#

Algebra 2/Trig
Unit 6 Notes Packet

Name:
Date:
Period:

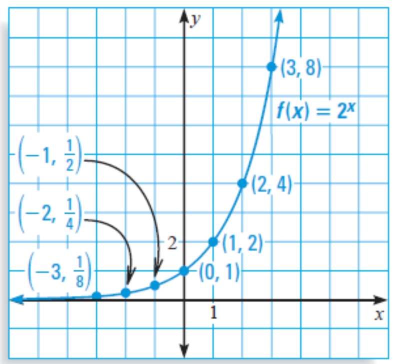
Exponential and Logarithmic Functions

1. Worksheet
2. Worksheet
3. Pg 483-484 #17-57 column; 61-73 column and 76-77 (need graph paper)
4. Pg 483-484 #20-60 column; 63-75 column and 78-80 (need graph paper)
5. Pg 490-491 #16-52 column and 56-74 column (need graph paper)
6. Pg 490-491 #19-55 column and 58-76 column (need graph paper)
7. Pg 496-497 #14-42 column; 46-56 column; 58-70 column
8. Pg 496-497 #17-45 column; 47-57 column; 61-73 column
9. Pg 505-506 #19-23 odd; 25-59 odd
10. Pg 505-506 #20-24 even; 26-60 even
11. Worksheet
12. Worksheet
13. Chapter Review

Exponential Function

An **exponential function** involves the expression b^x where the **base** *b* is a positive number other than 1. To see the basic shape of the graph of an exponential function such as $f(x) = 2^x$, you can make a table of values and plot points, as shown below.

x	$f(x) = 2^x$
-3	$2^{-3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



Notice the end behavior of the graph. As $x \to +\infty$, $f(x) \to +\infty$, which means that the graph moves up to the right. As $x \to -\infty$, $f(x) \to 0$, which means that the graph has the line $y = 0$ as an *asymptote*. An **asymptote** is a line that a graph approaches as you move away from the origin.

Exponential Growth Function

$y = ab^x$, where a > 0 and b > 1

- The graph passes through the point (0, a). That is, the y-intercept is a.
- The x-axis is an asymptote of the graph
- The domain is all real numbers
- The range is y > 0 if a > 0 and y < 0 if a < 0

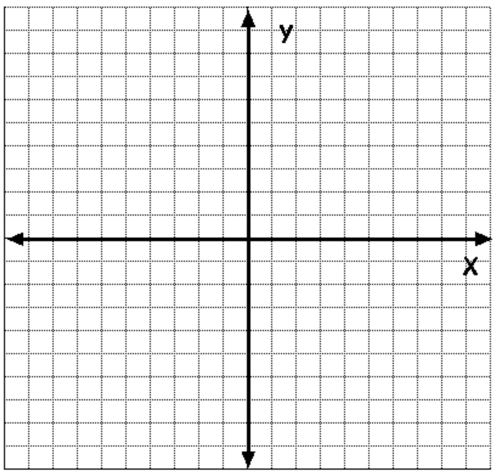
To graph a general exponential function

$$y = ab^{x-h} + k,$$

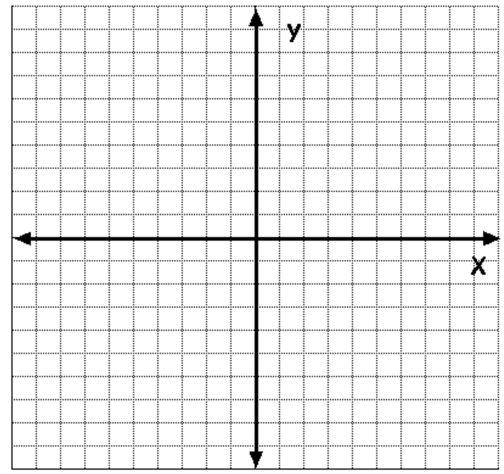
begin by sketching the graph of $y = ab^x$. Then translate the graph horizontally by *h* units and vertically by *k* units.

