3.2

Logarithmic Functions and Their Graphs

What you should learn

- Recognize and evaluate logarithmic functions with base *a*.
- Graph logarithmic functions.
- Recognize, evaluate, and graph natural logarithmic functions.
- Use logarithmic functions to model and solve real-life problems.

Why you should learn it

Logarithmic functions are often used to model scientific observations. For instance, in Exercise 89 on page 238, a logarithmic function is used to model human memory.



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Logarithmic Functions

In Section 1.9, you studied the concept of an inverse function. There, you learned that if a function is one-to-one-that is, if the function has the property that no horizontal line intersects the graph of the function more than once-the function must have an inverse function. By looking back at the graphs of the exponential functions introduced in Section 3.1, you will see that every function of the form $f(x) = a^x$ passes the Horizontal Line Test and therefore must have an inverse function. This inverse function is called the **logarithmic function with base** *a*.

Definition of Logarithmic Function with Base a

For x > 0, a > 0, and $a \neq 1$,

 $y = \log_a x$ if and only if $x = a^y$.

The function given by

 $f(x) = \log_a x$ Read as "log base *a* of *x*."

is called the logarithmic function with base a.

The equations

 $y = \log_a x$ and $x = a^{y}$

are equivalent. The first equation is in logarithmic form and the second is in exponential form. For example, the logarithmic equation $2 = \log_3 9$ can be rewritten in exponential form as $9 = 3^2$. The exponential equation $5^3 = 125$ can be rewritten in logarithmic form as $\log_5 125 = 3$.

When evaluating logarithms, remember that a logarithm is an exponent. This means that $\log_a x$ is the exponent to which a must be raised to obtain x. For instance, $\log_2 8 = 3$ because 2 must be raised to the third power to get 8.

Example 1

Evaluating Logarithms

Use the definition of logarithmic function to evaluate each logarithm at the indicated value of *x*.

a. $f(x) = \log_2 x, x = 32$	b. $f(x) = \log_3 x, x = 1$
c. $f(x) = \log_4 x, x = 2$	d. $f(x) = \log_{10} x$, $x = \frac{1}{100}$
Solution	
a. $f(32) = \log_2 32 = 5$	because $2^5 = 32$.
b. $f(1) = \log_3 1 = 0$	because $3^0 = 1$.
c. $f(2) = \log_4 2 = \frac{1}{2}$	because $4^{1/2} = \sqrt{4} = 2$.
d. $f\left(\frac{1}{100}\right) = \log_{10} \frac{1}{100} = -2$	because $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$.
CHECKPOINT Now try Exerci	se 17

STUDY TIP

Remember that a logarithm is an exponent. So, to evaluate the logarithmic expression $\log_a x$, you need to ask the question, "To what power must a be raised to obtain x?"

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Compare the two tables. What is the relationship between $f(x) = 10^{x}$ and $f(x) = \log x$?

The logarithmic function with base 10 is called the **common logarithmic** function. It is denoted by \log_{10} or simply by log. On most calculators, this function is denoted by [LOG]. Example 2 shows how to use a calculator to evaluate common logarithmic functions. You will learn how to use a calculator to calculate logarithms to any base in the next section.

Example 2 Evaluating Common Logarithms on a Calculator

Use a calculator to evaluate the function given by $f(x) = \log x$ at each value of x.

b. $x = \frac{1}{3}$ **a.** x = 10**c.** x = 2.5**d.** x = -2

Solution

	Function Value	Graphing Calculator Keystrokes	Display
a.	$f(10) = \log 10$	LOG 10 ENTER	1
b.	$f\left(\frac{1}{3}\right) = \log \frac{1}{3}$	LOG (1 ÷ 3) ENTER	-0.4771213
c.	$f(2.5) = \log 2.5$	LOG 2.5 ENTER	0.3979400
d.	$f(-2) = \log(-2)$	LOG (-) 2 ENTER	ERROR

Note that the calculator displays an error message (or a complex number) when you try to evaluate $\log(-2)$. The reason for this is that there is no real number power to which 10 can be raised to obtain -2.

CHECKPOINT Now try Exercise 23.

The following properties follow directly from the definition of the logarithmic function with base *a*.

