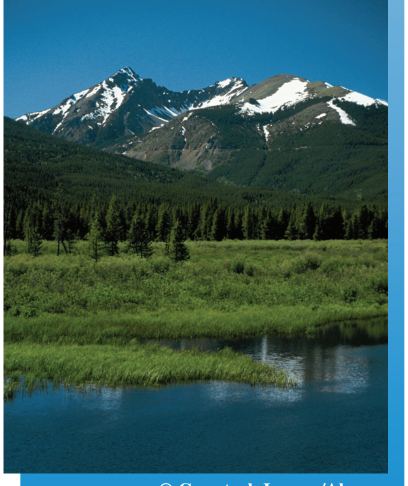
3.1 Exponential Functions and Their Graphs

What you should learn

- Recognize and evaluate exponential functions with base *a*.
- Graph exponential functions and use the One-to-One Property.
- Recognize, evaluate, and graph exponential functions with base *e*.
- Use exponential functions to model and solve real-life problems.

Why you should learn it

Exponential functions can be used to model and solve real-life problems. For instance, in Exercise 70 on page 228, an exponential function is used to model the atmospheric pressure at different altitudes.



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The *HM mathSpace®* CD-ROM and *Eduspace®* for this text contain additional resources related to the concepts discussed in this chapter.

Exponential Functions

So far, this text has dealt mainly with **algebraic functions,** which include polynomial functions and rational functions. In this chapter, you will study two types of nonalgebraic functions—*exponential functions* and *logarithmic functions*. These functions are examples of **transcendental functions.**

Definition of Exponential Function

The **exponential function *f* with base *a*** is denoted by

$$f(x) = a^x$$

where $a > 0$, $a \neq 1$, and x is any real number.

The base $a = 1$ is excluded because it yields $f(x) = 1^x = 1$. This is a constant function, not an exponential function.

You have evaluated a^x for integer and rational values of x. For example, you know that $4^3 = 64$ and $4^{1/2} = 2$. However, to evaluate 4^x for any real number x, you need to interpret forms with *irrational* exponents. For the purposes of this text, it is sufficient to think of

$$a^{\sqrt{2}} \quad (\text{where } \sqrt{2} \approx 1.41421356)$$

as the number that has the successively closer approximations

$$a^{1.4}, a^{1.41}, a^{1.414}, a^{1.4142}, a^{1.41421}, \ldots .$$

Example 1 Evaluating Exponential Functions

Use a calculator to evaluate each function at the indicated value of x.

	Function	*Value*
a.	$f(x) = 2^x$	$x = -3.1$
b.	$f(x) = 2^{-x}$	$x = \pi$
c.	$f(x) = 0.6^x$	$x = \frac{3}{2}$

Solution

	Function Value	*Graphing Calculator Keystrokes*	*Display*
a.	$f(-3.1) = 2^{-3.1}$	2 [^] [(−)] 3.1 [ENTER]	0.1166291
b.	$f(\pi) = 2^{-\pi}$	2 [^] [(−)] π [ENTER]	0.1133147
c.	$f\left(\frac{3}{2}\right) = (0.6)^{3/2}$	.6 [^] [(] 3 [÷] 2 [)] [ENTER]	0.4647580

✓CHECKPOINT Now try Exercise 1.

When evaluating exponential functions with a calculator, remember to enclose fractional exponents in parentheses. Because the calculator follows the order of operations, parentheses are crucial in order to obtain the correct result.

Exploration

Note that an exponential function $f(x) = a^x$ is a constant raised to a variable power, whereas a power function $g(x) = x^n$ is a variable raised to a constant power. Use a graphing utility to graph each pair of functions in the same viewing window. Describe any similarities and differences in the graphs.

a. $y_1 = 2^x, y_2 = x^2$

b. $y_1 = 3^x, y_2 = x^3$

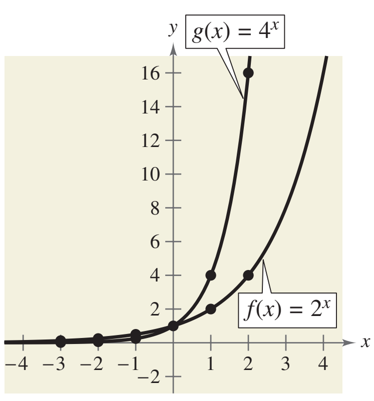

FIGURE 3.1

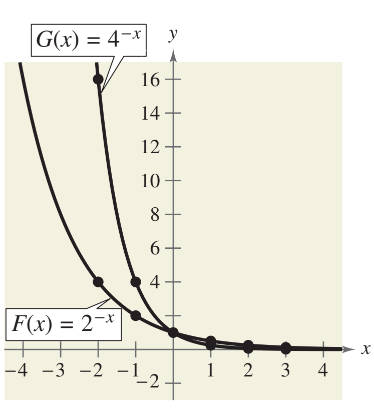

FIGURE 3.2

Graphs of Exponential Functions

The graphs of all exponential functions have similar characteristics, as shown in Examples 2, 3, and 5.

