# Graphing Rational Functions

**Algebra 2/Trigonometry** **Name:**

Unit 6 Graphing Rational Functions

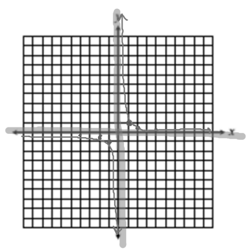
Warm Up

Graph $y = \frac{1}{x}$.

| x | y |
|---|---|
| -4 | $-\frac{1}{4}$ |
| -2 | $-\frac{1}{2}$ |
| 0 | $\frac{1}{0} = \emptyset$, undefined |
| 2 | $\frac{1}{2}$ |
| 4 | $\frac{1}{4}$ |

(1, 1)
(-1, -1)

$\frac{1}{x} = 0$
there is no x that I can plug in to get $y = 0$



Questions:

1) What value(s) can the denominator not equal? Why? $x \neq 0$ VA @ $x = 0$
$y \neq 0$ HA @ $y = 0$

**Asymptote definition:** A line that a curve approaches as it heads toward infinity.

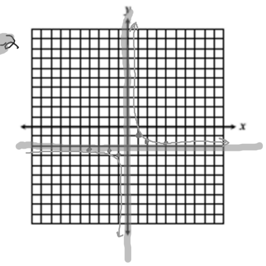
Graph $y = \frac{1}{x} - 2$

$-2 = \frac{1}{x} - 2$

$0 = \frac{1}{x}$ HA @ $y = -2$

| x | y |
|---|---|
| -4 | -2.25 |
| -2 | -2.5 |
| 0 | $\emptyset$ |
| 2 | -1.5 |
| 4 | -1.75 |

VA @ $x = 0$



Questions:

1) Where are the asymptotes here?

| Main Ideas/Questions | Notes/Examples |
|---|---|
| **RATIONAL FUNCTIONS** | A function of the form: $f(x) = \frac{p(x)}{q(x)}$   $p(x)$ and $q(x)$ are polynomial, $q(x) \neq 0$ |
| **STEPS TO GRAPH** | ① **SIMPLIFY** the function. factor & cancel |
| | ② Find the **x-intercept(s)** by setting the **numerator** equal to 0. |
| | ③ Find the **vertical asymptote(s)** by setting the **denominator** equal to 0. |
| | ④ Find the **horizontal asymptote** using the rules below. |
| | ⑤ Identify any **holes** in the function. |
| **WHAT IS A HOLE?** | A hole is a point **(x, y)** at which there is a **break** in the graph. A hole occurs when there is a **common factor** between the numerator and denominator. <br>» **To find the x-coodinate:** set common factor = 0. (aka cancel) $x = \_\_$ <br>» **To find the y-coordinate:** plug that x-value into simplified function. |

| | **CASE** | **HORIZONTAL ASYMPTOTE** |
|---|---|---|
| $\frac{x^3+1}{x}$ | $\frac{\text{big}}{\text{small}}$ | No H.A. |
| $\frac{3x+1}{x}$ | $\frac{\text{same}}{\text{same}}$ | $y$ = ratio of coeff. = $\frac{\text{L.C. of top}}{\text{L.C. of bottom}}$ |
| $\frac{x}{x^3+1}$ | $\frac{\text{small}}{\text{big}}$ | $y = 0$ |

plug in $x = 10{,}000$

$y = 3$

**Directions:** Graph each function and identify its key characteristics.

**1.** $f(x) = \frac{x-4}{x+4} = \frac{-5-4}{-5+4} = \frac{-9}{-1} = 9$

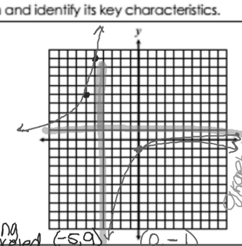
x-int: $x - 4 = 0$
$x = 4$ so $(4, 0)$

VA: $x + 4 = 0$
$x = -4$

HA: $\frac{\text{same}}{\text{same}} = y = 1$

Hole: none, b/c nothing canceled

(-5, 9)
(-6, 5)
(0, -1)



graph it

**x-int:** (4, 0)

**VA:** $x = -4$

**HA:** $y = 1$

**Hole:** none

**Domain:** $x \neq -4$
x's exclude VAs, x-va

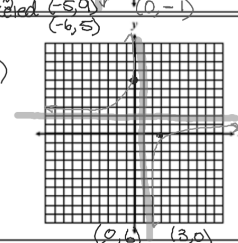
**Range:** $y \neq 1$
y's exclude HAs & y-va