E1. Graph the function

a. $y = \frac{1}{2} * 3^x$

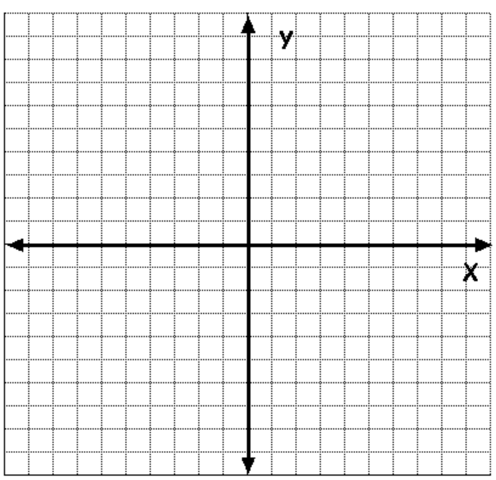


b. $y = -\left(\frac{3}{2}\right)^x$

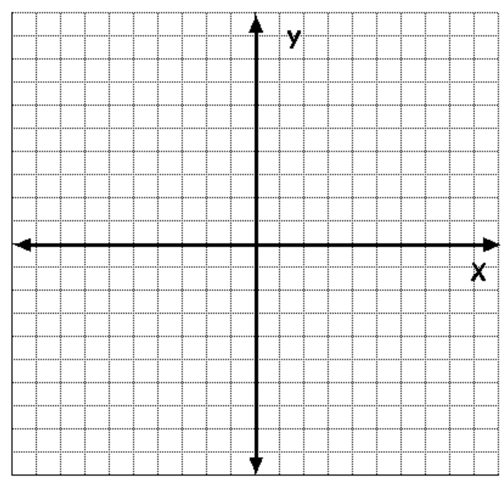


P1. Graph the function

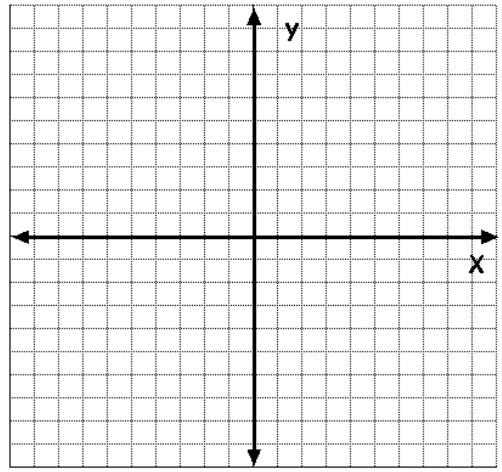
a. $y = \frac{2}{3} * 2^x$



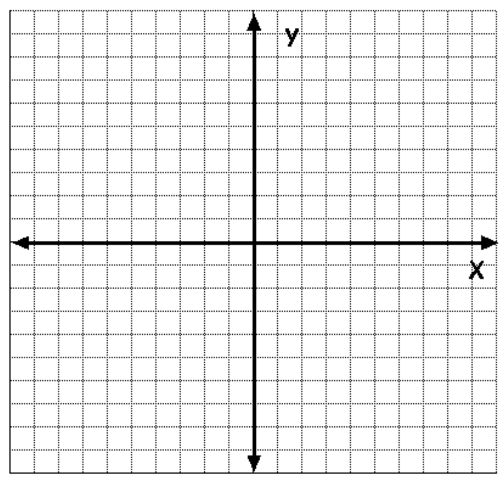
b. $y = -2(2)^x$



E2. Graph $y = 3 * 2^{x-1} - 4$



P2. Graph $y = 2 * 3^{x-2} + 1$



When a real-life quantity increases by a fixed percent each year (or other time period), the amount *y* of the quantity after *t* years can be modeled by this equation:

$$y = a(1 + r)^t$$

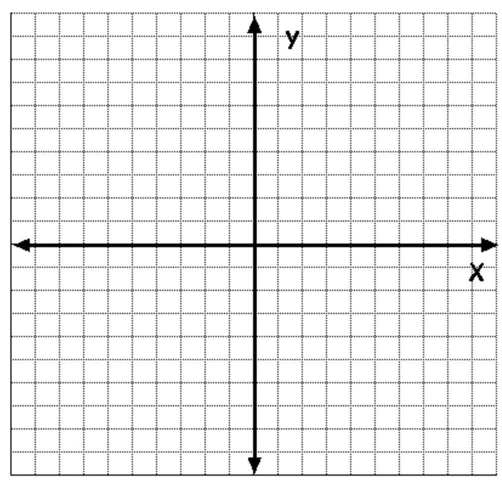
In this model, *a* is the initial amount and *r* is the percent increase expressed as a decimal. The quantity 1 + *r* is called the **growth factor.**

Error Alert: Do not confuse percent increase with growth factor!

E3. In January, 1993, there were about 1,313,000 Internet hosts. During the next five years, the number of hosts increased by about 100% per year.

a. Write a model giving the number *h* (in millions) of hosts *t* years after 1993. About how many hosts were there in 1996?

b. Graph the model.

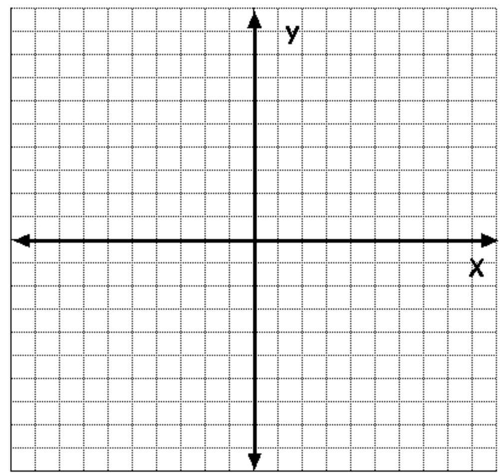


c. Use the graph to estimate the year when there were 30 million hosts.

