Unit 4 - Probability

Day 9: Review and Practice

Take out your homework...

Now try Practice Quiz A

1. A biology professor responds to some student questions by e-mail. The probability model below describes the number of e-mails that the professor may receive from students during a day.

e-mails received	0	1	2	3	4	5
Probability	0.05	0.10	0.20	0.25	0.30	0.10

a. How many e-mails should the professor expect to receive each day?

Let X = number of e-mails received.

$E(X) = 0(0.05) + 1(0.10) + 2(0.20) + 3(0.25) + 4(0.30) + 5(0.10) = 2.95$ e-mails per day

b. What is the standard deviation?

$$Var(X) = (0-2.95)^2(0.05) + (1-2.95)^2(0.10) + (2-2.95)^2(0.20)$$
$$+ (3-2.95)^2(0.25) + (4-2.95)^2(0.30) + (5-2.95)^2(0.10)$$
$$= 1.7475$$

$SD(X) = \sqrt{1.7475} = 1.32$ e-mails per day

c. If it takes the professor an average of ten minutes to respond to each e-mail, how much time should the professor expect to spend responding to student e-mails each day?

Let Y = amount of time spent responding to e-mails

$Y = X_1 + X_2 + \ldots + X_{10}$

$E(Y) = E(X_1 + X_2 + \ldots + X_{10}) = E(X_1) + E(X_2) + \ldots + E(X_{10}) = 2.95 + 2.95 + \ldots + 2.95 = 29.5$ minutes per day

2. The American Veterinary Association claims that the annual cost of medical care for dogs averages \$100 with a standard deviation of \$30, and for cats averages \$120 with a standard deviation of \$35.

a. Find the expected value for the annual cost of medical care for a person who has one dog and one cat.

$E(D+C) = E(D) + E(C) = \$100 + \$120 = \$220$

b. Find the standard deviation for the annual cost of medical care for a person who has one dog and one cat.

$Var(D+C) = Var(D) + Var(C) = 30^2 + 35^2 = 2125$, so $SD(D+C) = \sqrt{2125} = \$46.10$

c. Suppose that a couple owns four dogs.

i. Find the expected value for the annual cost of medical care for the couple's dogs.

$E(D_1 + D_2 + D_3 + D_4) = \$100 + \$100 + \$100 + \$100 = \$400$

ii. Find the standard deviation for the annual cost of medical care for the couple's dogs.

$Var(D_1 + D_2 + D_3 + D_4) = 30^2 + 30^2 + 30^2 + 30^2 = 3600$, so

$SD(D_1 + D_2 + D_3 + D_4) = \sqrt{3600} = \$60$

Take out your warm up packets…

Warm ups:

MC (part 2) #1, 2, 7, 8, 9,
11, 13, 14, 15, 16

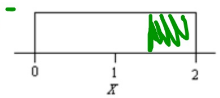
A psychologist studied the number of puzzles subjects were able to solve in a five-minute period while listening to soothing music. Let X be the number of puzzles completed successfully by a subject. The psychologist found that X had the following probability distribution:

Value of X	1	2	3	4
Probability	0.2	0.4	0.3	0.1

1. Referring to the information above, the probability that a randomly chosen subject completes *at least* three puzzles in the five-minute period while listening to soothing music is
A) 0.3 B) 0.4 C) 0.6 D) 0.9

2. Referring to the information above, $P(X < 3)$ has value
A) 0.3 B) 0.4 C) 0.6 D) 0.9

The probability density of a random variable X is given in the figure at the right



7. Referring to the information above, from this density, the probability that X is between 0.5 and 1.5 is
A) 1/3 B) 1/2 C) 3/4 D) 1

8. Referring to the information above, the probability that X is at least 1.5 is
A) 0 B) 1/4 C) 1/3 D) 1/2

9. Referring to the information above, the probability that $X = 1.5$ is
A) 0 B) 1/4 C) 1/3 D) 1/2

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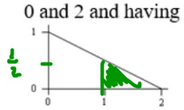
11. Suppose X is a continuous random variable taking values between 0 and 2 and having the probability density function below. $P(1 < X < 2)$ has value



A) 0.50 B) 0.33 C) 0.25 D) 0.00

Let the random variable X represent the profit made on a randomly selected day by a certain store. Assume X is normal with a mean of \$360 and standard deviation \$50.

13. Referring to the information above, the value of $P(X > \$400)$ is
A) 0.2881 B) 0.8450 C) 0.7881 D) 0.2119

14. Referring to the information above, the probability is approximately 0.6 that on a randomly selected day the store will make less than
A) \$347.40 B) \$0.30 C) \$361.30 D) \$372.60

In a particular game, a fair die is tossed. If the number of spots showing is either 4 or 5 you win \$1; if number of spots showing is 6 you win \$4; and if the number of spots showing is 1, 2, or 3 you win nothing. Let X be the amount that you win.

15. Referring to the information above, the expected value of X is
A) \$0 B) \$1 C) \$2.50 D) \$4

16. Referring to the information above, the variance of X is
A) 1.0 B) 3/2 C) 2.0 D) 13/6

X	0	1	4
P(X)	1/2	1/3	1/6

2000 AP® STATISTICS FREE-RESPONSE QUESTIONS

A random sample of 400 married couples was selected from a large population of married couples.

- Heights of married men are approximately normally distributed with mean 70 inches and standard deviation 3 inches.
- Heights of married women are approximately normally distributed with mean 65 inches and standard deviation 2.5 inches.
- There were 20 couples in which the wife was taller than her husband, and there were 380 couples in which the wife was shorter than her husband.

(b) Suppose that a married man is selected at random and a married woman is selected at random. Find the approximate probability that the woman will be taller than the man.