**2.** $f(x) = \frac{2x-6}{x-1} = \frac{2(x-3)}{x-1}$

x-int: $2x - 6 = 0$
$x = 3$ so $(3, 0)$

VA: $x - 1 = 0$
$x = 1$

HA: $\frac{\text{Same}}{\text{Same}} = y = 2$



(0, 6) (3, 0)

**x-int:** (3, 0)

**VA:** $x = 1$

**HA:** $y = 2$

**Hole:** none

**Domain:** $x \neq 1$

**Range:** $y \neq 2$

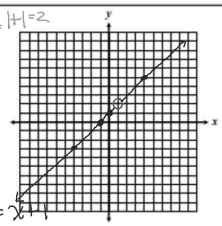
★ plug in a point to the left & right of each V.A.

3. $f(x) = \frac{x^2-1}{x-1} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \boxed{x+1}$ $1+1=2$

x-int: $x+1=0$, $x=-1$ $(-1,0)$

VA: there is no denom. $1 \neq 0$, no VA

Hole: $x-1=0$, $x=1$ $(1,2)$



$y=x+1$

x-int: $(-1,0)$

VA: none

HA: $\frac{\text{big}}{\text{small}}$ = none

Hole: $(1,2)$

Domain: $x \neq 1$

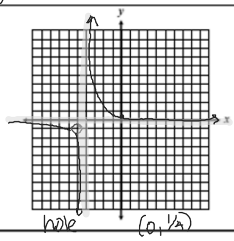
Range: $y \neq 2$

4. $f(x) = \frac{x+5}{x^2+9x+20} = \frac{\cancel{x+5}}{\cancel{(x+5)}(x+4)} = \boxed{\frac{1}{x+4}}$

VA: $x+4=0$, $x=-4$

HA: $\frac{\text{small}}{\text{big}} = y=0$

Hole: $x+5=0$, $x=-5$ $(-5,-1)$



hole $(0, \frac{1}{4})$

x-int: none

VA: $x=-4$

HA: $y=0$

Hole: $(-5,-1)$

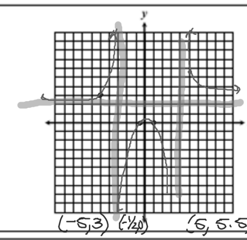
Domain: $x \neq -5, -4$

Range: $y \neq -1, 0$

5. $f(x) = \frac{2x^2-x-1}{x^2-x-12} = \frac{(2x+1)(x-1)}{(x-4)(x+3)}$

x-int: $2x+1=0$, $x=-\frac{1}{2}$; $x-1=0$, $x=1$

$x-4=0$, $x=4$; $x+3=0$, $x=-3$



$(-5,3)$ $(-\frac{1}{2},0)$ $(5,5.5)$

x-int: $(-\frac{1}{2},0),(1,0)$

VA: $x=4, x=-3$

HA: $\frac{\text{same}}{\text{same}}$ $y=2$

Hole: none

Domain:

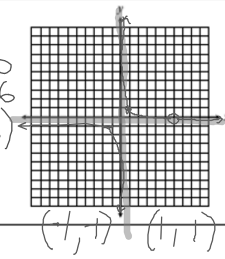
Range:

7. $f(x) = \frac{x-6}{x^2-6x} = \frac{0}{0}$

$= \frac{\cancel{x-6}}{x\cancel{(x-6)}}$ $x-6=0$, $x=6$

$= \boxed{\frac{1}{x}}$ $(6, \frac{1}{6})$

$1 \neq 0$ $x=0$



$(-1,-1)$ $(1,1)$

x-int: none

VA: $x=0$

HA: $y=0$

Hole: $(6, \frac{1}{6})$

Domain: $x \neq 0, 6$

exc. VA, x-value of hole

Range: $y \neq 0, \frac{1}{6}$

HA & y-value hole

Warm Up

RATIONAL FUNCTIONS

Graphing Calculator Reference Sheet

*Example:* $y = \frac{x^2-4x}{x^2-2x-8} = \frac{x(x-4)}{(x+2)(x-4)}$

FIND
- ❑ VERTICAL ASYMPTOTE
- ❑ HOLE
- ❑ HORIZONTAL ASYMPTOTE

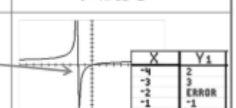
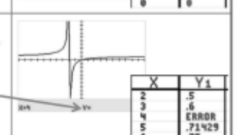
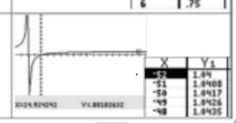
- take out your homework
- take out your graphing calculators