Example 2 Graphs of $y = a^x$

In the same coordinate plane, sketch the graph of each function.

a. $f(x) = 2^x$ **b.** $g(x) = 4^x$

Solution

The table below lists some values for each function, and Figure 3.1 shows the graphs of the two functions. Note that both graphs are increasing. Moreover, the graph of $g(x) = 4^x$ is increasing more rapidly than the graph of $f(x) = 2^x$.

x	-3	-2	-1	0	1	2
2^x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
4^x	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

CHECKPOINT Now try Exercise 11.

The table in Example 2 was evaluated by hand. You could, of course, use a graphing utility to construct tables with even more values.

Example 3 Graphs of $y = a^{-x}$

In the same coordinate plane, sketch the graph of each function.

a. $F(x) = 2^{-x}$ **b.** $G(x) = 4^{-x}$

Solution

The table below lists some values for each function, and Figure 3.2 shows the graphs of the two functions. Note that both graphs are decreasing. Moreover, the graph of $G(x) = 4^{-x}$ is decreasing more rapidly than the graph of $F(x) = 2^{-x}$.

x	-2	-1	0	1	2	3
2^{-x}	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
4^{-x}	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$

CHECKPOINT Now try Exercise 13.

In Example 3, note that by using one of the properties of exponents, the functions $F(x) = 2^{-x}$ and $G(x) = 4^{-x}$ can be rewritten with positive exponents.

$$F(x) = 2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x \quad \text{and} \quad G(x) = 4^{-x} = \frac{1}{4^x} = \left(\frac{1}{4}\right)^x$$

Comparing the functions in Examples 2 and 3, observe that

$$F(x) = 2^{-x} = f(-x) \qquad \text{and} \qquad G(x) = 4^{-x} = g(-x).$$

Consequently, the graph of F is a reflection (in the y-axis) of the graph of f. The graphs of G and g have the same relationship. The graphs in Figures 3.1 and 3.2 are typical of the exponential functions $y = a^x$ and $y = a^{-x}$. They have one y-intercept and one horizontal asymptote (the x-axis), and they are continuous. The basic characteristics of these exponential functions are summarized in Figures 3.3 and 3.4.

STUDY TIP

Notice that the range of an exponential function is $(0, \infty)$, which means that $a^x > 0$ for all values of x.

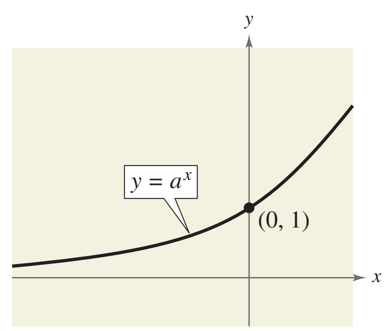

FIGURE 3.3

Graph of $y = a^x$, $a > 1$
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- Intercept: $(0, 1)$
- Increasing
- x-axis is a horizontal asymptote ($a^x \rightarrow 0$ as $x \rightarrow -\infty$)
- Continuous

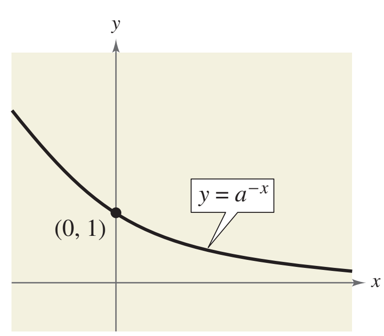

FIGURE 3.4

Graph of $y = a^{-x}$, $a > 1$
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- Intercept: $(0, 1)$
- Decreasing
- x-axis is a horizontal asymptote ($a^{-x} \rightarrow 0$ as $x \rightarrow \infty$)
- Continuous

From Figures 3.3 and 3.4, you can see that the graph of an exponential function is always increasing or always decreasing. As a result, the graphs pass the Horizontal Line Test, and therefore the functions are one-to-one functions. You can use the following **One-to-One Property** to solve simple exponential equations.

For $a > 0$ and $a \neq 1$, $a^x = a^y$ if and only if $x = y$. One-to-One Property

Example 4 Using the One-to-One Property

a. $9 = 3^{x+1}$ Original equation

$3^2 = 3^{x+1}$ $9 = 3^2$

$2 = x + 1$ One-to-One Property

$1 = x$ Solve for x.

b. $\left(\frac{1}{2}\right)^x = 8 \Longrightarrow 2^{-x} = 2^3 \Longrightarrow x = -3$

CHECKPOINT Now try Exercise 45.

