8.6 Permutations and Combinations

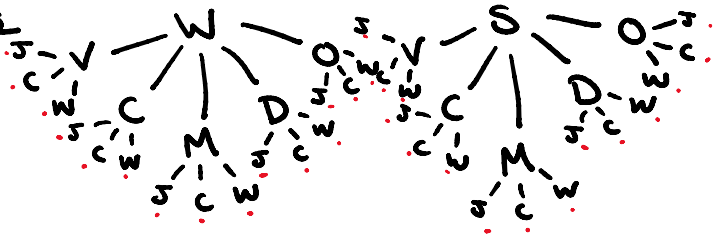
Wednesday, June 19, 2019 2:10 PM

Fundamental Counting Principle (FCP)

2 Cones: W, S

5 Flavors: V, C, M, D, O

3 Toppings: J, C, W



Total # = $\underset{\text{1 cone}}{2} \cdot \underset{\text{1 flavor}}{5} \cdot \underset{\text{1 topping}}{3} = 30$

2 scoops of flavors (can repeat flavors) = $\underset{\text{1 cone}}{2} \cdot \underset{\text{1st flavor}}{5} \cdot \underset{\text{2nd flavor}}{5} \cdot \underset{\text{1 topping}}{3} = 150$

2 scoops of flavors (no repeat flavor) = $2 \cdot 5 \cdot 4 \cdot 3 = 120$

Factorials: $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

PERMUTATIONS [order matters] vs. COMBINATIONS [order does not matter]

n = total #

r = subset of total

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

ex 8 horses in a race. How many ways can the top 3 horses finish?

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 336$$

ex 8 students. Pick 3 students to take a survey

$${}_8C_3 = \frac{8!}{(8-3)! \cdot 3!} = 56$$

Distinguishable Permutations:

(applicable when repeat items are present)

BANANA

n = 6 total letters

A occurs 3 times

N occurs 2 times

$$\#\text{ of total arrangements} = \frac{n!}{a! \cdot b! \cdot \ldots \cdot c!}$$

↑ ↑ ↑

the # of times a [illegible] item occurs

$$\frac{6!}{3! \cdot 2!} = \frac{\cancel{6} \cdot 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{2} \cdot \cancel{1}} = \boxed{60}$$

↑ ↑

↑ ↑ ↑
the # of times a repeated item occurs

$3! \cdot 2!$

↑ # of times A occurs

↑ # of times N occurs