# RATIONAL FUNCTIONS

## Graphing Calculator Reference Sheet

*Example:* $y = \frac{x^2-4x}{x^2-2x-8} = \frac{x(x-4)}{(x+2)(x-4)}$

FIND
- ❑ VERTICAL ASYMPTOTE
- ❑ HOLE
- ❑ HORIZONTAL ASYMPTOTE

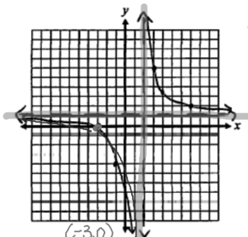
| STEPS | PICTURE |
|---|---|
| ❑ **STEP 1: Enter your equation into Y1 =** <br> * Be sure to have parentheses around the entire (numerator) and (denominator) | Y1=(X²-4X)/(X²-2X-8) **OR** <br> Y1=(X(X-4))/((X+2)(X-4)) |
| ❑ **STEP 2: WINDOW and Discontinuity Detection** <br> • ZOOM – Choose 6:Zstandard or change the settings in WINDOW <br> • WINDOW – Scroll down to Xres <br> • Xres = 1 (turn off detection) <br> • Xres = 2 (turn on detection) | WINDOW <br> Xmin=-10 <br> Xmax=10 <br> Xscl=1 <br> Ymin=-10 <br> Ymax=10 <br> Yscl=1 <br> Xres=2 |
| ❑ **STEP 3: Find the VERTICAL ASYMPTOTE** <br> • **GRAPH:** The discontinuity detection draws a vertical line for the vertical asymptote <br> • **TABLE:** 2ND – GRAPH – scroll to view when x = -2, the ERROR means there is no y-value, confirming the vertical asymptote |  |
| ❑ **STEP 4: Find the HOLE** <br> • **GRAPH:** 2ND – TRACE – Choose 1:value – x = 4 <br> • Enter the value you believe to be the hole in x = ____ <br> • No value for y = confirms the hole <br> • **TABLE:** 2ND – GRAPH – scroll to view when x = 4 the ERROR means there is no y-value, confirming the hole |  |
| ❑ **STEP 5: Find the HORIZONTAL ASYMPTOTE** <br> • **GRAPH:** TRACE – scroll as x → ∞ to see the value y approaches (or x → - ∞) <br> • **TABLE:** 2ND – GRAPH – scroll to large (or small) values of x to see the value y approaches |  |

VA: X = -2  HOLE: X = 4  HA: Y = 1

# Homework

**Graph each function. Identify the domain, range, asymptotes, and holes.**

**1.** $f(x) = \frac{x+3}{x-2}$

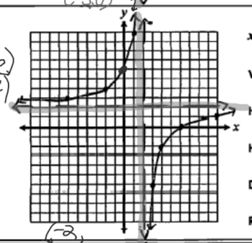



x-int: $(-3,0)$
VA: $x=2$
HA: $y=1$
Hole: none
Domain: $\{x \mid x \neq 2\}$
Range: $\{y \mid y \neq 1\}$

**2.** $f(x) = \frac{4x-24}{2x-4}$

$\frac{4(x-6)}{2(x-2)} = \frac{2(x-6)}{(x-2)}$

$x-2=0$
$x=2$



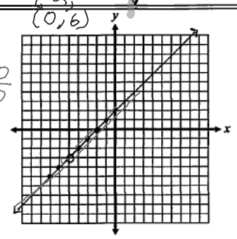

x-int: $(6,0)$
VA: $x=2$
HA: $y=2$
Hole: none
Domain: $\{x \mid x \neq 2\}$
Range: $\{y \mid y \neq 2\}$

**3.** $f(x) = \frac{x^2+7x+10}{x+5}$

$= \frac{(x+2)\cancel{(x+5)}}{\cancel{x+5}} = \frac{0}{0}$

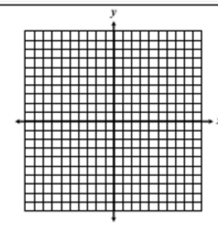
$= x+2$

$x+5=0$
$x=-5$



x-int: $(-2,0)$
VA: none
HA: none
Hole: $(-5,-3)$
Domain: $\{x \mid x \neq -5\}$
Range: $\{y \mid y \neq -3\}$

