Graphing Review

Key.

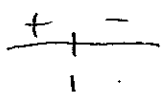
I. Graph each of the following

1. $y = x^4 - 4x^3 + 4x^2$
2. $y = 1 - 9x - 6x^2 - x^3$
3. $y = 1 - (x+1)^3$
4. $y = x^4 + 2x^3$
5. $y = 2x - 3x^{2/3}$
6. $y = \frac{1}{x^2 - 9}$
7. $y = \frac{x^2}{x-2}$
8. $y = \frac{x^2 - 3}{2x}$
9. $y = \frac{x-2}{x-5}$

II. Sketch a graph with the following Properties

10. $f(1) = 2$; $f(-3) = -1$; $f(-5) = -3$

$f'(x) = 0$ when $x = 1$



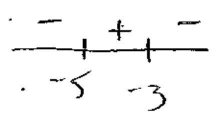
$f'(x) > 0$ when $x < 1$

$f'(x) < 0$ when $x > 1$

$f''(x) = 0$ when $x = -3, -5$

$f''(x) > 0$ when $-5 < x < -3$

$f''(x) < 0$ when $x > -3$ and $x < -5$



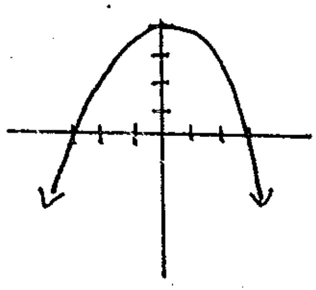
11. $f(3) = -2$

$f(-3) = -2$

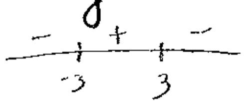
$f(-6) = 4$

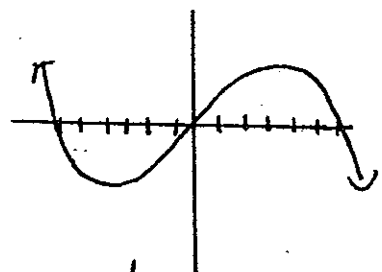
$f(0) = -8$

$f(6) = 4$

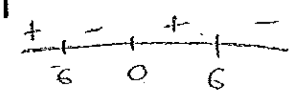


y''

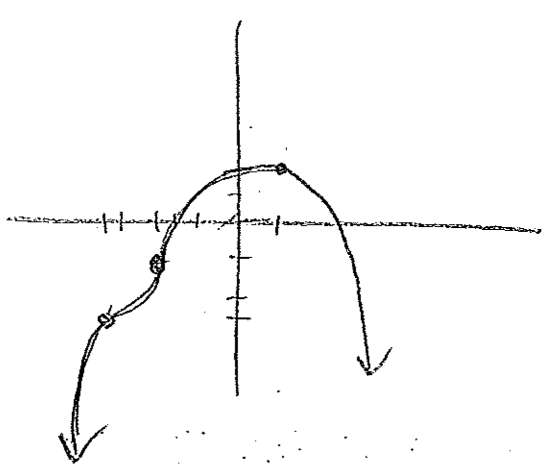




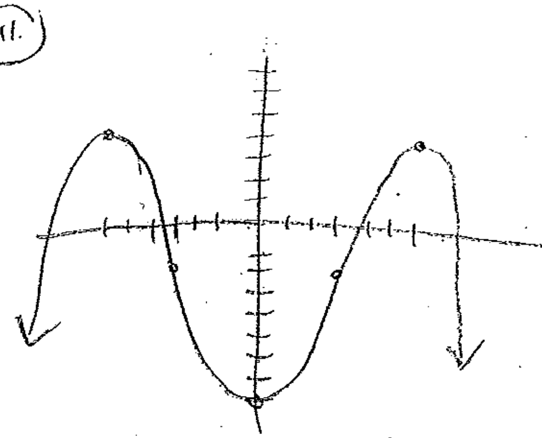
y'



(10.)



(11.)



1. $y = x^4 - 4x^3 + 4x^2$

$y' = 4x^3 - 12x^2 + 8x$

$0 = 4x(x^2 - 3x + 2)$

$0 = \cancel{4x(x-1)(x-2)}$

$x = 0, 1, 2$

$y'' = 12x^2 - 24x + 8$

$0 = 4(3x^2 - 6x + 2)$

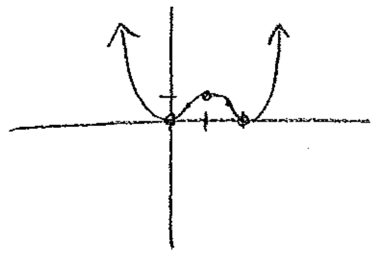
$x = \dfrac{6 \pm \sqrt{36 - 4(3)(2)}}{6} = \dfrac{6 \pm \sqrt{12}}{6}$