In the following example, notice how the graph of $y = a^x$ can be used to sketch the graphs of functions of the form $f(x) = b \pm a^{x+c}$.

Example 5 Transformations of Graphs of Exponential Functions

Each of the following graphs is a transformation of the graph of $f(x) = 3^x$.

a. Because $g(x) = 3^{x+1} = f(x + 1)$, the graph of g can be obtained by shifting the graph of f one unit to the *left*, as shown in Figure 3.5.

b. Because $h(x) = 3^x - 2 = f(x) - 2$, the graph of h can be obtained by shifting the graph of f *downward* two units, as shown in Figure 3.6.

c. Because $k(x) = -3^x = -f(x)$, the graph of k can be obtained by *reflecting* the graph of f in the x-axis, as shown in Figure 3.7.

d. Because $j(x) = 3^{-x} = f(-x)$, the graph of j can be obtained by *reflecting* the graph of f in the y-axis, as shown in Figure 3.8.

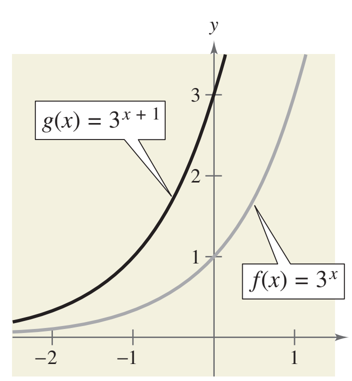

FIGURE 3.5 *Horizontal shift*

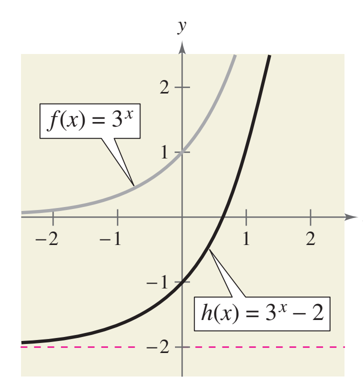

FIGURE 3.6 *Vertical shift*

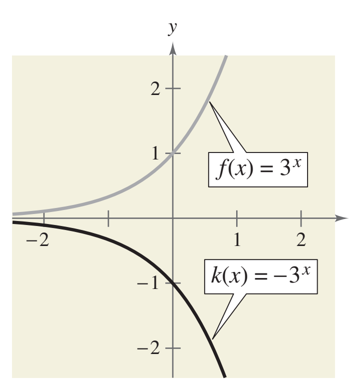

FIGURE 3.7 *Reflection in x-axis*

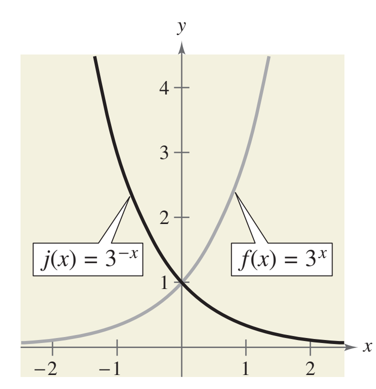

FIGURE 3.8 *Reflection in y-axis*

CHECKPOINT Now try Exercise 17.

Notice that the transformations in Figures 3.5, 3.7, and 3.8 keep the x-axis as a horizontal asymptote, but the transformation in Figure 3.6 yields a new horizontal asymptote of $y = -2$. Also, be sure to note how the y-intercept is affected by each transformation.

The Natural Base e

In many applications, the most convenient choice for a base is the irrational number

$$e \approx 2.718281828 \ldots .$$

This number is called the **natural base.** The function given by $f(x) = e^x$ is called the **natural exponential function.** Its graph is shown in Figure 3.9. Be sure you see that for the exponential function $f(x) = e^x$, e is the constant $2.718281828 \ldots$, whereas x is the variable.

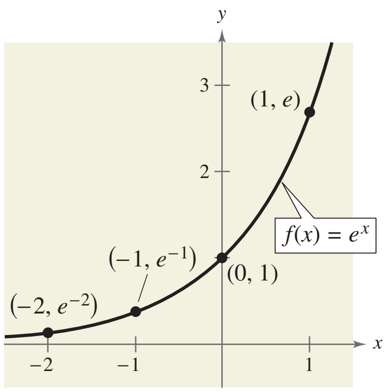

FIGURE 3.9

> **Exploration**
>
> Use a graphing utility to graph $y_1 = (1 + 1/x)^x$ and $y_2 = e$ in the same viewing window. Using the *trace* feature, explain what happens to the graph of y_1 as x increases.

