8.8 Definite Integrals

Wednesday, June 12, 2019 7:15 AM

Indefinite Integrals: $\int f(x)\,dx = F(x) + C$

Definite Integrals: $\int_a^b f(x)\,dx = F(b) - F(a) \leftarrow$ no $+C$

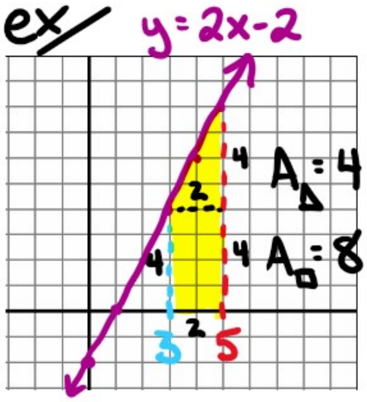
b ← upper bound

lower bound → a

$F(b)$: antiderivative evaluated at b

$F(a)$: antiderivative evaluated at a

ex/



$$\int_3^5 (2x-2)\,dx = x^2 - 2x\Big|_3^5 = \boxed{12}$$

$$= \underbrace{\left[(5)^2 - 2(5)\right]}_{F(5)} - \underbrace{\left[(3)^2 - 2(3)\right]}_{F(3)}$$

$$\quad\;\; 25 - 10 \qquad\qquad 9 - 6$$

$$= 15 - 3$$

ex/ $$\int_0^\pi \sin x\,dx = -\cos x\Big|_0^\pi = \left[-\cos(\pi)\right] - \left[-\cos(0)\right] = \boxed{2}$$

$$1 \quad - \quad -1$$

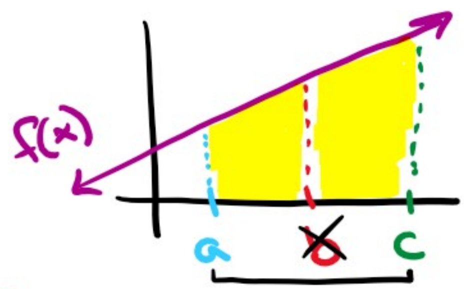
Properties of Definite Integrals:

① $\int_a^a f(x)\,dx = 0$ because $F(a) - F(a) = 0$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ because $F(b) - F(a) = -\left[F(a) - F(b)\right]$

③ $\int_a^b k \cdot f(x)\,dx = k \cdot \int_a^b f(x)\,dx$ because $kF(b) - kF(a) = k\left[F(b) - F(a)\right]$

④ $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



$$\left[\cancel{F(b)} - F(a)\right] + \left[F(c) \cancel{- F(b)}\right] = F(c) - F(a)$$