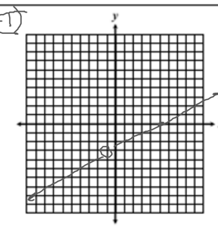
**8.** $f(x) = \frac{x^2+3x-28}{x^2+12x+35}$



x-int:
VA:
HA:
Hole:
Domain:
Range:

**9.** $f(x) = \frac{x^2-4x-5}{2x+2}$ $\frac{(x-5)\cancel{(x+1)}}{2\cancel{(x+1)}}$ $\frac{x-5}{2}$

hole: $x+1=0$
$x=-1$
$(-1,-3)$ $\frac{-1-5}{2}$ $\frac{-6}{2}$



x-int: $(5,0)$
VA: none
HA: none
Hole: $(-1,-3)$
Domain: $x \neq -1$
Range: $y \neq -3$

With a partner, graph this function on you r graphing calculator:

$$y = \frac{x^2+4x}{2x-1}$$

What do you notice about the asymptotes?

| Main Ideas & Questions | Notes & Examples | |
|---|---|---|
| | Case | Horizontal Asymptote |
| **HAs** $\frac{1}{x^3}$ | $\frac{\text{small}}{\text{big}}$ | $y=0$ |
| $\frac{x^2+1}{x^2}$ | $\frac{\text{same}}{\text{same}}$ | $y = \frac{\text{ratio of}}{\text{coeff.}} = \frac{\text{L.C. top}}{\text{L.C. bottom}}$ |
| $\frac{x^3}{1}$ | $\frac{\text{big}}{\text{small}}$ | no HA<br>However |
| **SAs**<br>- subtract<br>- need parenthases<br>- place holder | IF there is NO horizontal asymptote AND the degree of the numerator is ONLY ONE degree greater than the degree of the denominator, check for a **slant asymptote (SA)** using the simplified equation.<br>a) Use long or synthetic division.<br>b) If the remainder is zero, then there is NO SA.<br>$\frac{\text{big}}{\text{small}}$ no **HA, SA** if $\frac{x^2}{x}, \frac{x^3}{x^2}, \frac{x^4}{x^3} \ldots$ | |
| **Polynomial Division** | | |
| **Synthetic Division** | | |

## HW Review p. 543 #14-22

14. $y=1, x=3$
D: $x \neq 3$
R: $y \neq 1$

15. $y=2/3, x=-1/3$
D: $x \neq -1/3$
R: $y \neq 2/3$

16. $y=3/4, x=-5/4$
$x \neq -5/4$
$y \neq 3/4$

17. $y=-17, x=-43$
D: $x \neq -43$
R: $y \neq -17$

18. $y=\frac{17}{8}, x=-\frac{1}{4}$
D: $x \neq -1/4$
R: $y \neq 17/8$

19. $y=19, x=6$
D: $x \neq 6$
R: $y \neq 19$

20. B  21. C  22. A

Divide $2x^4+3x^3+5x-1$ by $x^2-2x+2$
$+0x^2$

$$
\begin{array}{r}
2x^2+7x+10 \\
x^2-2x+2 \,\big)\, 2x^4+3x^3+0x^2+5x-1 \\
+(-2x^4+4x^3-4x^2) \\
\hline
7x^3-4x^2+5x \\
+(-7x^3+14x^2-14x) \\
\hline
10x^2-9x-1 \\
+(-10x^2+20x-20) \\
\hline
\end{array}
$$

Remainder $11x-21$

#1) $(x^2+7x-5)\div(x-2)$

#2) $(2x^4+7)\div(x^2-1)$

#3) $(6x^2+x-7)\div(2x+3)$

**Long Division of Polynomials**

**Steps:** 1.) Divide 2.) Multiply 3.) Subtract 4.) Drop 5.) Repeat

**** Remember to add in place holding zeros!!!

1.) $(x^2-x-22)\div(x-5)$

- parentheses
- subtract

$$
\begin{array}{r}
x+4 \quad R\text{-}2 \\
x-5 \,\big)\, x^2-x-22 \\
+(-x^2+5x) \\
\hline
4x-22 \\
+(-4x+20) \\
\hline
\boxed{-2}
\end{array}
$$