Example 6 Evaluating the Natural Exponential Function

Use a calculator to evaluate the function given by $f(x) = e^x$ at each indicated value of x.

a. $x = -2$ **b.** $x = -1$ **c.** $x = 0.25$ **d.** $x = -0.3$

Solution

Function Value	*Graphing Calculator Keystrokes*	*Display*
a. $f(-2) = e^{-2}$	e^x (−) 2 ENTER	0.1353353
b. $f(-1) = e^{-1}$	e^x (−) 1 ENTER	0.3678794
c. $f(0.25) = e^{0.25}$	e^x 0.25 ENTER	1.2840254
d. $f(-0.3) = e^{-0.3}$	e^x (−) 0.3 ENTER	0.7408182

CHECKPOINT Now try Exercise 27.

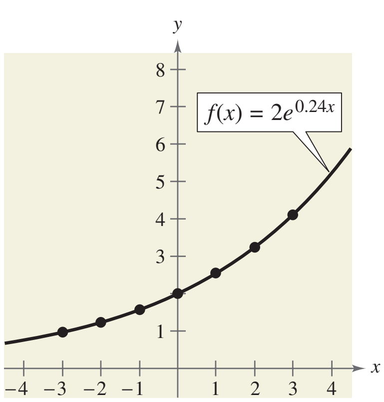

FIGURE 3.10

Example 7 Graphing Natural Exponential Functions

Sketch the graph of each natural exponential function.

a. $f(x) = 2e^{0.24x}$ **b.** $g(x) = \frac{1}{2}e^{-0.58x}$

Solution

To sketch these two graphs, you can use a graphing utility to construct a table of values, as shown below. After constructing the table, plot the points and connect them with smooth curves, as shown in Figures 3.10 and 3.11. Note that the graph in Figure 3.10 is increasing, whereas the graph in Figure 3.11 is decreasing.

x	-3	-2	-1	0	1	2	3
$f(x)$	0.974	1.238	1.573	2.000	2.542	3.232	4.109
$g(x)$	2.849	1.595	0.893	0.500	0.280	0.157	0.088

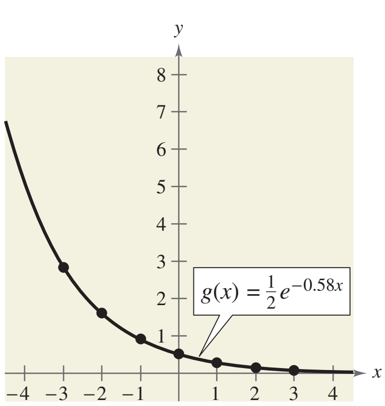

FIGURE 3.11

CHECKPOINT Now try Exercise 35.

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Applications

One of the most familiar examples of exponential growth is that of an investment earning *continuously compounded interest*. Using exponential functions, you can *develop* a formula for interest compounded n times per year and show how it leads to continuous compounding.

Suppose a principal P is invested at an annual interest rate r, compounded once a year. If the interest is added to the principal at the end of the year, the new balance P_1 is

$$\begin{aligned} P_1 &= P + Pr \\ &= P(1 + r). \end{aligned}$$

This pattern of multiplying the previous principal by $1 + r$ is then repeated each successive year, as shown below.

Year	*Balance After Each Compounding*
0	$P = P$
1	$P_1 = P(1 + r)$
2	$P_2 = P_1(1 + r) = P(1 + r)(1 + r) = P(1 + r)^2$
3	$P_3 = P_2(1 + r) = P(1 + r)^2(1 + r) = P(1 + r)^3$
$\vdots$	$\vdots$
t	$P_t = P(1 + r)^t$

To accommodate more frequent (quarterly, monthly, or daily) compounding of interest, let n be the number of compoundings per year and let t be the number of years. Then the rate per compounding is r/n and the account balance after t years is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}. \qquad \text{Amount (balance) with } n \text{ compoundings per year}$$

If you let the number of compoundings n increase without bound, the process approaches what is called **continuous compounding.** In the formula for n compoundings per year, let $m = n/r$. This produces

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{nt} && \text{Amount with } n \text{ compoundings per year} \\ &= P\left(1 + \frac{r}{mr}\right)^{mrt} && \text{Substitute } mr \text{ for } n. \\ &= P\left(1 + \frac{1}{m}\right)^{mrt} && \text{Simplify.} \\ &= P\left[\left(1 + \frac{1}{m}\right)^{m}\right]^{rt}. && \text{Property of exponents} \end{aligned}$$

As m increases without bound, the table at the left shows that $[1 + (1/m)]^m \to e$ as $m \to \infty$. From this, you can conclude that the formula for continuous compounding is

$$A = Pe^{rt}. \qquad \text{Substitute } e \text{ for } (1 + 1/m)^m.$$

Exploration

Use the formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

to calculate the amount in an account when $P = \$3000$, $r = 6\%$, $t = 10$ years, and compounding is done (a) by the day, (b) by the hour, (c) by the minute, and (d) by the second. Does increasing the number of compoundings per year result in unlimited growth of the amount in the account? Explain.

