

Section 1: Use the properties of logarithms to expand the following.

1. $\log_2 5x =$

$$\log_2 5 + \log_2 x$$

2. $\log_8 x^4 =$

$$4 \log_8 x$$

3. $\log_3 \frac{5}{x} =$

$$\log_3 5 - \log_3 x$$

4. $\ln \sqrt{z} =$

$$\frac{1}{2} \ln z$$

5. $\ln \sqrt{z}(z-1)^2 =$

$$\frac{1}{2} \ln z + 2 \ln(z-1)$$

6. $\log_7 \frac{x^2}{y^2 z^3} =$

$$\text{or } 2 \log_7 x - 2 \log_7 y - 3 \log_7 z$$

$$2 \log_7 x - (2 \log_7 y + 3 \log_7 z)$$

7. $\log_x \frac{\sqrt{a} y^4}{z^4} =$

$$\frac{1}{2} \log_x a + 4 \log_x y - 4 \log_x z$$

8. $\log \left(\frac{x^2 - 1}{x^3} \right)^3 =$

$$\text{or } 3 \log(x+1) + 3 \log(x-1) - 9 \log x$$

$$3(\log(x+1) + \log(x-1) - \log x)$$

9. $\ln \frac{x}{\sqrt{x^2 + 1}} =$

$$\ln x - \frac{1}{2} \ln(x^2 + 1)$$

10. $\log(x^2 - 8x + 15) =$

$$\log(x-5) + \log(x-3)$$

Section 2: Use the properties of logarithms to rewrite the following as a single logarithm.

11. $\ln x + \ln 2 =$

$$\ln(2x)$$

12. $2 \ln 8 + 5 \ln z =$

$$\ln(64z^5)$$

13. $\log_4 z - \log_4 y =$

$$\log_4 \left(\frac{z}{y} \right)$$

14. $3 \ln x + 2 \ln y - 4 \ln z =$

$$\ln \left(\frac{x^3 y^2}{z^4} \right)$$

15. $2 \log_2 (x+4) =$

$$\log_2 (x+4)^2$$

$$\log_2 (x^2 + 8x + 16)$$

16. $4[\ln z + \ln(z+5)] - 2 \ln(z-5) =$

$$\ln \left(\frac{(z(z+5))^4}{(z-5)^2} \right) \text{ or } \ln \frac{z^4(z+5)^4}{(z-5)^2}$$

17. $\frac{1}{3} \log_3 5x =$

$$\log_3 \sqrt[3]{5x}$$

18. $\ln x - 2[\ln(x+2) + \ln(x-2)] =$

$$\ln \left(\frac{x}{(x^2-4)^2} \right)$$

19. $\log_3 (x-2) - \log_3 (x+2) =$

$$\log_3 \left(\frac{x-2}{x+2} \right)$$

20. $\frac{3}{2} \log_4 5t^6 - \frac{3}{4} \log_4 t^4 =$

$$\log_4 5t^6 \sqrt{5}$$

21. $\log_8 6 + \log_8 x - \log_8 5 + \log_8 y - \log_8 z =$

$$\log_8 \left(\frac{6xy}{5z} \right)$$

22. $1 - 2(\log x + 3 \log y) + \frac{1}{2} \log z =$

[Hint: all terms must be logarithms in order to condense. Think properties of logs.]

$$\log \frac{10\sqrt{z}}{x^2 y^6}$$