Properties of Logarithms

1.	$\log_a 1 = 0$ because $a^0 = 1$.	
2.	$\log_a a = 1$ because $a^1 = a$.	
3.	$\log_a a^x = x$ and $a^{\log_a x} = x$	Inverse Properties
4.	If $\log_a x = \log_a y$, then $x = y$.	One-to-One Property

Using Properties of Logarithms Example 3

a. Simplify: log₄ 1

```
b. Simplify: \log_{\sqrt{7}} \sqrt{7} c. Simplify: 6^{\log_6 20}
```

Solution

- **a.** Using Property 1, it follows that $\log_4 1 = 0$.
- **b.** Using Property 2, you can conclude that $\log_{\sqrt{7}} \sqrt{7} = 1$.
- **c.** Using the Inverse Property (Property 3), it follows that $6^{\log_6 20} = 20$.

CHECKPOINT Now try Exercise 27.

You can use the One-to-One Property (Property 4) to solve simple logarithmic equations, as shown in Example 4.

The logarithmic function can be one of the most difficult concepts for students to understand. Remind students that a logarithm is an exponent. Converting back and forth from logarithmic form to exponential form supports this concept.

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Example 4 Using the One-to-One Property

a.	$\log_3 x = \log_3 12$	Original equation
	x = 12	One-to-One Property
b.	$\log(2x+1) = \log x =$	$\Rightarrow 2x + 1 = x \implies x = -1$
c.	$\log_4(x^2 - 6) = \log_4 10$	$\Rightarrow x^2 - 6 = 10 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$
	CHECKPOINT Now try	Exercise 79.

Graphs of Logarithmic Functions

To sketch the graph of $y = \log_a x$, you can use the fact that the graphs of inverse functions are reflections of each other in the line y = x.

Example 5 Graphs of Exponential and Logarithmic Functions

In the same coordinate plane, sketch the graph of each function.

a.
$$f(x) = 2^x$$
 b. $g(x) = \log_2 x$

a. For $f(x) = 2^x$, construct a table of values. By plotting these points and con-

x	-2	-1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

b. Because $g(x) = \log_2 x$ is the inverse function of $f(x) = 2^x$, the graph of g is obtained by plotting the points (f(x), x) and connecting them with a smooth curve. The graph of g is a reflection of the graph of f in the line y = x, as shown in Figure 3.13.

CHECKPOINT Now try Exercise 31.

5 Vertical asymptote: x = 04 3 $f(x) = \log x$ 2 1 9

 $f(x) = 2^x$

v = x

FIGURE 3.14

10

8

6

4

.2

FIGURE 3.13

Example 6 Sketching the Graph of a Logarithmic Function

Sketch the graph of the common logarithmic function $f(x) = \log x$. Identify the vertical asymptote.

Solution

Begin by constructing a table of values. Note that some of the values can be obtained without a calculator by using the Inverse Property of Logarithms. Others require a calculator. Next, plot the points and connect them with a smooth curve, as shown in Figure 3.14. The vertical asymptote is x = 0 (y-axis).

	Without calculator				lator With calculator		
x	$\frac{1}{100}$	$\frac{1}{10}$	1	10	2	5	8
$f(x) = \log x$	-2	-1	0	1	0.301	0.699	0.903



CHECKPOINT Now try Exercise 37.

Solution

necting them with a smooth curve, you obtain the graph shown in Figure 3.13.

x	-2	-1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

 $g(x) = \log_2 x$ 10 8 6

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The nature of the graph in Figure 3.14 is typical of functions of the form $f(x) = \log_a x, a > 1$. They have one x-intercept and one vertical asymptote. Notice how slowly the graph rises for x > 1. The basic characteristics of logarithmic graphs are summarized in Figure 3.15.



The basic characteristics of the graph of $f(x) = a^x$ are shown below to illustrate the inverse relation between $f(x) = a^x$ and $g(x) = \log_a x$.

- Domain: $(-\infty, \infty)$ Range: $(0, \infty)$
- *y*-intercept: (0,1) • *x*-axis is a horizontal asymptote $(a^x \rightarrow 0 \text{ as } x \rightarrow -\infty)$.

In the next example, the graph of $y = \log_a x$ is used to sketch the graphs of functions of the form $f(x) = b \pm \log_a(x + c)$. Notice how a horizontal shift of the graph results in a horizontal shift of the vertical asymptote.

Example 7 Shifting Graphs of Logarithmic Functions

The graph of each of the functions is similar to the graph of $f(x) = \log x$.

- **a.** Because $g(x) = \log(x 1) = f(x 1)$, the graph of g can be obtained by shifting the graph of f one unit to the right, as shown in Figure 3.16.
- **b.** Because $h(x) = 2 + \log x = 2 + f(x)$, the graph of h can be obtained by shifting the graph of f two units upward, as shown in Figure 3.17.