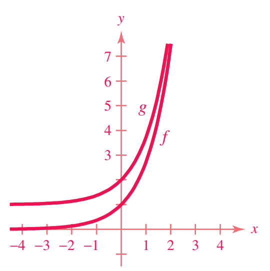
m	$\left(1 + \frac{1}{m}\right)^m$
1	2
10	2.59374246
100	2.704813829
1,000	2.716923932
10,000	2.718145927
100,000	2.718268237
1,000,000	2.718280469
10,000,000	2.718281693
$\downarrow$	$\downarrow$
∞	e

STUDY TIP

Be sure you see that the annual interest rate must be written in decimal form. For instance, 6% should be written as 0.06.

Activities

1. Sketch the graphs of the functions $f(x) = e^x$ and $g(x) = 1 + e^x$ on the same coordinate system.



2. Determine the balance A at the end of 20 years if \$1500 is invested at 6.5% interest and the interest is compounded (a) quarterly and (b) continuously.
   Answer: (a) \$5446.73 (b) \$5503.95
3. The number of fruit flies in an experimental population after t hours is given by $Q(t) = 20e^{0.03t}$, $t \geq 0$.
   a. Find the initial number of fruit flies in the population.
   b. How large is the population of fruit flies after 72 hours?
   Answer: (a) 20 flies (b) 173 flies

Group Activity

The sequence 3, 6, 9, 12, 15, . . . is given by $f(n) = 3n$ and is an example of linear growth. The sequence 3, 9, 27, 81, 243, . . . is given by $f(n) = 3^n$ and is an example of exponential growth. Explain the difference between these two types of growth. For each of the following sequences, indicate whether the sequence represents linear growth or exponential growth, and find a linear or exponential function that represents the sequence. Give several other examples of linear and exponential growth.

a. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$
b. 4, 8, 12, 16, 20, . . .
c. $\frac{2}{3}, \frac{4}{3}, 2, \frac{8}{3}, \frac{10}{3}, 4, \ldots$
d. 5, 25, 125, 625, . . .

Formulas for Compound Interest

After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas.

1. For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

2. For continuous compounding: $A = Pe^{rt}$

Example 8 Compound Interest

A total of \$12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years if it is compounded

a. quarterly.

b. monthly.

c. continuously.

Solution

a. For quarterly compounding, you have $n = 4$. So, in 5 years at 9%, the balance is

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad \text{Formula for compound interest}$$

$$= 12{,}000\left(1 + \frac{0.09}{4}\right)^{4(5)} \qquad \text{Substitute for } P, r, n, \text{ and } t.$$

$$\approx \$18{,}726.11. \qquad \text{Use a calculator.}$$

b. For monthly compounding, you have $n = 12$. So, in 5 years at 9%, the balance is

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad \text{Formula for compound interest}$$

$$= 12{,}000\left(1 + \frac{0.09}{12}\right)^{12(5)} \qquad \text{Substitute for } P, r, n, \text{ and } t.$$

$$\approx \$18{,}788.17. \qquad \text{Use a calculator.}$$

c. For continuous compounding, the balance is

$$A = Pe^{rt} \qquad \text{Formula for continuous compounding}$$

$$= 12{,}000e^{0.09(5)} \qquad \text{Substitute for } P, r, \text{ and } t.$$

$$\approx \$18{,}819.75. \qquad \text{Use a calculator.}$$

CHECKPOINT Now try Exercise 53.

In Example 8, note that continuous compounding yields more than quarterly or monthly compounding. This is typical of the two types of compounding. That is, for a given principal, interest rate, and time, continuous compounding will always yield a larger balance than compounding n times a year.

Example 9 Radioactive Decay

In 1986, a nuclear reactor accident occurred in Chernobyl in what was then the Soviet Union. The explosion spread highly toxic radioactive chemicals, such as plutonium, over hundreds of square miles, and the government evacuated the city and the surrounding area. To see why the city is now uninhabited, consider the model

$$P = 10\left(\frac{1}{2}\right)^{t/24,100}$$

which represents the amount of plutonium P that remains (from an initial amount of 10 pounds) after t years. Sketch the graph of this function over the interval from $t = 0$ to $t = 100,000$, where $t = 0$ represents 1986. How much of the 10 pounds will remain in the year 2010? How much of the 10 pounds will remain after 100,000 years?