P3. In 1980 about 2,180,000 US workers worked at home. During the next ten years, the number of workers working at home increased 5% per year.

a. Write a model giving the number w (in millions) of workers working at home *t* years after 1980.

b. Graph the model.



c**.** Use the graph to estimate the year when there were about 3.22 million workers who worked at home.

COMPOUND INTEREST

Exponential growth functions are used in real-life situations involving *compound interest.* Compound interest is interest paid on the initial investment, called the *principal,* and on previously earned interest. (Interest paid only on the principal is called *simple interest*.) Although interest earned is expressed as an *annual* percent, the interest is usually compounded more frequently than once per year. Therefore, the formula $y = a(1 + r)t$ must be modified for compound interest problems.

$$A = P(1 + \frac{r}{n})^{nt}$$

P: initial principal deposited
r: annual interest rate (expressed as a decimal)
n: number of times that it is compounded a year
t: years
A: amount of money

$$A = P(1 + \frac{r}{n})^{nt}$$

P: initial principal deposited
r: annual interest rate (expressed as a decimal)
n: number of times that it is compounded a year
t: years
A: amount of money

E4. You deposit $1000 in an account that pays 8% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a. annually

b. quarterly

c. daily

P4. You deposit $1500 in an account that pays 6% annual interest. Find the balance after 1 year if the interest is compounded with the given frequency.

a. annually

b. semiannually

c. quarterly

Exponential Decay Function

$y = ab^x$, where a > 0 and 0 < b < 1

- The graph passes through the point (0, a). That is, the y-intercept is a.
- The x-axis is an asymptote of the graph
- The domain is all real numbers
- The range is y > 0 if a > 0 and y < 0 if a < 0

To graph a general exponential function

$$y = ab^{x-h} + k,$$

begin by sketching the graph of $y = ab^x$. Then translate the graph horizontally by *h* units and vertically by *k* units.

E1. State whether f(x) is an exponential growth or exponential decay function.

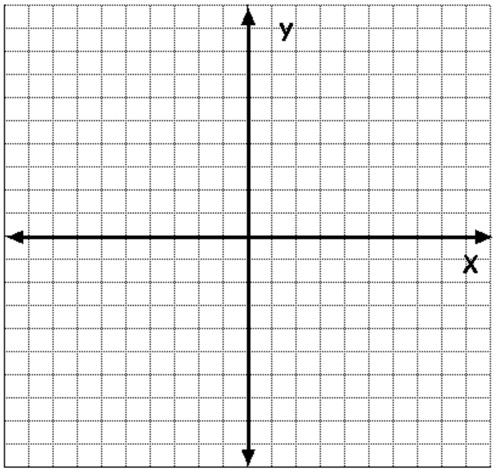
a. $f(x) = 5\left(\frac{2}{3}\right)^x$ b. $f(x) = 8\left(\frac{3}{2}\right)^x$ c. $f(x) = 10(3)^{-x}$

P1. State whether f(x) is an exponential growth or exponential decay function.

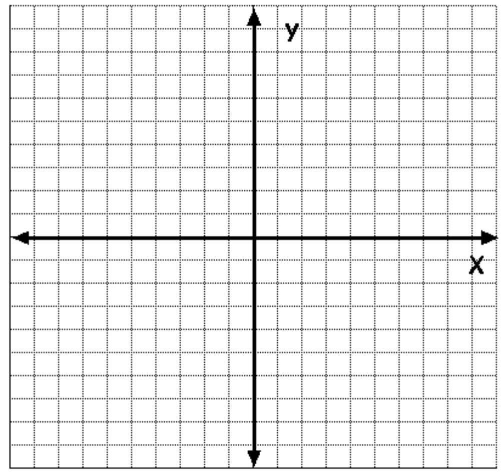
a. $f(x) = \frac{1}{3}(2)^{-x}$ b. $f(x) = 8\left(\frac{3}{2}\right)^x$ c. $f(x) = 4\left(\frac{5}{8}\right)^x$