**Long Division of Polynomials**

**Steps:** 1.) Divide 2.) Multiply 3.) Subtract 4.) Drop 5.) Repeat

****** Remember to add in place holding zeros!!!**

2.) $(2x^2 + 17x + 21) \div (2x + 3)$

$$\begin{array}{r} x + 7 \\ 2x+3 \overline{)\,2x^2 + 17x + 21} \\ + \; -(2x^2 \mp 3x) \\ 14x + 21 \\ + \; -(14x \mp 21) \\ \hline \boxed{0} \end{array}$$

Synthetic: – always ÷ (x−k)
– place holders
– add

5.) $(x^3 - 3x^2 - 7x + 6) \div (x + 2)$ $x+2=0$ $x=-2$

$x-(-2)$

$$\begin{array}{r|rrrr} k & & & & \\ -2 & 1 & -3 & -7 & 6 \\ + & \downarrow & -2 & 10 & -6 \\ \hline & 1x^2 & -5x & 3 & \boxed{0} \end{array}$$

R

$y = x^2 - 5x + 3$

6.) $(5x^4 - 2x^3 - 3x^2 + 5x + 1) \div (x - 1)$

$$\begin{array}{r|rrrrr} 1 & 5 & -2 & -3 & 5 & 1 \\ + & \downarrow & 5 & 3 & 0 & 5 \\ \hline & 5x^3 & +3x^2 & 0x & +5 & \boxed{6} \end{array}$$

$y = 5x^3 + 3x^2 + 5$ R6

7.) $(x^4 - 5x^3 + 4x - 17) \div (x - 5)$

$x^4 - 5x^3 + 0x^2 + 4x - 17$

$$\begin{array}{r|rrrrr} 5 & 1 & -5 & 0 & 4 & -17 \\ + & \downarrow & 5 & 0 & 0 & 20 \\ \hline & 1x^3 & 0x^2 & 0x & 4 & \boxed{3} \end{array}$$

$\boxed{y = x^3 + 4}$ R3

8.) $(x^3 - 2) \div (x + 1)$

# *Follow these examples*

**Long Division of Polynomials**

**Steps:** 1.) Divide 2.) Multiply 3.) Subtract 4.) Drop 5.) Repeat

****** Remember to add in place holding zeros!!!**

1.) $(x^2 - x - 22) \div (x - 5)$

$\boxed{x + 4 \text{ R } -2}$

$$\begin{array}{r} x + 4 \\ x-5 \overline{)\, x^2 - x - 22} \\ x^2 - 5x \\ \hline 4x - 22 \\ 4x - 20 \\ \hline -2 \end{array}$$

**Synthetic Division of Polynomials**

****** Remember to add in place holding zeros!!!**

5.) $(x^3 - 3x^2 - 7x + 6) \div (x + 2)$

$\boxed{x^2 - 5x + 3}$

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -7 & 6 \\ & \downarrow & -2 & 10 & -6 \\ \hline & 1 & -5 & 3 & 0 \end{array}$$

1.) $(x^2 - x - 22) \div (x - 5)$
$x + 4$ R $-2$

2.) $(2x^2 + 17x + 21) \div (2x + 3)$
$x + 7$

3.) $(x^3 + 6x^2 - 5x - 10) \div (x^2 - 5)$
$x + 6$ R 20

4.) $(4x^2 - 8) \div (2x + 1)$
$2x - 1$ R $-7$

5.) $(x^3 - 3x^2 - 7x + 6) \div (x + 2)$
$x^2 - 5x + 3$

6.) $(5x^4 - 2x^3 - 3x^2 + 5x + 1) \div (x - 1)$
$5x^3 + 3x^2 + 5$ R 6

7.) $(x^4 - 5x^3 + 4x - 17) \div (x - 5)$
$x^3 + 4$ R 3

8.) $(x^3 - 2) \div (x + 1)$
$x^2 - x + 1$ R $-3$

9.) $(2x^3 + 4x^2 - 70x) \div (x - 5)$
$2x^2 + 14x$

10.) $(x^2 + 5x + 6) \div (x + 3)$
$x + 2$

**Long Division of Polynomials**

**Steps:** 1.) Divide 2.) Multiply 3.) Subtract 4.) Drop 5.) Repeat

****** Remember to add in place holding zeros!!!**

3.) $(x^3 + 6x^2 - 5x - 10) \div (x^2 - 5)$

$x^2 + 0x - 5$

$$
\begin{array}{r}
x + 6 \quad R20 \\
x^2 + 0x - 5 \overline{)\, x^3 + 6x^2 - 5x - 10} \\
+ (-x^3 \mp 0x^2 + 5x) \downarrow \\
\hline
6x^2 + 0x - 10 \\
+ (-6x^2 \mp 0x + 30) \\
\hline
\boxed{20}
\end{array}
$$