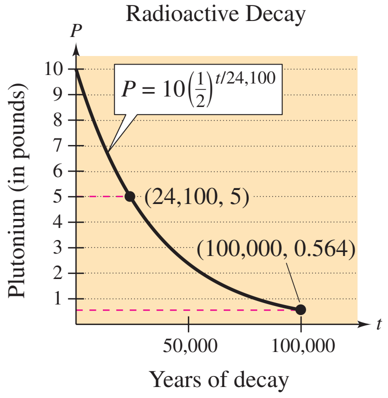


FIGURE 3.12

Solution

The graph of this function is shown in Figure 3.12. Note from this graph that plutonium has a *half-life* of about 24,100 years. That is, after 24,100 years, *half* of the original amount will remain. After another 24,100 years, one-quarter of the original amount will remain, and so on. In the year 2010 ($t = 24$), there will still be

$$P = 10\left(\frac{1}{2}\right)^{24/24,100} \approx 10\left(\frac{1}{2}\right)^{0.0009959} \approx 9.993 \text{ pounds}$$

of plutonium remaining. After 100,000 years, there will still be

$$P = 10\left(\frac{1}{2}\right)^{100,000/24,100} \approx 10\left(\frac{1}{2}\right)^{4.1494} \approx 0.564 \text{ pound}$$

of plutonium remaining.

CHECKPOINT Now try Exercise 67.

Writing About Mathematics Suggestion:

One way your students might approach this problem is to create a table, covering x-values from -2 through 3, for each of the functions and compare this table with the given tables. If this method is used, you might consider dividing your class into groups of three or six and having the groups assign one or two functions to each member. They should then pool their results and work cooperatively to determine that each function has a y-intercept of $(0, 8)$.

Another approach is a graphical one: the groups can create scatter plots of the data shown in the table and compare them with sketches of the graphs of the given functions. Consider assigning students to groups of four and giving the responsibility for sketching three graphs to each group member.

WRITING ABOUT MATHEMATICS

Identifying Exponential Functions Which of the following functions generated the two tables below? Discuss how you were able to decide. What do these functions have in common? Are any of them the same? If so, explain why.

a. $f_1(x) = 2^{(x+3)}$ **b.** $f_2(x) = 8\left(\frac{1}{2}\right)^x$ **c.** $f_3(x) = \left(\frac{1}{2}\right)^{(x-3)}$

d. $f_4(x) = \left(\frac{1}{2}\right)^x + 7$ **e.** $f_5(x) = 7 + 2^x$ **f.** $f_6(x) = (8)2^x$

x	-1	0	1	2	3
$g(x)$	7.5	8	9	11	15

x	-2	-1	0	1	2
$h(x)$	32	16	8	4	2

Create two different exponential functions of the forms $y = a(b)^x$ and $y = c^x + d$ with y-intercepts of $(0, -3)$.

3.1 Exercises

The *HM mathSpace®* CD-ROM and *Eduspace®* for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

VOCABULARY CHECK: Fill in the blanks.

1. Polynomials and rational functions are examples of ________ functions.
2. Exponential and logarithmic functions are examples of nonalgebraic functions, also called ________ functions.
3. The exponential function given by $f(x) = e^x$ is called the ________ ________ function, and the base e is called the ________ base.
4. To find the amount A in an account after t years with principal P and an annual interest rate r compounded n times per year, you can use the formula ________.
5. To find the amount A in an account after t years with principal P and an annual interest rate r compounded continuously, you can use the formula ________.

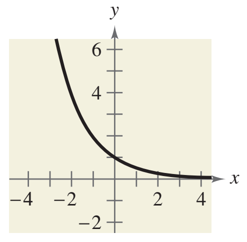
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–6, evaluate the function at the indicated value of x. Round your result to three decimal places.

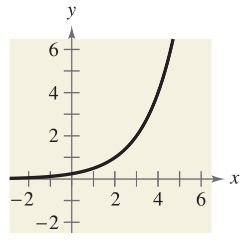
	Function	*Value*
1.	$f(x) = 3.4^x$	$x = 5.6$
2.	$f(x) = 2.3^x$	$x = \frac{3}{2}$
3.	$f(x) = 5^x$	$x = -\pi$
4.	$f(x) = \left(\frac{2}{3}\right)^{5x}$	$x = \frac{3}{10}$
5.	$g(x) = 5000(2^x)$	$x = -1.5$
6.	$f(x) = 200(1.2)^{12x}$	$x = 24$

In Exercises 7–10, match the exponential function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

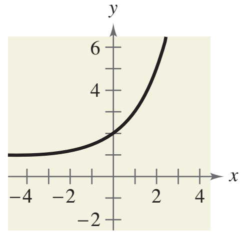
(a)



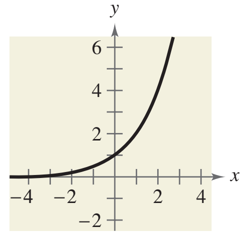
(b)



(c)



(d)



7. $f(x) = 2^x$ **8.** $f(x) = 2^x + 1$

9. $f(x) = 2^{-x}$ **10.** $f(x) = 2^{x-2}$

In Exercises 11–16, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

11. $f(x) = \left(\frac{1}{2}\right)^x$ **12.** $f(x) = \left(\frac{1}{2}\right)^{-x}$

13. $f(x) = 6^{-x}$ **14.** $f(x) = 6^x$

15. $f(x) = 2^{x-1}$ **16.** $f(x) = 4^{x-3} + 3$

In Exercises 17–22, use the graph of f to describe the transformation that yields the graph of g.