STUDY TIP

You can use your understanding of transformations to identify vertical asymptotes of logarithmic functions. For instance, in Example 7(a) the graph of g(x) = f(x - 1) shifts the graph of f(x) one unit to the right. So, the vertical asymptote of g(x) is x = 1, one unit to the right of the vertical asymptote of the graph of f(x).



The Natural Logarithmic Function

By looking back at the graph of the natural exponential function introduced in Section 3.1 on page 388, you will see that $f(x) = e^x$ is one-to-one and so has an inverse function. This inverse function is called the **natural logarithmic function** and is denoted by the special symbol ln *x*, read as "the natural log of *x*" or "el en of *x*." Note that the natural logarithm is written without a base. The base is understood to be *e*.

The Natural Logarithmic Function

The function defined by

 $f(x) = \log_e x = \ln x, \quad x > 0$

is called the natural logarithmic function.

The definition above implies that the natural logarithmic function and the natural exponential function are inverse functions of each other. So, every logarithmic equation can be written in an equivalent exponential form and every exponential equation can be written in logarithmic form. That is, $y = \ln x$ and $x = e^y$ are equivalent equations.

Because the functions given by $f(x) = e^x$ and $g(x) = \ln x$ are inverse functions of each other, their graphs are reflections of each other in the line y = x. This reflective property is illustrated in Figure 3.18.

On most calculators, the natural logarithm is denoted by \boxed{LN} , as illustrated in Example 8.

Example 8 Evaluating the Natural Logarithmic Function

Use a calculator to evaluate the function given by $f(x) = \ln x$ for each value of x.

a. x = 2 **b.** x = 0.3 **c.** x = -1 **d.** $x = 1 + \sqrt{2}$

Solution

	Function Value	Graphing Calculator Keystrokes	Display
a.	$f(2) = \ln 2$	LN 2 ENTER	0.6931472
b.	$f(0.3) = \ln 0.3$	LN .3 ENTER	-1.2039728
c.	$f(-1) = \ln(-1)$	LN (-) 1 ENTER	ERROR
d.	$f(1+\sqrt{2}) = \ln(1+\sqrt{2})$	LN (1 + \checkmark 2) ENTER	0.8813736

CHECKPOINT Now try Exercise 61.

In Example 8, be sure you see that $\ln(-1)$ gives an error message on most calculators. (Some calculators may display a complex number.) This occurs because the domain of $\ln x$ is the set of positive real numbers (see Figure 3.18). So, $\ln(-1)$ is undefined.

The four properties of logarithms listed on page 230 are also valid for natural logarithms.



Reflection of graph of $f(x) = e^x$ about the line y = xFIGURE 3.18

STUDY TIP

Notice that as with every other logarithmic function, the domain of the natural logarithmic function is the set of *positive real numbers*—be sure you see that ln *x* is not defined for zero or for negative numbers.

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Chapter 3 **Exponential and Logarithmic Functions**

Properties of Natural Logarithms 1. $\ln 1 = 0$ because $e^0 = 1$. **2.** $\ln e = 1$ because $e^1 = e$. **3.** $\ln e^x = x$ and $e^{\ln x} = x$ **Inverse Properties** 4. If $\ln x = \ln y$, then x = y. One-to-One Property

Example 9 **Using Properties of Natural Logarithms**

Use the properties of natural logarithms to simplify each expression.

a.
$$\ln \frac{1}{e}$$
 b. $e^{\ln 5}$ **c.** $\frac{\ln 1}{3}$ **d.** $2 \ln e$
Solution
a. $\ln \frac{1}{e} = \ln e^{-1} = -1$ Inverse Property **b.** $e^{\ln 5} = 5$ Inverse Property
c. $\frac{\ln 1}{3} = \frac{0}{3} = 0$ Property 1 **d.** $2 \ln e = 2(1) = 2$ Property 2

Example 10 Finding the Domains of Logarithmic Functions

Find the domain of each function.

a.
$$f(x) = \ln(x - 2)$$
 b. $g(x) = \ln(2 - x)$ **c.** $h(x) = \ln x^2$

Solution

- **a.** Because $\ln(x 2)$ is defined only if x 2 > 0, it follows that the domain of f is $(2, \infty)$. The graph of f is shown in Figure 3.19.
- **b.** Because $\ln(2 x)$ is defined only if 2 x > 0, it follows that the domain of g is $(-\infty, 2)$. The graph of g is shown in Figure 3.20.
- **c.** Because $\ln x^2$ is defined only if $x^2 > 0$, it follows that the domain of h is all real numbers except x = 0. The graph of h is shown in Figure 3.21.