**Long Division of Polynomials**

**Steps:** 1.) Divide 2.) Multiply 3.) Subtract 4.) Drop 5.) Repeat

****** Remember to add in place holding zeros!!!**

4.) $(4x^2 - 8) \div (2x + 1)$

$4x^2 + 0x - 8$

$$
\begin{array}{r}
2x - 1 \quad R{-7} \\
2x + 1 \overline{)\, 4x^2 + 0x - 8} \\
+ (-4x^2 \mp 2x) \downarrow \\
\hline
-2x - 8 \\
+ (+2x + 1) \\
\hline
\boxed{-7}
\end{array}
$$

9.) $(2x^3 + 4x^2 - 70x) \div (x - 5)$

10.) $(x^2 + 5x + 6) \div (x + 3)$

| Main Ideas & Questions | Notes & Examples | |
|---|---|---|
| | Case | Horizontal Asymptote |
| **HAs** $\frac{1}{x^3}$ | $\frac{\text{small}}{\text{big}}$ | $y = 0$ |
| $\frac{x^2+1}{x^2}$ | $\frac{\text{same}}{\text{same}}$ | $y =$ ratio of coeff. $= \frac{\text{L.C. top}}{\text{L.C. bottom}}$ |
| $\frac{x^3}{1}$ | $\frac{\text{big}}{\text{small}}$ | no HA |
| **SAs** <br> - subtract <br> - need parentheses <br> - place holder | However, If there is NO horizontal asymptote AND the degree of the numerator is ONLY ONE degree greater than the degree of the denominator, check for a **slant asymptote (SA)** using the simplified equation: <br> a) Use long or synthetic division. <br> b) If the remainder is zero, then there is NO SA. we want a remainder <br> $\frac{\text{big}}{\text{small}}$ no **HA, SA** if $\frac{x^2}{x}, \frac{x^3}{x^2}, \frac{x^4}{x^3} \ldots$ | |
| **Polynomial Division** | | |
| **Synthetic Division** | | |

**Examples: Horizontal Asymptotes (HA)**

1. $y = \frac{3x+1}{2x-2}$ HA: $y = \frac{3}{2}$

2. $y = \frac{4x}{1+x^2}$ HA: $y = 0$

3. $y = \frac{8x-3}{4x+8}$ HA: $y = 2$

4. $y = \frac{5x^2+3x+2}{2x^3-8x}$ HA: $y = 0$

**Examples: Slant Asymptotes (SA)**

5. $y = \frac{x^2+8x+15}{x+2}$ SA: $y = x+6$

$$\begin{array}{r|rrr} -2 & 1 & 8 & 15 \\ + & \downarrow & -2 & -12 \\ \hline & 1x & 6 & \boxed{3} \end{array}$$

6. $y = \frac{5x^2-10x+1}{x-2}$ SA:

7. $y = \frac{x^2}{x-1}$ SA:

8. $y = \frac{x^2-9}{x+3}$ SA: ~~None~~

9. $y = \frac{6x^2-x+4}{3x+1}$ SA:

10. $y = \frac{x^2-1}{2x+4}$ SA: $y = \frac{1}{2}x - 1$

$$\begin{array}{r} \frac{1}{2}x - 1 \\ 2x+4 \overline{) x^2 + 0x - 1} \\ + \underline{(-x^2 \mp 2x)} \\ -2x - 1 \\ + \underline{(+2x + 4)} \\ \boxed{3} \end{array}$$

11. $y = \frac{x^3+6x^2-5x-10}{x^2-5}$ SA:

12. $y = \frac{4x^3+2x-1}{x-7}$ SA:

# Warm Up

- Quiz: This is a formative assessment. You WILL be graded on i[t] but you WILL be able to redo it until you get 100%.
- Do YOUR best. Be prepared to learn. You will succeed!

- HW on your desks

Also, check your grades! If you have a missing, you NEED to complete that TODAY after school. Otherwise, it's a ZERO.