E2. Graph the function

a. $y = 3 * (\frac{1}{4})^x$

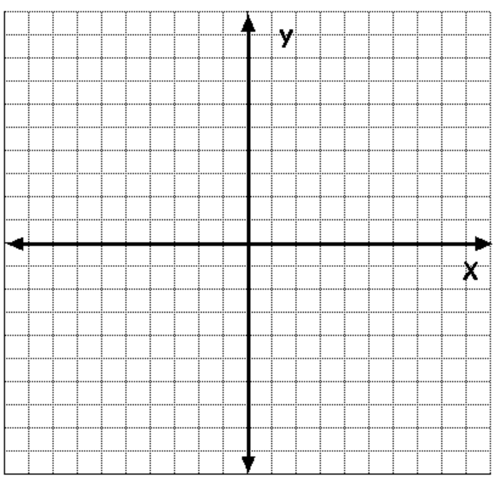


b. $y = -5\left(\frac{2}{3}\right)^x$

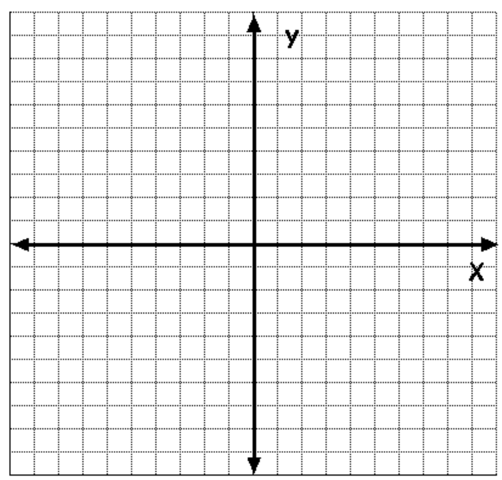


P2. Graph the function

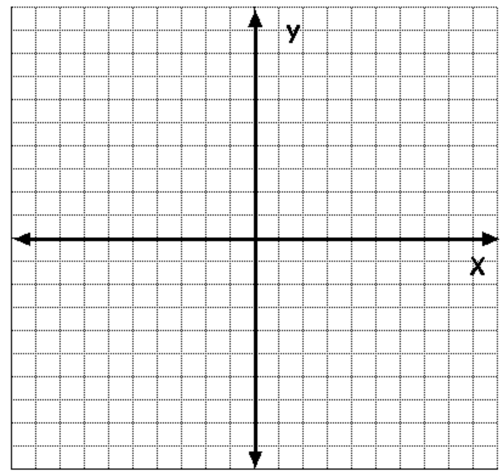
a. $y = -2\left(\frac{1}{3}\right)^x$



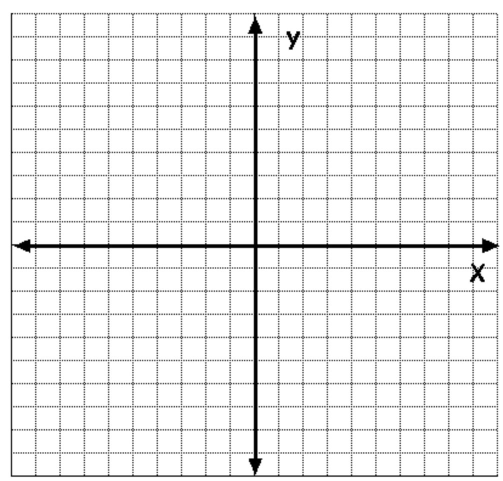
b. $y = 4\left(\frac{2}{5}\right)^x$



E3. Graph $y = -3\left(\frac{1}{2}\right)^{x+2} + 1$



P2. Graph $y = 5\left(\frac{1}{8}\right)^{x+1} - 2$



When a real-life quantity decreases by a fixed percent each year (or other time period), the amount *y* of the quantity after *t* years can be modeled by the equation

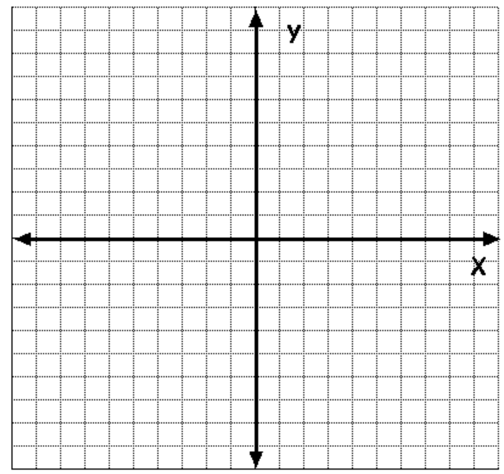
$$y = a(1 - r)^t$$

where *a* is the initial amount and *r* is the percent decrease expressed as a decimal. The quantity 1 – r is called the **decay factor.**

E4. You buy a new car for \$24,000. The value y of the car decreases by 16% each year.

a. Write an exponential decay model for the value of the car. Use the model to estimate the value after 2 years.

b. Graph the model.