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(b*) Suppose that a married **couple** were chosen at random. What is the probability that the woman is taller than the man?

$\frac{20}{400} = \boxed{.05}$

Things to work on...

- AP FRQs
- Chapter 16 Practice Quiz B
- extra practice multiple choice
- p. 405 #1-3, 5-7, 18, 19, 21, 26, 28, 35-37, 40-41
- WS - probability rules

A fast food restaurant just leased a new freezer and food fryer for three years. The service contract for the freezer offers unlimited repairs for a fee of \$125 a year plus a \$35 service charge for each repair needed. The restaurant's research suggested that during a given year 80% of these freezers need no repairs, 11% needed to be serviced once, 5% twice, 4% three times, and none required more than three repairs.

1. Find the expected number of repairs this kind of freezer is expected to need each year. Show your work.
   $E(X) = 0(0.80) + 1(0.11) + 2(0.05) + 3(0.04) = 0.33$ repairs

2. Find the standard deviation of the number of repairs each year.
   $Var(X) = (0 - 0.33)^2(0.80) + (2 - 0.33)^2(0.05) + (3 - 0.33)^2(0.04) = 0.561$
   Standard deviation $= \sqrt{0.561} = 0.749$

3. What are the mean and standard deviation of the restaurant's annual expense for the service contract?
   Let C = \$125 + \$35X; E(C) = \$125 + \$35(0.33) = \$136.55
   Standard deviation(C) = \$35(0.749) = \$26.22

4. How many times should the restaurant expect to have to get this freezer repaired over the three-year term of the lease?
   $E(X_1 + X_2 + X_3) = 0.33 + 0.33 + 0.33 = 0.99$ repairs

5. What is the standard deviation of the number of repairs that may be required during the three-year term of the lease? On what assumption does your calculation rest? Do you think this assumption is reasonable?
   $Var(X_1 + X_2 + X_3) = 0.561 + 0.561 + 0.561 = 1.683$, so standard deviation$(C) = 1.297$
   The assumption is that the number of repairs is independent from year to year. This might be incorrect because some freezers might need more service than others.

6. The yearly service contract for the food fryer estimates a mean annual cost of \$140 with a standard deviation of \$40. What is the expected value and standard deviation of the total cost for the service contracts for the freezer and the food fryer?
   E(freezer + fryer) = \$136.55 + \$140 = \$276.55
   Var(freezer + fryer) = $(\$26.22)^2 + (\$40)^2 = 2287.49$, so standard deviation = \$47.83

7. Which service contract should the restaurant expect to cost more each year? How much more? With what standard deviation?
   The food fryer's service contract is expected to cost more.
   E(fryer – freezer) = \$140 - \$136.55 = \$3.45 more
   Var(fryer – freezer) = \$47.83 (same as the sum in problem 6)

2016 AP® STATISTICS FREE-RESPONSE QUESTIONS

4. A company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.

   A company engineer develops a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process.

   Step 1: One super igniter is selected at random and used in a rocket.

   Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.

   Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

   (a) If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?

   [redacted]

   (b) Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first or the thirty-second super igniter tested if the failure rate of the super igniters is 15 percent?

   [redacted]

   (c) Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.

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[redacted]

3. A shopping mall has three automated teller machines (ATMs). Because the machines receive heavy use, they sometimes stop working and need to be repaired. Let the random variable X represent the number of ATMs that are working when the mall opens on a randomly selected day. The table shows the probability distribution of X.

Number of ATMs working when the mall opens	0	1	2	3
Probability	0.15	0.21	0.40	0.24

(a) What is the probability that at least one ATM is working when the mall opens?

[redacted]

(b) What is the expected value of the number of ATMs that are working when the mall opens?

[redacted]

(c) What is the probability that all three ATMs are working when the mall opens, given that at least one ATM is working?

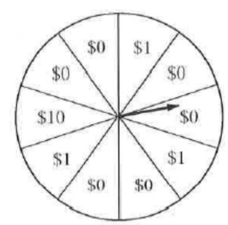
[redacted]

(d) Given that at least one ATM is working when the mall opens, would the expected value of the number of ATMs that are working be less than, equal to, or greater than the expected value from part (b) ? Explain.

[redacted]

2012 AP® STATISTICS FREE-RESPONSE QUESTIONS

2. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.



A donation of $2 is required to play the game. For each $2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

x	$2	$1	–$8
$P(x)$	[redacted]		

(b) What is the expected value of the net contribution to the charity for one play of the game?

[redacted]

(c) The charity would like to receive a net contribution of $500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least $500 ?

The expected contribution after n plays is 0.70n$. Setting this to be at least $500 and solving for n gives:

[redacted]

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least $500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are $700 and $92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least $500 in 1,000 plays of the game.

[redacted]

2013

3. Each full carton of Grade A eggs consists of 1 randomly selected empty cardboard container and 12 randomly selected eggs. The weights of such full cartons are approximately normally distributed with a mean of 840 grams and a standard deviation of 7.9 grams.

(a) What is the probability that a randomly selected full carton of Grade A eggs will weigh more than 850 grams?

[redacted]

(b) The weights of the empty cardboard containers have a mean of 20 grams and a standard deviation of 1.7 grams. It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton.

[redacted]

(b) The weights of the empty cardboard containers have a mean of 20 grams and a standard deviation of 1.7 grams. It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton.

Let the random variable X be the weight of a single randomly selected Grade A egg.

i) What is the mean of X ?

[redacted]

ii) What is the standard deviation of X ?

[redacted]

Homework - STUDY!

- extra practice multiple choice
- p. 405 #1-3, 5-7, 18, 19, 21, 26, 28, 35-37, 40-41