**HW: Text pg 356-357**

**Divide using polynomial long division.**

**4.** $(2x^3 - 7x^2 - 17x - 3) \div (2x + 3)$

$\boxed{x^2 - 5x - 1}$

$$\begin{array}{r} \boxed{x^2 - 5x - 1} \\ 2x+3 \overline{) 2x^3 - 7x^2 - 17x - 3} \\ \underline{-2x^3 + 3x^2} \\ -10x^2 - 17x \\ \underline{+10x^2 + 15x} \\ -2x - 3 \\ \underline{+2x + 3} \\ 0 \end{array}$$

**USING LONG DIVISION** **Divide using polynomial long division.**

**21.** $(2x^4 + 7) \div (x^2 - 1)$

$\boxed{2x^2 + 2 + \frac{9}{x^2-1}}$

$$\begin{array}{r} \boxed{2x^2 + 2 + \frac{9}{x^2-1}} \\ x^2 + 0x - 1 \overline{) 2x^4 + 0x^3 + 0x^2 + 0x + 7} \\ \underline{-2x^4 + 0x^3 + 2x^2} \\ 2x^2 + 0x + 7 \\ \underline{-2x^2 + 0x + 2} \\ 9 \end{array}$$

**USING SYNTHETIC DIVISION** **Divide using synthetic division.**

**28.** $(x^3 - 14x + 8) \div (x + 4)$ $k = -4$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -14 & 8 \\ + & \downarrow & -4 & 16 & -8 \\ \hline & 1 & -4 & 2 & \boxed{0} \end{array}$$

$\boxed{x^2 - 4x + 2}$

**31.** $(2x^2 + 7x + 8) \div (x - 2)$ $k = 2$

$$\begin{array}{r|rrr} 2 & 2 & 7 & 8 \\ + & \downarrow & 4 & 22 \\ \hline & 2 & 11 & \boxed{30} \end{array}$$

$\boxed{2x + 11 + \frac{30}{x-2}}$

**FACTORING** **Factor the polynomial given that $f(k) = 0$.**

**39.** $f(x) = x^3 - 5x^2 - 2x + 24$; $k = -2$

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -2 & 24 \\ + & \downarrow & -2 & 14 & -24 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$

$(x+2)(x^2 - 7x + 12)$

$\boxed{(x+2)(x-4)(x-3)}$

**41.** $f(x) = x^3 - 12x^2 + 12x + 80$; $k = 10$

$$\begin{array}{r|rrrr} 10 & 1 & -12 & 12 & 80 \\ + & \downarrow & 10 & -20 & -80 \\ \hline & 1 & -2 & -8 & \boxed{0} \end{array}$$

$(x-10)(x^2 - 2x - 8)$

$\boxed{(x-10)(x-4)(x+2)}$

**FACTORING** Factor the polynomial given that $f(k) = 0$.

**43.** $f(x) = x^3 - x^2 - 21x + 45;\ k = -5$

$$\begin{array}{r|rrrr} -5 & 1 & -1 & -21 & 45 \\ + & \downarrow & -5 & 30 & -45 \\ \hline & 1 & -6 & 9 & \boxed{0} \end{array}$$

$(x+5)(x^2-6x+9)$

$\boxed{(x+5)(x-3)(x-3)}$

**45.** $f(x) = 4x^3 - 4x^2 - 9x + 9;\ k = 1$

$$\begin{array}{r|rrrr} 1 & 4 & -4 & -9 & 9 \\ + & \downarrow & 4 & 0 & -9 \\ \hline & 4 & 0 & -9 & \boxed{0} \end{array}$$

$(x-1)(4x^2-9)$

$\boxed{(x-1)(2x+3)(2x-3)}$

**Putting it all together now:**

**13.** $y = \dfrac{x-4}{x^2-16}$

$= \dfrac{\cancel{(x-4)}}{\cancel{(x-4)}(x+4)} = \boxed{\dfrac{1}{x+4}}$

Hole: $(4, \frac{1}{8})$
VA: $x=-4$
HA: $y=0$
SA:

**14.** $y = \dfrac{x^2-3x-4}{2x^2+x-1}$

$\dfrac{(x-4)\cancel{(x+1)}}{(2x-1)\cancel{(x+1)}}$

Hole: $(-1, \frac{5}{3})$
VA: $x=\frac{1}{2}$
HA: $y=\frac{1}{2}$
SA:

**15.** $y = \dfrac{2x^2-5x+5}{x-2}$

DNF

Hole:
VA: $x=2$
HA:
SA: $y=2x-1$

$$\begin{array}{r|rrr} 2 & 2 & -5 & 5 \\ + & \downarrow & 4 & -2 \\ \hline & 2 & -1 & \boxed{3} \end{array}$$

- We will only ever have an HA or an SA- never both!
- We will only ever have ONE HA/SA
- We can, however, have multiple VAs

$f(x) = \dfrac{x^3+3x^2}{x^2+2x-3}$

$= \dfrac{x^2\cancel{(x+3)}}{(x-1)\cancel{(x+3)}}$

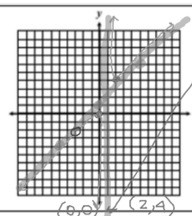
$\boxed{= \dfrac{x^2}{x-1}}$

$x+3=0$
$x=-3$

$\frac{4}{1}$

$x^2=0$
$x=0$

$x-1=0$
$x=1$




$(-3, -\frac{9}{4})$

x-int: $(0,0)$
VA: $x=1$
HA:
Hole: $(-3, -\frac{9}{4})$
Domain: $x \neq -3, 1$
Range: $y \neq -\frac{9}{4}$

Slant: $y = x+1$

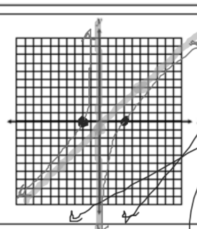
$\dfrac{x^2+0x+0}{x-1}$

$$\begin{array}{r|rrr} 1 & 1 & 0 & 0 \\ + & \downarrow & 1 & 1 \\ \hline & 1x & +1 & \boxed{1} \end{array}$$

Since we have a remainder we also have an SA

$f(x) = \dfrac{x^2-x-6}{x}$

$= \dfrac{(x-3)(x+2)}{x}$




x-int: $(3,0), (-2,0)$
VA: $x=0$
HA:
Hole:
Domain: $x \neq 0$
Range: $\mathbb{R}$

$x-3=0$
$x=3$

$x+2=0$
$x=-2$

$x=0$

Slant:
$y = x-1$

$mx+b$

$$\begin{array}{r} y = x - 1 \\ x+0 \overline{\big)\, x^2 - x - 6} \\ -(x^2+0x) \\ \hline -x - 6 \\ -(-x - 0) \\ \hline \boxed{-6} \end{array}$$

I have a SA

I can write the equation of a function with the given characteristics.

**VAs** @#
↳ when the denom. = 0
$\frac{1}{(x-\#)}$

**HAs**
$\frac{sm}{big}$ : y = 0
$\frac{same}{same}$ : y = ratio
$\frac{big}{small}$ : none

**Holes** @#
cancel factors out of top & bottom
$\frac{(x-\#)}{(x-\#)}$

# Examples:

1) VA at x = -1, a hole at x = 3, HA at y = 2

same / same

$$\frac{2x(x-3)}{(x+1)(x-3)}$$

2) VA at x=0, hole at x=1, HA at y=0

sm / big

$$\frac{(x-1)}{x(x-1)}$$

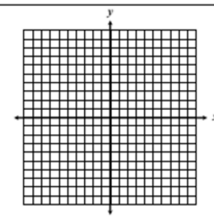
12) Holes at x = -1 and x = 3 and resembles the graph y = x

x+1=0   x-3=0

resembles → add to the top

$$\frac{x(x+1)(x-3)}{(x+1)(x-3)}$$

$f(x) = \frac{3x^2-5x-2}{x^2-3x+2}$



x-int: ____________

VA: ____________

HA: ____________

Hole: ____________

Domain: ____________

Range: ____________

Create a function of the form y = f(x) with:

19) VA at x = 4, hole at x = 0, HA at y = 3

same / same

$$\frac{\cancel{2x}\,3x\,x}{x(x-4)}$$

20) VA at x = -5 and x = 1, hole at x = -1

$$\frac{(x+1)}{(x+5)(x-1)(x+1)}$$

21) holes at x = 3 and x = -7, resembles y = x

x-3=0   x+7

$$\frac{x(x-3)(x+7)}{(x-3)(x+7)}$$

# Homework:

1. Text pg 356-357
   -#4, 21, 28, 31,

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