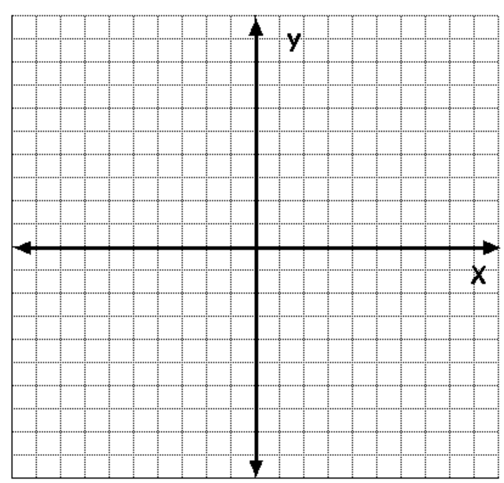


c. Use the graph to estimate when the car will have a value of \$12,000.

P4. There are 40,000 homes in your city. Each year 10% of the homes are expected to disconnect from septic systems and connect to the sewer system

a. Write an exponential decay model for the number of homes that still use septic systems. Use the model to estimate the number of homes using septic systems after 5 years.

b. Graph the model.



c. Use the graph to estimate when about 17,200 homes will still not be connected to the sewer system.

The Natural Base *e*

The natural base *e* is irrational. It is defined as follows:

$$\text{As n approaches } +\infty, \left(1+\frac{1}{n}\right)^n \text{ approaches } e \approx 2.718281828459$$

E1. Simplify the expression

a. $e^3 * e^4$ b. $\frac{10e^3}{5e^2}$ c. $(3e^{-4x})^2$

P1. Simplify the expression

a. $e^5 * e^6$ b. $\frac{24e^8}{8e^5}$ c. $(2e^{-5x})^{-2}$

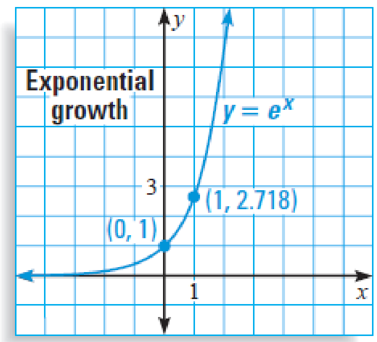
E2. Use a calculator to evaluate the expression

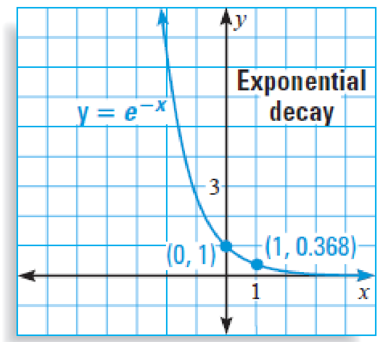
a. e^2 b. $e^{-0.06}$

P2. Use a calculator to evaluate the expression

a. e^3 b. $e^{-0.12}$

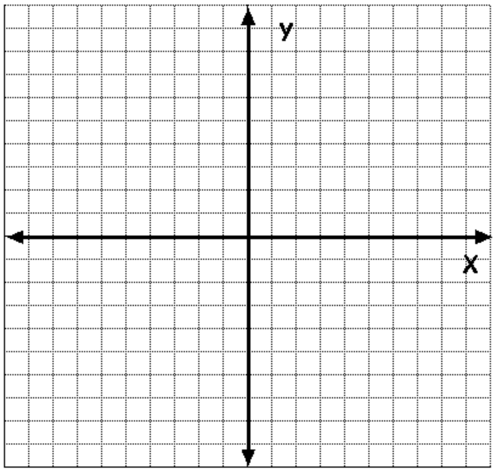
A function of the form $f(x) = ae^{rx}$ is called a *natural base exponential function*. If *a* > 0 and *r* > 0, the function is an exponential growth function, and if *a* > 0 and *r* < 0, the function is an exponential decay function. The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown below.



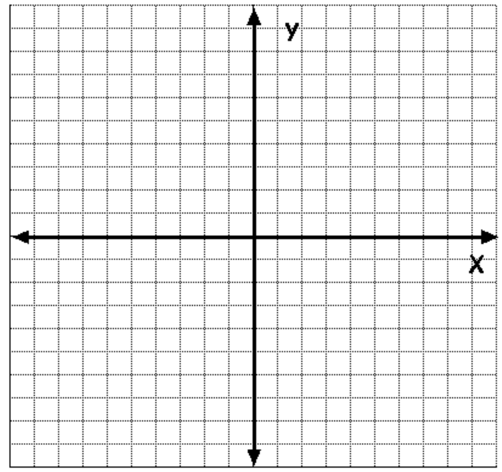


E3. Graph the function. State the domain and range.

