5.7 Slant Asymptotes and Discontinuities

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HA @ $y = \#$ VA @ $x = \#$ SA @ $y = mx + b$

Slant Asymptotes:

Rule - A slant asymptote exists if and only if the degree of the numerator is exactly one greater than the degree of the denominator.

* HA and SA cannot coexist (impossible to have both)

$$f(x) = \frac{3x^4 - 4x + 5}{x^2 + x - 1}$$ HA @ y = none, SA @ y = none

To find the $y = mx + b$ equation of a slant asymptote, we must long divide the numerator by the denominator.

ex

$$f(x) = \frac{4x^3 + 6x^2 - 3x + 5}{x^2 - 1}$$

0x

$y = mx + b$

$$\begin{array}{r} 4x + 6 \\ x^2 + 0x - 1 \,\big)\, \overline{4x^3 + 6x^2 - 3x + 5} \\ -4x^3 \mp 0x^2 + 4x \\ \hline 6x^2 + x + 5 \end{array}$$

STOP

SA @ $y = 4x + 6$

ex

$$f(x) = \frac{x^3 + 4x^2 - x - 4}{2x^2 + 8x - 10}$$

$2(x^2 + 4x - 5)$

① HA @ y = NONE (TOP-HEAVY)

② SA @ $y = \frac{1}{2}x$

③ Linear Factorization $$f(x) = \frac{(x+4)(x+1)\cancel{(x-1)}}{\boxed{2}\boxed{(x+5)}\cancel{(x-1)}}$$

x=1

④ Disc. @ $\left(\underset{x}{1}, \underset{y}{\tfrac{5}{6}}\right)$

⑤ VA @ $x = -5$

$$\frac{5 \cdot 2}{2 \cdot 6}$$

$$\begin{array}{r} \tfrac{1}{2}x + 0 \\ 2x^2 + 8x - 10 \,\big)\, \overline{x^3 + 4x^2 - x - 4} \\ -x^3 \mp 4x^2 + 5x \\ \hline 0x^2 + 4x - 4 \end{array}$$

STOP

Discontinuities:

A hole on the graph that has (x, y) coordinates.

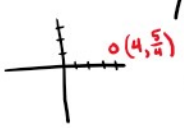
Disc. exist at any factors that are common in both the numerator and denominator.

Steps:

① Factor the numerator and denominator (Linear Factorization of f(x))

② Disc occur at any factors that are common in the numerator and denominator

③ Cancel the common factors and then plug in x-value to find the corresponding y-value



ex $$f(x) = \frac{x^2 - 3x - 4}{x^3 - 7x^2 + 12x}$$

① $$f(x) = \frac{\cancel{(x-4)}(x+1)}{x\,\cancel{(x-4)}(x-3)}$$

VA @ x=0 ~~VA~~ x=3

② Disc. @ x=4

③ $$f(x) = \frac{x+1}{x(x-3)}$$

$$f(4) = \frac{4+1}{4(4-3)} = \frac{5}{4}$$

Disc. @ $\left(4, \frac{5}{4}\right)$