[redacted]

[redacted]

| $t$ (minutes) | 0 | 12 | 20 | 24 | 40 |
|---|---|---|---|---|---|
| $v(t)$ (meters per minute) | 0 | 200 | 240 | –220 | 150 |

[redacted] Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function $v$. Selected values of $v(t)$, where $t$ is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

(c) Bob is riding his bicycle along the same path. For $0 \le t \le 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where $t$ is measured in minutes and $B(t)$ is measured in meters per minute. Find Bob's acceleration at time $t = 5$.

(d) Based on the model $B$ from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

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[redacted]

[redacted]

[redacted] The rate at which rainwater flows into a drainpipe is modeled by the function $R$, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, $t$ is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time $t$, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time $w$ when the pipe will begin to overflow.

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[redacted] Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of $f$ at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

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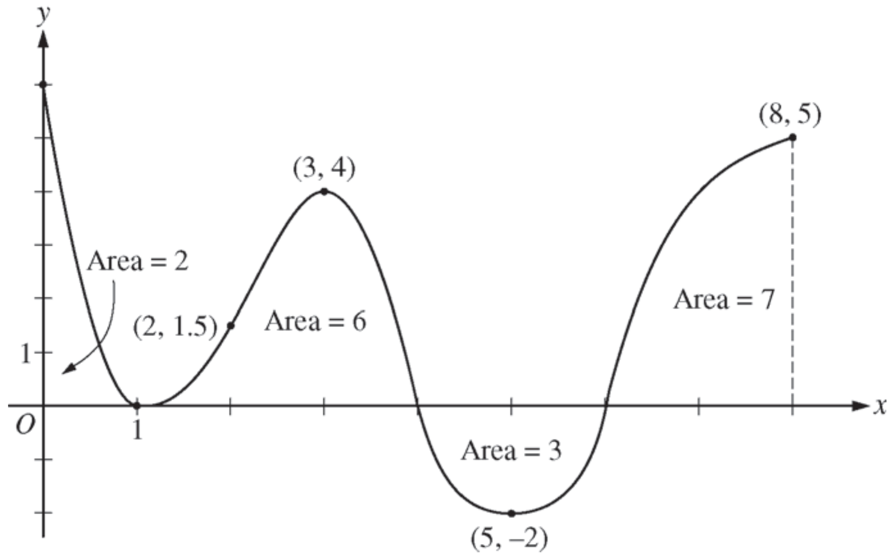
[redacted]

[redacted]




Graph of $f'$

[redacted] The figure above shows the graph of $f'$, the derivative of a twice-differentiable function $f$, on the closed interval $0 \le x \le 8$. The graph of $f'$ has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of $f'$ and the $x$-axis are labeled in the figure. The function $f$ is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of $x$ on the open interval $0 < x < 8$ for which the function $f$ has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of $f$ on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of $f$ both concave down and increasing? Explain your reasoning.

(d) The function $g$ is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of $g$ at $x = 3$.

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