a. $y = 2e^{0.75x}$

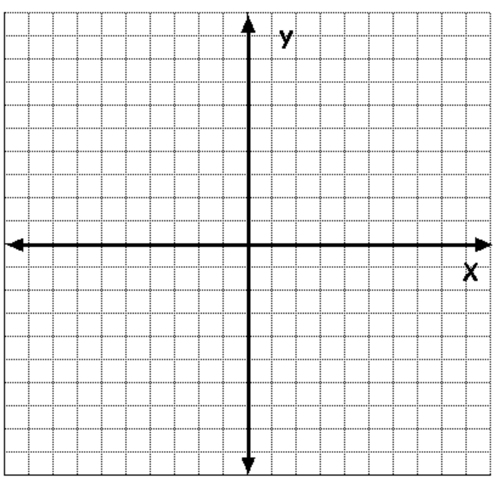


b. $y = e^{-0.5(x-2)} + 1$

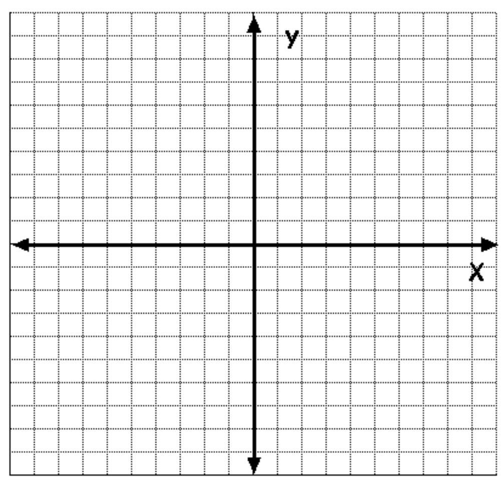


P3. Graph the function. State the domain and range.

a. $y = -3e^{0.5x}$



b. $y = e^{0.4(x+1)} - 2$



The amount A in an account earning interest compounded n times per year for t years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where P is the principal and r is the annual interest rate expressed as a decimal. As n approaches positive infinity, the compound interest formula approximates the following formula for *continuously compounded interest:*

$$A = Pe^{rt}$$

E4. You deposit $1000 in an account that pays 8% annual interest compounded continuously. What is the balance after 1 year?

P4. You deposit $1500 in an account that pays 7.5% annual interest compounded continuously. What is the balance after 1 year?

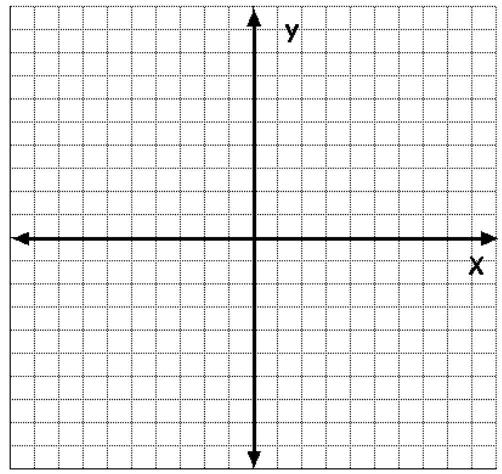
E5. Since 1972 the U.S. Fish and Wildlife Service has kept a list of endangered species in the United States. For the years 1972–1998, the number s of species on the list can be modeled by

$$s = 119.6e^{0.0917t}$$

where t is the number of years since 1972.

a. What was the number of endangered species in 1972?

b. Graph the model.



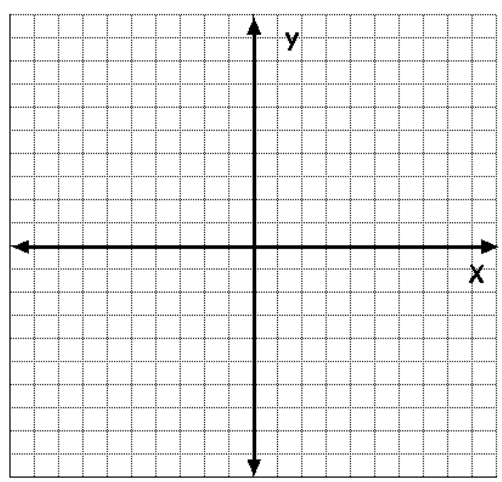
c. Use the graph to estimate when the number of endangered species reached 1000.

P5. The atmospheric pressure P (in pounds per square inch) of an object d miles above sea level can be modeled by

$$P = 14.7e^{-0.21d}$$

a. How much pressure per square inch would you experience at the summit of Mount Washington, 6288 feet above sea level?

b. Graph the model.



c. Use the graph to estimate your height above sea level if you experience 13.23 lb/in^2 of pressure.

