4.4 Rational Zeros

Wednesday, June 19, 2019 1:28 PM

Steps

1. Try to factor $f(x)$
2. Apply the Rational Root Test (identify all rational roots/zeros)
3. Create a fraction of the form: $\dfrac{\pm \text{ factors of constant term}}{\text{only } + \text{ factors of leading coefficient}}$
4. Create a list of possible zeros by combining each value in the numerator with each value in the denominator
5. Test each possible zero by applying synthetic division

Remainder = 0 the value is a root

Remainder ≠ 0 the value is not a root

ex $f(x) = 4x^3 + 5x^2 - 23x - 6$

3 zeros/roots

① DNF ☹

② RRT ☺

③ $\dfrac{1, -1, 2, -2, 3, -3, 6, -6}{1, 2, 4}$

④ possible zeros: $\cancel{1}, \cancel{-1}, 2, -2, 3, -3, 6, -6,$

$\frac{1}{2}, \frac{1}{4}, -\frac{1}{2}, -\frac{1}{4}, \frac{3}{2}, \frac{3}{4}, -\frac{3}{2}, -\frac{3}{4}$

16 possible zeros

⑤
$$\begin{array}{r|rrrr} 1 & 4 & 5 & -23 & -6 \\ & \downarrow & 4 & 9 & -14 \\ \hline & 4 & 9 & -14 & -20 \end{array} \leftarrow ☹$$

$$\begin{array}{r|rrrr} -1 & 4 & 5 & -23 & -6 \\ & \downarrow & -4 & -1 & 24 \\ \hline & 4 & 1 & -24 & 18 \end{array} \leftarrow ☹$$

$$\begin{array}{r|rrrr} 2 & 4 & 5 & -23 & -6 \\ & \downarrow & 8 & 26 & 6 \\ \hline & 4 & 13 & 3 & \boxed{0} \\ & x^2 & x & c & R \end{array} \leftarrow ☺$$

$4x^2 + 13x + 3$

$x = 2$ is a root/zero

$(x-2)$ is a factor

$f(x) = 4x^3 + 5x^2 - 23x - 6$

$(x-2)(4x^2 + 13x + 3)$

$(x+3)(4x+1)$

12

12 | 1

$4x^2 + 13x + 3$

$(4x^2 + 12x) + (1x + 3)$

$4x(x+3) + 1(x+3)$

$(x+3)(4x+1)$

Linear Factorization: $f(x) = (x-2)(x+3)(4x+1)$

Zeros/roots: $x = 2, -3, -\frac{1}{4}$

ex $f(x) = 1x^4 - x^3 + x^2 - 3x - 6$

$\dfrac{1, -1, 2, -2, 3, -3, 6, -6}{1}$

possible zeros: $\cancel{1}, -1, 2, -2, 3, -3, 6, -6$

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & 1 & -3 & -6 \\ & & 1 & 0 & 1 & -2 \\ \hline & 1 & 0 & 1 & -2 & -8 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 1 & -3 & -6 \\ & & -1 & 2 & -3 & 6 \\ \hline & 1 & -2 & 3 & -6 & 0 \\ & x^3 & x^2 & x & c & R \end{array}$$

$(x+1)(x^3 - 2x^2 + 3x - 6)$

$f(x) = (x+1)((x^3 - 2x^2) + (3x - 6))$

$x^2(x-2) + 3(x-2)$

$(x-2)(x^2+3)$

$f(x) = (x+1)(x-2)(x^2+3)$

$\underbrace{x = -1 \quad x = 2}_{\text{2 rational roots}}$

$x^2 + 3 = 0$

$-3 \quad -3$

$\sqrt{x^2} = \pm\sqrt{-3}$

$x = \pm\sqrt{-3}$

$x = \pm\sqrt{3}\, i$

2 complex roots