$\approx 1.6, .42$

y': $-$ | 0 | $+$ | 1 | $-$ | 2 | $+$

y'': $+$ | .42 | $-$ | 1.6 | $+$

(0,0)
(1,1)
(2,0)



min (0,0), (2,0)
max (1,1)
Inc $[0,1] \cup [2,\infty)$
Dec $(-\infty,0] \cup [1,2]$
IP (.4,.4), (1.6,.4)
CD (.4,1.6)
CU $(-\infty,.4) \cup (1.6,\infty)$

2. $y = 1 - 9x - 6x^2 - x^3$

$y' = -9 - 12x - 3x^2$

$0 = -3(3 + 4x + x^2)$

$0 = -3(x+3)(x+1)$

$x = -3, -1$

$y'' = -12 - 6x$

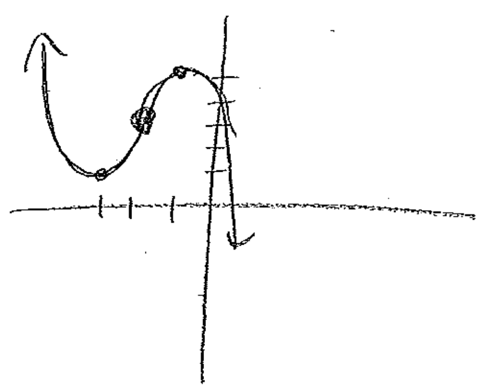
$0 = -12 - 6x$

$x = -2$

y'': $+$ | -2 | $-$

y': $-$ | -3 | $+$ | -1 | $-$

(-1,5)
(-3,1)
(-2,3)



min (-3,1)
max (-1,5)
Inc [-3,-1]
Dec $(-\infty,-3] \cup [-1,\infty)$
IP (-2,3)
CU $(-\infty,-2)$
CD $(-2,\infty)$

(3.)

$y = 1 - (x+1)^3$

$y' = -3(x+1)^2$

$0 = -3(x+1)^2$

$x = -1$

$-$ $-$

-1

$y'' = -6(x+1)$

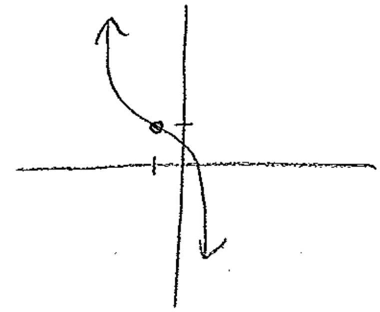
$0 = -6(x+1)$

$x = -1$

$+$ $-$

-1

$(-1, 1)$



min —
max —
Inc —
Dec $(-\infty, \infty)$
IP $(-1, 1)$
CU $(-\infty, -1)$
CD $(-1, \infty)$

(4.) See Notes!

(5.)

$y = 2x - 3x^{2/3}$

$y' = 2 - 2x^{-\frac{1}{3}}$

$\frac{-2}{1} = \frac{-2}{x^{1/3}}$

$-\cancel{2}x^{1/3} = -\cancel{2}$

$x = 1$ $x = 0$ und.

und   min

$+$ $-$ $+$

0 1

$(1, -1)$

$(0, 0)$

$y'' = \frac{2}{3}x^{-\frac{4}{3}}$

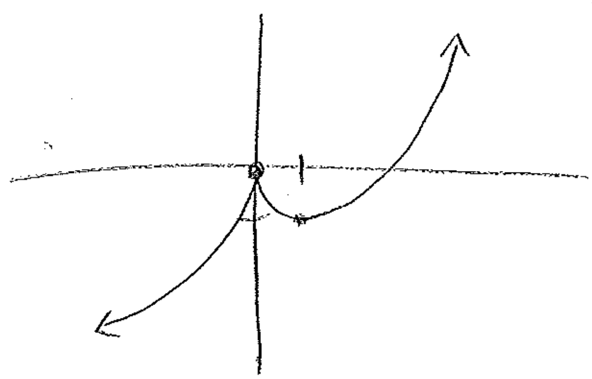
$0 = \frac{2}{3x^{4/3}}$

$x \neq 0$

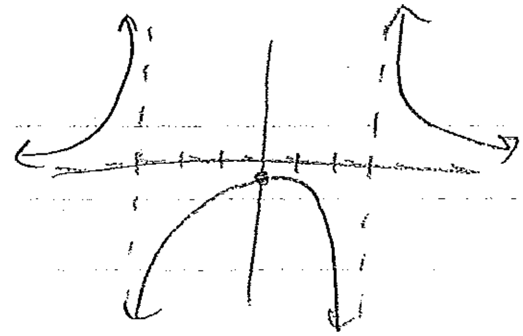
$+$ $+$

0

min $(1, -1)$
max —
Inc $(-\infty, 0] \cup [1, \infty)$
Dec $[0, 1]$
IP —
CU $(-\infty, \infty)$
CD —



