t = 33/14 \text{ or } \boxed{2\frac{5}{14} \text{ days}}$$

8) It takes Leah 21 hours to paint a kitchen. Together, Leah and Hank can complete the same job in 12 hours. How long does it take Hank to complete the job if he works alone?

$$\frac{1}{21}t + rt = 1 \Rightarrow \frac{1}{21}(12) + 12r = 1$$

$$\frac{12}{21} + 12r = 1$$

$$12r = 9/21$$

$$\frac{12r}{12} = \frac{3/7}{12}$$

$$r = 3/84 = 1/28$$

$$\boxed{28 \text{ hrs.}}$$

Section 4 - Simplifying Complex Fractions

$$1) \frac{x-4}{\left(\frac{x}{4} - \frac{4}{x}\right)} \quad \frac{x \cdot x}{4 \cdot x} - \frac{4 \cdot 4}{x \cdot 4} \quad \frac{x^2-16}{4x} \quad (x-4) \div \frac{x^2-16}{4x} \rightarrow x-4 \cdot \frac{4x}{x^2-16}$$

$$(x-4) \cdot \frac{4x}{(x-4)(x+4)} = \boxed{\frac{4x}{x+4}}$$

$$2) \frac{\left(1 + \frac{7}{x-2}\right)}{\left(1 + \frac{3}{x+2}\right)} \quad 1 + \frac{7}{x-2} \quad 1 + \frac{3}{x+2} \quad \frac{x+5}{x-2} \div \frac{x+5}{x+2}$$

$$\frac{x-2}{x-2} + \frac{7}{x-2} \quad \frac{x+2}{x+2} + \frac{3}{x+2} \quad \frac{x+5}{x-2} \cdot \frac{x+2}{x+5} = \boxed{\frac{x+2}{x-2}}$$

$$\left(\frac{x+5}{x-2}\right) \div \left(\frac{x+5}{x+2}\right)$$

$$3) \frac{(3 - \frac{2}{x})}{\left(\frac{1}{x^2} - \frac{1}{4}\right)} \quad 3 - \frac{2}{x} \quad \frac{1}{x^2} - \frac{1}{4} \quad \frac{3x-2}{x} \div \frac{-x^2+4}{4x^2}$$

$$\frac{3x}{x} - \frac{2}{x} \quad \frac{4}{4x^2} - \frac{x^2}{4x^2} \quad \frac{3x-2}{x} \cdot \frac{4x^2}{-1(x^2-4)} = \boxed{\frac{4(3x-2)}{-1(x+2)(x-2)}}$$

$$\left(\frac{3x-2}{x}\right) \div \left(\frac{4-x^2}{4x^2}\right)$$

$$4) \frac{\left(\frac{x+2}{x^2-9}\right)}{\left(\frac{-x}{x^2-6x+9} + \frac{4}{x-3}\right)} \quad \frac{x}{(x-3)(x-3)} + \frac{4}{x-3}$$

$$\frac{x+4(x-3)}{(x-3)(x-3)} \quad \frac{x+2}{(x+3)(x-3)} \div \frac{5x-12}{(x-3)(x-3)}$$

$$\frac{5x-12}{(x-3)(x-3)} \quad \frac{(x+2)}{(x+3)(x-3)} \cdot \frac{(x-3)(x-3)}{(5x-12)}$$

$$\boxed{\frac{(x+2)(x-3)}{(x+3)(5x-12)}}$$

Section 5 → Identify the domain in interval notation

$$1) f(x) = \frac{x+2}{x^2+2x-15} \rightarrow \frac{x+2}{(x+5)(x-3)} \quad x+5 \neq 0 \quad x-3 \neq 0 \quad x \neq -5 \quad x \neq 3 \quad \boxed{(-\infty, -5) \cup (-5, 3) \cup (3, \infty)}$$

$$2) f(x) = \frac{1}{x^2-5} \quad x^2-5 \neq 0 \quad x \neq \pm\sqrt{5} \quad \boxed{(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, \infty)}$$

$$3) f(x) = \frac{9x-3}{3x+12} \quad \frac{3(3x-1)}{3(x+4)} \quad x+4 \neq 0 \quad x \neq -4 \quad \boxed{(-\infty, -4) \cup (-4, \infty)}$$