FIGURE 3.22

Application





Students participating in a psychology experiment attended several lectures on a subject and were given an exam. Every month for a year after the exam, the students were retested to see how much of the material they remembered. The average scores for the group are given by the *human memory model*

$$f(t) = 75 - 6 \ln(t+1), \quad 0 \le t \le 12$$

where *t* is the time in months. The graph of *f* is shown in Figure 3.22.

a. What was the average score on the original (t = 0) exam?

Human Memory Model

- **b.** What was the average score at the end of t = 2 months?
- c. What was the average score at the end of t = 6 months?

Solution

a. The original average score was

$f(0) = 75 - 6\ln(0 + 1)$	Substitute 0 for <i>t</i> .
$= 75 - 6 \ln 1$	Simplify.
= 75 - 6(0)	Property of natural logarithms
= 75.	Solution

b. After 2 months, the average score was

$f(2) = 75 - 6\ln(2 + 1)$	Substitute 2 for <i>t</i> .
$= 75 - 6 \ln 3$	Simplify.
$\approx 75 - 6(1.0986)$	Use a calculator.
≈ 68.4.	Solution

c. After 6 months, the average score was

 $f(6) = 75 - 6 \ln(6 + 1)$ Substitute 6 for t. = 75 - 6 ln 7 Simplify. $\approx 75 - 6(1.9459)$ Use a calculator. $\approx 63.3.$ Solution

CHECKPOINT Now try Exercise 89.

WRITING ABOUT MATHEMATICS

Analyzing a Human Memory Model Use a graphing utility to determine the time in months when the average score in Example 11 was 60. Explain your method of solving the problem. Describe another way that you can use a graphing utility to determine the answer.

Alternative Writing About Mathematics

Use a graphing utility to graph $f(x) = \ln x$. How will the graphs of $h(x) = \ln x + 5$, $j(x) = \ln(x - 3)$, and $l(x) = \ln x - 4$ differ from the graph of f?

How will the basic graph of *f* be affected when a constant *c* is introduced: $g(x) = c \ln x$? Use a graphing utility to graph *g* with several different positive values of *c*, and summarize the effect of *c*.

3.2 Exercises

VOCABULARY CHECK: Fill in the blanks.

- 1. The inverse function of the exponential function given by $f(x) = a^x$ is called the _____ function with base a.
- 2. The common logarithmic function has base ______.
- 3. The logarithmic function given by $f(x) = \ln x$ is called the ______ logarithmic function and has base ______
- **4.** The Inverse Property of logarithms and exponentials states that $\log_a a^x = x$ and _____
- 5. The One-to-One Property of natural logarithms states that if $\ln x = \ln y$, then _____

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at www.Eduspace.com.

In Exercises 1–8, write the logarithmic equation in exponential form. For example, the exponential form of $\log_5 25 = 2$ is $5^2 = 25$.

1.
$$\log_4 64 = 3$$
 2. $\log_3 81 = 4$

 3. $\log_7 \frac{1}{49} = -2$
 4. $\log \frac{1}{1000} = -3$

 5. $\log_{32} 4 = \frac{2}{5}$
 6. $\log_{16} 8 = \frac{3}{4}$

 7. $\log_{26} 6 = \frac{1}{2}$
 8. $\log_8 4 = \frac{2}{3}$

In Exercises 9–16, write the exponential equation in logarithmic form. For example, the logarithmic form of $2^3 = 8$ is $\log_2 8 = 3$.

9.
$$5^3 = 125$$
 10. $8^2 = 64$

 11. $81^{1/4} = 3$
 12. $9^{3/2} = 27$

 13. $6^{-2} = \frac{1}{36}$
 14. $4^{-3} = \frac{1}{64}$

 15. $7^0 = 1$
 16. $10^{-3} = 0.001$

In Exercises 17–22, evaluate the function at the indicated value of *x* without using a calculator.

Function	Value
17. $f(x) = \log_2 x$	<i>x</i> = 16
18. $f(x) = \log_{16} x$	x = 4
19. $f(x) = \log_7 x$	x = 1
20. $f(x) = \log x$	x = 10
21. $g(x) = \log_a x$	$x = a^2$
22. $g(x) = \log_b x$	$x = b^{-3}$

In Exercises 23–26, use a calculator to evaluate $f(x) = \log x$ at the indicated value of x. Round your result to three decimal places.