(6) $y = \dfrac{1}{x^2-9}$ VA: $x = \pm 3$
HA: $y = 0$



$y = (x^2-9)^{-1}$

$y' = -1(x^2-9)^{-2} \cdot 2x$

$0 = \dfrac{-2x}{(x^2-9)^2}$

$x = 0$
und $x = \pm 3$

+ und + − und −
−3 0 3

max $(0, -\frac{1}{9})$

$y'' = \dfrac{(x^2-9)^{2}\cdot -2 + -2x(2(x^2-9)2x)}{(x^2-9)^{4}}$

$0 = \dfrac{-2x^2+18 + 8x^2}{(x^2-9)^3} = \dfrac{6x^2+18}{(x^2-9)^3}$

$x = \emptyset$
und $x = \pm 3$

+ und − − und +
−3 0 3

$\lim\limits_{x \to 3^+} f(x) = \dfrac{1}{{}^{+}0} = \infty$

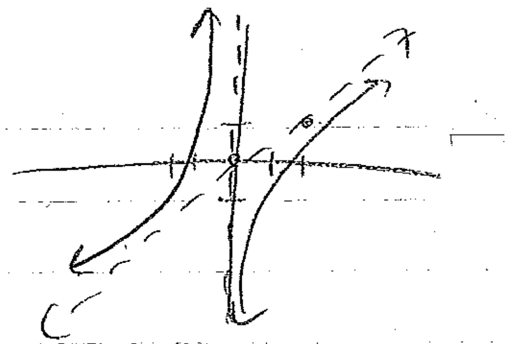
$\lim\limits_{x \to 3^-} f(x) = \dfrac{1}{{}^{-}0} = -\infty$

$\lim\limits_{x \to -3^+} f(x) = \dfrac{1}{{}^{-}0} = -\infty$

$\lim\limits_{x \to -3^-} f(x) = \dfrac{1}{{}^{+}0} = \infty$

8.

$$y = \frac{x^2-3}{2x} \qquad VA: x=0 \qquad SA: y = \frac{1}{2}x$$



$$y' = \frac{2x(2x) - (x^2-3)2}{4x^2}$$

$$0 = \frac{4x^2 - 2x^2 + 6}{4x^2} = \frac{2x^2+6}{4x^2}$$

und $x=0$

$$\begin{array}{ccc} + & \text{und} & + \\ & 0 & \end{array}$$

$$y'' = \frac{4x^2(4x) - (2x^2+6)8x}{16x^4}$$

$$0 = \frac{16x^3 - 16x^3 - 48x}{16x^4}$$

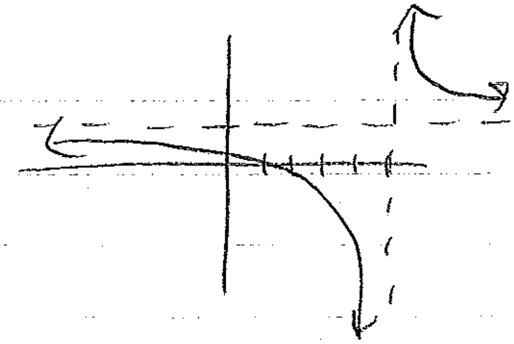
$$0 = \frac{-48x}{16x^4} = \frac{-48}{16x^3}$$

$$\begin{array}{ccc} + & \text{und} & - \\ & 0 & \end{array}$$

$$\lim_{x \to 0^+} f(x) = \frac{-3}{^{+}0} = -\infty$$

$$\lim_{x \to 0^-} f(x) = \frac{-3}{^{-}0} = \infty$$

9. $y = \dfrac{x-2}{x-5}$ VA: $x=5$ HA: $y=1$



$$y' = \frac{(x-5) + (x+2)}{(x-5)^2}$$

$$0 = \frac{-3}{(x-5)^2} = -3(x-5)^{-2}$$

und $x=5$

$-$ und $-$

5

$$y'' = +6(x-5)^{-3}$$

$$0 = \frac{6}{(x-5)^3}$$

und $x=5$

$-$ und $+$

5

$$\lim_{x \to 5^+} f(x) = \frac{3}{^+0} = \infty$$

$$\lim_{x \to 5^-} f(x) = \frac{3}{^-0} = -\infty$$