Logarithm Form **Exponential Form**

$\log_b y = x$ is equivalent to $b^x = y$

$\log_b y = x$ is read "log base b of y equals x"

E1. Rewrite Logarithmic Equations

a. $\log_2 32 = 5$ b. $\log_5 1 = 0$ c. $\log_{10} 10 = 1$ d. $\log_{10} 0.1 = -1$ e. $\log_{1/2} 2 = -1$

$2^5 = 32$ $5^0 = 1$ $10^1 = 10$ $10^{-1} = 0.1$ $\left(\frac{1}{2}\right)^{-1} = 2$

P1. Rewrite Logarithmic Equations

a. $\log_3 9 = 2$ c. $\log_8 1 = 0$ e. $\log_5\left(\frac{1}{25}\right) = -2$

Special Logarithm Values

Let b be a positive real number such that b ≠ 0

Logarithm of 1: $\log_b 1 = 0 \; because \; b^{0=1}$

Logarithm of base b $\log_b b = 1 \; because \; b^1 = b$

E2. Evaluate the expression

a. $\log_3 81$ b. $\log_5 0.04$ c. $\log_{1/2} 8$ d. $\log_9 3$

$3^x = 81$ $5^x = .04$ $\left(\frac{1}{2}\right)^x = 8$ $9^x = 3$

$x = 4$ $5^x = \frac{1}{25}$ $x = -3$ $x = \frac{1}{2}$

$x = -2$

P2. Evaluate the expression

a. $\log_4 64$ b. $\log_2 0.125$ c. $\log_{1/4} 256$ d. $\log_{32} 2$

Common Logarithm **Natural Logarithm**

$\log_{10} x = \log x$ $\log_e x = \ln x$

E3. Evaluating Common and Natural Logarithms

a. log 5 b. ln 0.1

.69897 -2.30

.70

P3. Evaluate

a. log 7 b. ln 0.25

Inverse Functions

$g(x) = \log_b x$ and $f(x) = b^x$ are inverse functions. This means:

$$g(f(x)) = \log_b b^x = x \text{ and } f(g(x)) = b^{\log_b x} = x$$

In other words, exponential functions and logarithmic functions "undo" each other.

E4. Simplify the expression

a. $10^{\log 2} = 10^{\log_{10} 2}$

2

b. $\log_3 9^x = \log_3 3^{2x}$

$2x$

P4. Simplify the expression

a. $10^{\log x}$

b. $\log_5 125^x$

E5. Find the inverse of the function

a. $y = \log_3 x$

$x = \log_3 y$

$3^x = y$

b. $y = \ln(x + 1)$

$x = \ln(y+1)$

$e^x = y+1$

$\boxed{e^x - 1 = y}$

P5. Find the inverse of the function

a. $y = \log_8 x$

b. $y = \ln(x - 3)$

Graph of Logarithmic Functions

$$y = \log_b(x - h) + k$$

- The line $x = h$ is a vertical asymptote.
- The domain is $x > h$, and the range is all real numbers.
- If $b > 1$, the graph moves up to the right. If $0 < b < 1$, the graph moves down to the right.

E6. Graph the function. State the domain and range.