23.	$x = \frac{4}{5}$	24. $x = \frac{1}{50}$	$\overline{0}$
25.	x = 12.5	26. $x = 75$	5.25

In Exercises 27–30, use the properties of logarithms to simplify the expression.

27.	$\log_3 3^4$	28.	$\log_{1.5}1$
29.	$\log_{\pi} \pi$	30.	$9^{\log_9 15}$

In Exercises 31–38, find the domain, *x*-intercept, and vertical asymptote of the logarithmic function and sketch its graph.

31. $f(x) = \log_4 x$	32. $g(x) = \log_6 x$
33. $y = -\log_3 x + 2$	34. $h(x) = \log_4(x - 3)$
35. $f(x) = -\log_6(x+2)$	36. $y = \log_5(x - 1) + 4$
$37. \ y = \log\left(\frac{x}{5}\right)$	38. $y = \log(-x)$

In Exercises 39–44, use the graph of $g(x) = \log_3 x$ to match the given function with its graph. Then describe the relationship between the graphs of f and g. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]





39. $f(x) = \log_3 x + 2$	40. $f(x) =$
41. $f(x) = -\log_3(x+2)$	42. $f(x) =$
43. $f(x) = \log_3(1 - x)$	44. $f(x) =$

In Exercises 45–52, write the logarithmic equation in exponential form.

 $\log_3(x-1) \\ -\log_3(-x)$

45. $\ln \frac{1}{2} = -0.693 \dots$	46. $\ln \frac{2}{5} = -0.916 \dots$
47. $\ln 4 = 1.386 \dots$	48. $\ln 10 = 2.302 \dots$
49. $\ln 250 = 5.521 \dots$	50. $\ln 679 = 6.520 \dots$
51. $\ln 1 = 0$	52. $\ln e = 1$

In Exercises 53–60, write the exponential equation in logarithmic form.

53. $e^3 = 20.0855 \dots$	54. $e^2 = 7.3890 \dots$
55. $e^{1/2} = 1.6487 \dots$	56. $e^{1/3} = 1.3956$
57. $e^{-0.5} = 0.6065 \dots$	58. $e^{-4.1} = 0.0165 \dots$
59. $e^x = 4$	60. $e^{2x} = 3$

In Exercises 61–64, use a calculator to evaluate the function at the indicated value of x. Round your result to three decimal places.

Function	Value
61. $f(x) = \ln x$	x = 18.42
62. $f(x) = 3 \ln x$	x = 0.32
63. $g(x) = 2 \ln x$	x = 0.75
64. $g(x) = -\ln x$	$x = \frac{1}{2}$

In Exercises 65–68, evaluate $g(x) = \ln x$ at the indicated value of x without using a calculator.

65.	$x = e^3$	66.	$x = e^{-2}$
67.	$x = e^{-2/3}$	68.	$x = e^{-5/2}$

In Exercises 69–72, find the domain, *x*-intercept, and vertical asymptote of the logarithmic function and sketch its graph.

69. $f(x) = \ln(x - 1)$	70. $h(x) = \ln(x+1)$
71. $g(x) = \ln(-x)$	72. $f(x) = \ln(3 - x)$

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In Exercises 73–78, use a graphing utility to graph the function. Be sure to use an appropriate viewing window.

73. $f(x) = \log(x + 1)$	74. $f(x) = \log(x - 1)$
75. $f(x) = \ln(x - 1)$	76. $f(x) = \ln(x + 2)$
77. $f(x) = \ln x + 2$	78. $f(x) = 3 \ln x - 1$

In Exercises 79–86, use the One-to-One Property to solve the equation for *x*.

79. $\log_2(x+1) = \log_2 4$	80. $\log_2(x-3) = \log_2 9$
81. $\log(2x + 1) = \log 15$	82. $\log(5x + 3) = \log 12$
83. $\ln(x+2) = \ln 6$	84. $\ln(x - 4) = \ln 2$
85. $\ln(x^2 - 2) = \ln 23$	86. $\ln(x^2 - x) = \ln 6$

Model It

87. *Monthly Payment* The model

$$t = 12.542 \ln\left(\frac{x}{x - 1000}\right), \quad x > 1000$$

approximates the length of a home mortgage of \$150,000 at 8% in terms of the monthly payment. In the model, t is the length of the mortgage in years and x is the monthly payment in dollars (see figure).