17. $f(x) = 3^x$, $g(x) = 3^{x-4}$

18. $f(x) = 4^x$, $g(x) = 4^x + 1$

19. $f(x) = -2^x$, $g(x) = 5 - 2^x$

20. $f(x) = 10^x$, $g(x) = 10^{-x+3}$

21. $f(x) = \left(\frac{7}{2}\right)^x$, $g(x) = -\left(\frac{7}{2}\right)^{-x+6}$

22. $f(x) = 0.3^x$, $g(x) = -0.3^x + 5$

In Exercises 23–26, use a graphing utility to graph the exponential function.

23. $y = 2^{-x^2}$ **24.** $y = 3^{-|x|}$

25. $y = 3^{x-2} + 1$ **26.** $y = 4^{x+1} - 2$

In Exercises 27–32, evaluate the function at the indicated value of x. Round your result to three decimal places.

	Function	*Value*
27.	$h(x) = e^{-x}$	$x = \frac{3}{4}$
28.	$f(x) = e^x$	$x = 3.2$
29.	$f(x) = 2e^{-5x}$	$x = 10$
30.	$f(x) = 1.5e^{x/2}$	$x = 240$
31.	$f(x) = 5000e^{0.06x}$	$x = 6$
32.	$f(x) = 250e^{0.05x}$	$x = 20$

In Exercises 33–38, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

33. $f(x) = e^x$

34. $f(x) = e^{-x}$

35. $f(x) = 3e^{x+4}$

36. $f(x) = 2e^{-0.5x}$

37. $f(x) = 2e^{x-2} + 4$

38. $f(x) = 2 + e^{x-5}$

In Exercises 39–44, use a graphing utility to graph the exponential function.

39. $y = 1.08^{-5x}$

40. $y = 1.08^{5x}$

41. $s(t) = 2e^{0.12t}$

42. $s(t) = 3e^{-0.2t}$

43. $g(x) = 1 + e^{-x}$

44. $h(x) = e^{x-2}$

In Exercise 45–52, use the One-to-One Property to solve the equation for *x*.

45. $3^{x+1} = 27$

46. $2^{x-3} = 16$

47. $2^{x-2} = \dfrac{1}{32}$

48. $\left(\dfrac{1}{5}\right)^{x+1} = 125$

49. $e^{3x+2} = e^3$

50. $e^{2x-1} = e^4$

51. $e^{x^2-3} = e^{2x}$

52. $e^{x^2+6} = e^{5x}$

Compound Interest **In Exercises 53–56, complete the table to determine the balance *A* for *P* dollars invested at rate *r* for *t* years and compounded *n* times per year.**

n	1	2	4	12	365	Continuous
A						

53. $P = \$2500, r = 2.5\%, t = 10$ years

54. $P = \$1000, r = 4\%, t = 10$ years

55. $P = \$2500, r = 3\%, t = 20$ years

56. $P = \$1000, r = 6\%, t = 40$ years

Compound Interest **In Exercises 57–60, complete the table to determine the balance *A* for $12,000 invested at rate *r* for *t* years, compounded continuously.**

t	10	20	30	40	50
A					

57. $r = 4\%$

58. $r = 6\%$

59. $r = 6.5\%$

60. $r = 3.5\%$

61. ***Trust Fund*** On the day of a child's birth, a deposit of $25,000 is made in a trust fund that pays 8.75% interest, compounded continuously. Determine the balance in this account on the child's 25th birthday.

62. ***Trust Fund*** A deposit of $5000 is made in a trust fund that pays 7.5% interest, compounded continuously. It is specified that the balance will be given to the college from which the donor graduated after the money has earned interest for 50 years. How much will the college receive?

63. ***Inflation*** If the annual rate of inflation averages 4% over the next 10 years, the approximate costs C of goods or services during any year in that decade will be modeled by $C(t) = P(1.04)^t$, where t is the time in years and P is the present cost. The price of an oil change for your car is presently $23.95. Estimate the price 10 years from now.

64. ***Demand*** The demand equation for a product is given by

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

where p is the price and x is the number of units.

(a) Use a graphing utility to graph the demand function for $x > 0$ and $p > 0$.

(b) Find the price p for a demand of $x = 500$ units.

(c) Use the graph in part (a) to approximate the greatest price that will still yield a demand of at least 600 units.

65. ***Computer Virus*** The number V of computers infected by a computer virus increases according to the model $V(t) = 100e^{4.6052t}$, where t is the time in hours. Find (a) $V(1)$, (b) $V(1.5)$, and (c) $V(2)$.