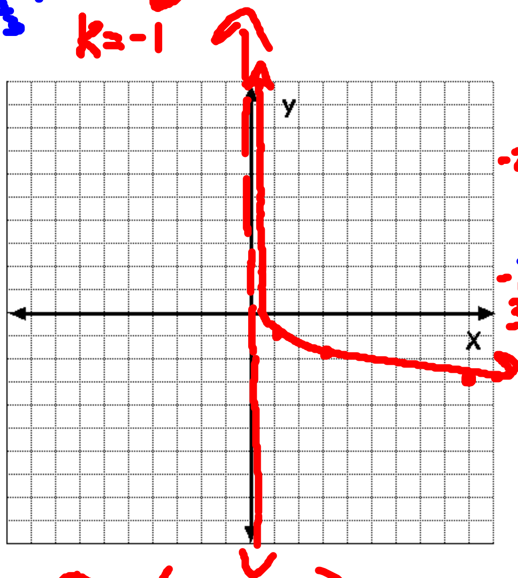
a. $y = \log_{1/3} x - 1$ V.A. $x=0$

Parent $y = \log_{1/3} x$

$\left(\frac{1}{3}\right)^y = x$

$k=-1$

x	y	−1
27	−3	−4
9	−2	−3
3	−1	−2
1	0	−1
1/3	1	0



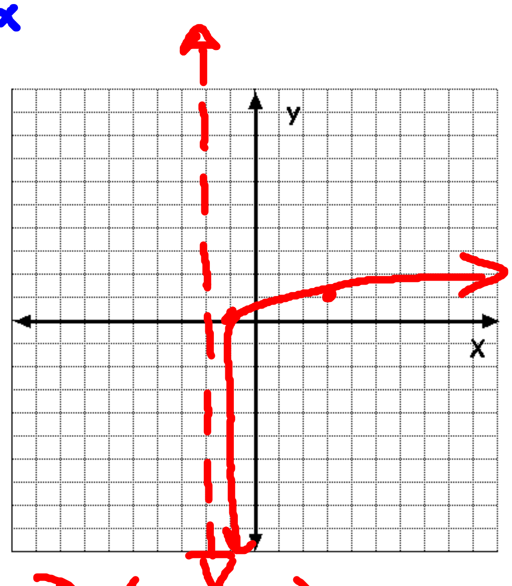
D: $(0, \infty)$
R: $(-\infty, \infty)$

b. $y = \log_5(x + 2)$ VA $x=-2$

Parent $y = \log_5 x$

$5^y = x$

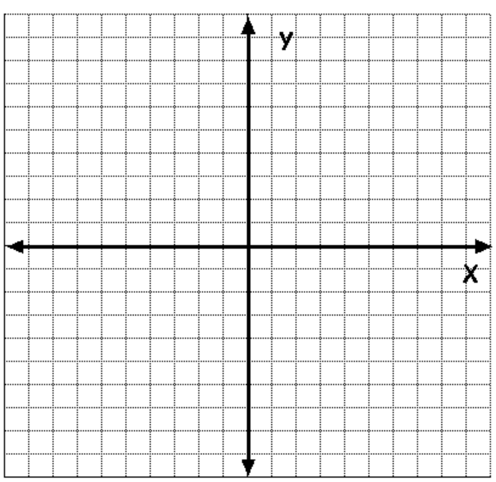
−2	x	y
	1/25	−2
	1/5	−1
−1	1	0
3	5	1
	25	2



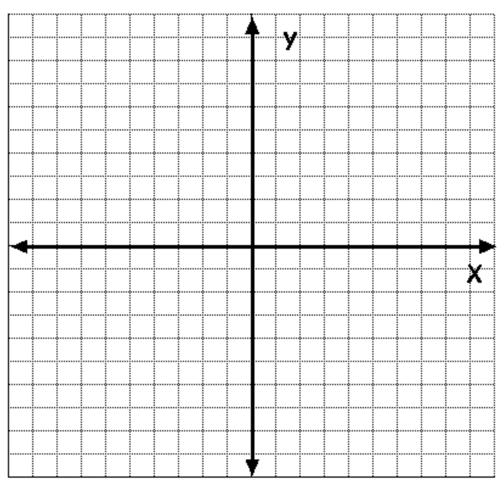
D. $(-2, \infty)$
R. $(-\infty, \infty)$

P6. Graph the function. State the domain and range.

a. $y = \log_{1/2} x + 4$



b. $y = \log_3(x - 2)$



Properties of Logarithms

Let b, u and v be positive numbers such that b ≠ 1.

Product Property $\log_b uv = \log_b u + \log_b v$

Quotient Property $\log_b \frac{u}{v} = \log_b u - \log_b v$

Power Property $\log_b u^n = n \log_b u$

Change of Base Formula

Let u, b, and c be positive numbers with b ≠ 1 and c ≠ 1. Then:

$$\log_c u = \frac{\log_b u}{\log_b c}$$

In particular, $\log_c u = \frac{\log u}{\log c}$ and $\log_c u = \frac{\ln u}{\ln c}$

E1. Use $\log_5 3 \approx 0.683$ *and* $\log_5 7 \approx 1.209$ to approximate the following

a. $\log_5 \frac{3}{7}$ b. $\log_5 21$ c. $\log_5 49$

P1. Use $\log_9 5 \approx 0.732$ *and* $\log_9 11 \approx 1.091$ to approximate the following

a. $\log_9 \frac{5}{11}$ b. $\log_9 55$ c. $\log_9 25$

E2. Expand $\log_2 \frac{7x^3}{y}$, assume x and y are positive.

P2. Expand $\log_5 2x^6$, assume x and y are positive.

E3. Condense $\log 6 + 2\log 2 - \log 3$.

P3. Condense $2\log_3 7 - 5\log_3 x$.

E4. Evaluate the expression $\log_3 7$ using common and natural logarithms.

P4. Evaluate the expression $\log_4 8$ using common and natural logarithms.

Solving Exponential and Logarithmic Equations

1. Check to see if the bases are equal (This can save time)
2. Use inverse operations
   - take the logarithm of both sides (exponential equation)
   - exponentiate each side (logarithmic equation)
3. Check for extraneous solutions

E1. Solve: $4^{3x} = 8^{x+1}$

P1. Solve: $2^{4x} = 32^{x-1}$

E2. Solve: $2^x = 7$

P2. Solve: $4^x = 15$

E3. Solve: $10^{2x-3} + 4 = 21$

P3. Solve: $5^{x+2} + 3 = 25$

E4. Solve: $\log_3(5x-1)=\log_3(x+7)$

P4. Solve: $\log_4(x+3)=\log_4(8x+17)$

E5. Solve: $\log_5(3x+1)=2$

P5. Solve: $\log_4(x+3)=2$

E6. Solve: $\log 5x+\log(x-1)=2$

P6. Solve: $\log_2 x+\log_2(x-7)=3$