- (a) Use the model to approximate the lengths of a \$150,000 mortgage at 8% when the monthly payment is \$1100.65 and when the monthly payment is \$1254.68.
- (b) Approximate the total amounts paid over the term of the mortgage with a monthly payment of \$1100.65 and with a monthly payment of \$1254.68.
- (c) Approximate the total interest charges for a monthly payment of \$1100.65 and for a monthly payment of \$1254.68.
- (d) What is the vertical asymptote for the model? Interpret its meaning in the context of the problem.

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- **88.** Compound Interest A principal *P*, invested at $9\frac{1}{2}\%$ and compounded continuously, increases to an amount *K* times the original principal after *t* years, where *t* is given by $t = (\ln K)/0.095$.
 - (a) Complete the table and interpret your results.



- (b) Sketch a graph of the function.
- **89.** *Human Memory Model* Students in a mathematics class were given an exam and then retested monthly with an equivalent exam. The average scores for the class are given by the human memory model $f(t) = 80 17 \log(t + 1)$, $0 \le t \le 12$ where *t* is the time in months.
- (a) Use a graphing utility to graph the model over the specified domain.
 - (b) What was the average score on the original exam (t = 0)?
 - (c) What was the average score after 4 months?
 - (d) What was the average score after 10 months?
- **90.** *Sound Intensity* The relationship between the number of decibels β and the intensity of a sound *I* in watts per square meter is

$$\beta = 10 \log \left(\frac{I}{10^{-12}} \right).$$

- (a) Determine the number of decibels of a sound with an intensity of 1 watt per square meter.
- (b) Determine the number of decibels of a sound with an intensity of 10^{-2} watt per square meter.
- (c) The intensity of the sound in part (a) is 100 times as great as that in part (b). Is the number of decibels 100 times as great? Explain.

Synthesis

True or False? In Exercises 91 and 92, determine whether the statement is true or false. Justify your answer.

- **91.** You can determine the graph of $f(x) = \log_6 x$ by graphing $g(x) = 6^x$ and reflecting it about the *x*-axis.
- **92.** The graph of $f(x) = \log_3 x$ contains the point (27, 3).

In Exercises 93–96, sketch the graph of f and g and describe the relationship between the graphs of f and g. What is the relationship between the functions f and g?

93.
$$f(x) = 3^x$$
, $g(x) = \log_3 x$
94. $f(x) = 5^x$, $g(x) = \log_5 x$
95. $f(x) = e^x$, $g(x) = \ln x$
96. $f(x) = 10^x$, $g(x) = \log x$

97. *Graphical Analysis* Use a graphing utility to graph f and g in the same viewing window and determine which is increasing at the greater rate as x approaches $+\infty$. What can you conclude about the rate of growth of the natural logarithmic function?

(a)
$$f(x) = \ln x$$
, $g(x) = \sqrt{x}$
(b) $f(x) = \ln x$, $g(x) = \frac{4}{\sqrt{x}}$

(b)
$$f(x) = \ln x$$
, $g(x) = \sqrt[3]{x}$

98. (a) Complete the table for the function given by

f(x)	=	$\frac{\ln x}{x}$	

x	1	5	10	10 ²	104	106
f(x)						

- (b) Use the table in part (a) to determine what value f(x) approaches as *x* increases without bound.
- (c) Use a graphing utility to confirm the result of part (b).
- **99.** *Think About It* The table of values was obtained by evaluating a function. Determine which of the statements may be true and which must be false.

x	1	2	8	
y	0	1	3	

(a) y is an exponential function of x.

- (b) *y* is a logarithmic function of *x*.
- (c) x is an exponential function of y.
- (d) *y* is a linear function of *x*.
- **100.** Writing Explain why $\log_a x$ is defined only for 0 < a < 1 and a > 1.

In Exercises 101 and 102, (a) use a graphing utility to graph the function, (b) use the graph to determine the intervals in which the function is increasing and decreasing, and (c) approximate any relative maximum or minimum values of the function.

101. $f(x) = |\ln x|$ **102.** $h(x) = \ln(x^2 + 1)$

Skills Review

In Exercises 103–108, evaluate the function for f(x) = 3x + 2 and $g(x) = x^3 - 1$.

103. (f + g)(2) **104.** (f - g)(-1)

- **105.** (fg)(6) **106.** $\left(\frac{f}{g}\right)(0)$
- **107.** $(f \circ g)(7)$ **108.** $(g \circ f)(-3)$