66. ***Population*** The population P (in millions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-0.0039t}$, where t represents the year, with $t = 6$ corresponding to 1996. (Source: Census Bureau, International Data Base)

(a) According to the model, is the population of Russia increasing or decreasing? Explain.

(b) Find the population of Russia in 1998 and 2000.

(c) Use the model to predict the population of Russia in 2010.

67. ***Radioactive Decay*** Let Q represent a mass of radioactive radium (^{226}Ra) (in grams), whose half-life is 1599 years. The quantity of radium present after t years is $Q = 25\left(\frac{1}{2}\right)^{t/1599}$.

(a) Determine the initial quantity (when $t = 0$).

(b) Determine the quantity present after 1000 years.

(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 5000$.

68. ***Radioactive Decay*** Let Q represent a mass of carbon 14 (^{14}C) (in grams), whose half-life is 5715 years. The quantity of carbon 14 present after t years is $Q = 10\left(\frac{1}{2}\right)^{t/5715}$.

(a) Determine the initial quantity (when $t = 0$).

(b) Determine the quantity present after 2000 years.

(c) Sketch the graph of this function over the interval $t = 0$ to $t = 10{,}000$.

Model It

69. ***Data Analysis: Biology*** To estimate the amount of defoliation caused by the gypsy moth during a given year, a forester counts the number x of egg masses on $\frac{1}{40}$ of an acre (circle of radius 18.6 feet) in the fall. The percent of defoliation y the next spring is shown in the table. (Source: USDA, Forest Service)

Egg masses, x	Percent of defoliation, y
0	12
25	44
50	81
75	96
100	99

A model for the data is given by

$$y = \frac{100}{1 + 7e^{-0.069x}}.$$

(a) Use a graphing utility to create a scatter plot of the data and graph the model in the same viewing window.

(b) Create a table that compares the model with the sample data.

(c) Estimate the percent of defoliation if 36 egg masses are counted on $\frac{1}{40}$ acre.

(d) You observe that $\frac{2}{3}$ of a forest is defoliated the following spring. Use the graph in part (a) to estimate the number of egg masses per $\frac{1}{40}$ acre.

70. ***Data Analysis: Meteorology*** A meteorologist measures the atmospheric pressure P (in pascals) at altitude h (in kilometers). The data are shown in the table.

Altitude, h	Pressure, P
0	101,293
5	54,735
10	23,294
15	12,157
20	5,069

A model for the data is given by $P = 107{,}428e^{-0.150h}$.

(a) Sketch a scatter plot of the data and graph the model on the same set of axes.

(b) Estimate the atmospheric pressure at a height of 8 kilometers.

Synthesis

True or False? **In Exercises 71 and 72, determine whether the statement is true or false. Justify your answer.**

71. The line $y = -2$ is an asymptote for the graph of $f(x) = 10^x - 2$.

72. $e = \dfrac{271{,}801}{99{,}990}$.

Think About It **In Exercises 73–76, use properties of exponents to determine which functions (if any) are the same.**

73. $f(x) = 3^{x-2}$
$g(x) = 3^x - 9$
$h(x) = \frac{1}{9}(3^x)$

74. $f(x) = 4^x + 12$
$g(x) = 2^{2x+6}$
$h(x) = 64(4^x)$

75. $f(x) = 16(4^{-x})$
$g(x) = \left(\frac{1}{4}\right)^{x-2}$
$h(x) = 16(2^{-2x})$

76. $f(x) = e^{-x} + 3$
$g(x) = e^{3-x}$
$h(x) = -e^{x-3}$

77. Graph the functions given by $y = 3^x$ and $y = 4^x$ and use the graphs to solve each inequality.

(a) $4^x < 3^x$ (b) $4^x > 3^x$

78. Use a graphing utility to graph each function. Use the graph to find where the function is increasing and decreasing, and approximate any relative maximum or minimum values.

(a) $f(x) = x^2e^{-x}$ (b) $g(x) = x2^{3-x}$

79. ***Graphical Analysis*** Use a graphing utility to graph

$$f(x) = \left(1 + \frac{0.5}{x}\right)^x \quad \text{and} \quad g(x) = e^{0.5}$$

in the same viewing window. What is the relationship between f and g as x increases and decreases without bound?

80. ***Think About It*** Which functions are exponential?

(a) $3x$ (b) $3x^2$ (c) 3^x (d) 2^{-x}

Skills Review

In Exercises 81 and 82, solve for y.

81. $x^2 + y^2 = 25$

82. $x - |y| = 2$

In Exercises 83 and 84, sketch the graph of the function.

83. $f(x) = \dfrac{2}{9 + x}$

84. $f(x) = \sqrt{7 - x}$

85. **Make a Decision** To work an extended application analyzing the population per square mile of the United States, visit this text's website at *college.hmco.com*. *(Data Source: U.S. Census Bureau)*