Calculus 2

Name: KEY

Chapter 7 Practice Test

Expand the logarithmic expression:

1. $\ln\frac{x^4(y-2)^2}{\sqrt{z+4}}$ $4\ln x + 2\ln(y-2) - \frac{1}{2}\ln(z+4)$

Write the expression as a single logarithm:

2. $4\ln 2 - \frac{1}{3}(x-1) + 5\ln y$ $\ln\frac{16y^5}{\sqrt[3]{x-1}}$

Solve:

3. $\ln 4 + 2\ln x = 2$
$\ln 4\cdot x^2 = 2$ $x = \frac{1}{2}e$
$e^2 = 4x^2$

4. $5e^{-3t} = 11$
$-3t = \ln\frac{11}{5}$
$t = -.26$

5. $2^{3x+4} = 8$
$(3x+4)\ln 2 = \ln 8$ or $2^{3x+4} = 2^3$
$x = -\frac{1}{3}$ $3x+4 = 3$
$x = -\frac{1}{3}$

Find $\frac{dy}{dx}$ for each of the following:

6. $y = \ln(x^3 - 2x)$ $y' = \frac{3x^2-2}{x^3-2x}$

7. $y = \ln\frac{(2x+1)^2}{\sqrt[3]{3-2x}} = 2\ln(2x+1) - \frac{1}{3}\ln(3-2x)$
$y' = \frac{2\cdot 2}{2x+1} - \frac{-2}{3\cdot(3-2x)} = \frac{4}{2x+1} + \frac{2}{9-6x}$

8. $y = 2\ln\frac{3}{x^4} = 2\ln 3 - 8\ln x$
$y' = \frac{-8}{x}$

9. $y = \ln\frac{e^{2x}}{1+e^{-x}} = 2x\ln e - \ln(1+e^{-x})$
$y' = 2 - \frac{-e^{-x}}{1+e^{-x}} = 2 + \frac{e^{-x}}{1+e^{-x}}$

10. $y = e^{-x^2+1}$ $y' = -2xe^{-x^2+1}$

11. $y = \frac{x^5}{e^x}$ $y' = \frac{e^x\cdot 5x^4 - x^5\cdot e^x}{e^{2x}} = \frac{5x^4 - x^5}{e^x}$

12. $y = 3xe^{\cos 4x}$ $y' = 3x\cdot -4\sin 4x\, e^{\cos 4x} + 3e^{\cos 4x}$
$= 3e^{\cos 4x}(-4x\sin 4x + 1)$

13. $y = (e^{2x} - e^{-x})^3$ $y' = 3(e^{2x}-e^{-x})^2\cdot(2e^{2x}+e^{-x})$
$= 3(2e^{2x}+e^{-x})(e^{2x}-e^{-x})^2$

Find $\frac{dy}{dx}$ for each of the following using logarithmic differentiation:

14. $y = \sqrt[3]{\frac{x^5}{(1-2x)^2}}$
$\ln y = \frac{5}{3}\ln x - \frac{2}{3}\ln(1-2x)$
$\frac{1}{y}\frac{dy}{dx} = \frac{5}{3x} - \frac{2\cdot -2}{3(1-2x)}$
$\frac{dy}{dx} = \left(\frac{5}{3x} + \frac{4}{3-6x}\right)\sqrt[3]{\frac{x^5}{(1-2x)^2}}$

15. $y = x^{\sin x}$
$\ln y = \sin x \ln x$
$\frac{1}{y}\frac{dy}{dx} = \sin x\cdot\frac{1}{x} + \ln x\cos x$
$\frac{dy}{dx} = \left(\frac{\sin x}{x} + \ln x\cos x\right)x^{\sin x}$

Find the equation of the tangent line to the graph of the function at the given point:

16. $y = \ln\sqrt{x}$; $x = 4$ $y = \ln 2$, $m = \frac{1}{8}$
$= \frac{1}{2}\ln x$ $y = \frac{1}{8}x - \frac{1}{2} + \ln 2$
$y' = \frac{1}{2x}$

17. $y = xe^{2x}$; $x = 0$, $y = 0$, $m = 1$
$y' = x\cdot 2e^{2x} + e^{2x}$ $y = x$

Find $\frac{dy}{dx}$ using implicit differentiation:

18. $x^2y = y^2 + e^x$
$x^2\frac{dy}{dx} + 2xy = 2y\frac{dy}{dx} + e^x$
$\frac{dy}{dx} = \frac{e^x - 2xy}{x^2 - 2y}$

Find the x coordinates of any maxima, minima and points of inflection:

19. $y = xe^{-x}$
max: $x = 1$
PI: $x = 2$
$y' = -xe^{-x} + e^{-x}$
$e^{-x}(-x+1) = 0$ + − (1)
$x = 1$
$y'' = xe^{-x} - e^{-x} - e^{-x}$
$e^{-x}(x-2) = 0$ − + (2)
$x = 2$

20. $y = \ln\cancel{(x^2-1)}$ $x^2\ln x$
max: $x = 0$
min: $x = e^{-\frac{1}{2}}$
PI: $x = e^{-\frac{3}{2}}$
$y' = \frac{x^2}{x} + \ln x\cdot 2x = x + 2x\ln x$
$x(1+2\ln x) = 0$ x − + (0, $e^{-\frac{1}{2}}$)
$x = 0$ $\ln x = -\frac{1}{2} \to x = e^{-\frac{1}{2}}$
$y'' = 1 + \frac{2x}{x} + \ln x\cdot 2$ x − + (0, $e^{-\frac{3}{2}}$)
$= 3 + 2\ln x = 0 \Rightarrow \ln x = -\frac{3}{2}$
$x = e^{-\frac{3}{2}}$

Integrate:

21. $\int \frac{4}{2x+1}\,dx$ $2\ln|2x+1| + c$

22. $\int \frac{-3\sec^2 x}{\tan x}\,dx$ $-3\ln|\tan x| + c$

23. $\int \frac{(\ln x)^3}{x}\,dx$ $\frac{1}{4}(\ln x)^4 + c$

24. $\int \frac{e^{3x}+e^{x}-3}{e^{2x}}\,dx$ $e^x - e^{-x} + \frac{3}{2}e^{-2x} + c$

$= e^x + e^{-x} - 3e^{-2x}$

25. $\int \frac{e^{5x}+e^{-5x}}{e^{5x}-e^{-5x}}\,dx$ $\frac{1}{5}\ln\left|e^{5x}-e^{-5x}\right| + c$

26. $\int x^2 e^{4-3x^3}\,dx$ $-\frac{1}{9}e^{4-3x^3} + c$

Evaluate the definite integrals:

27. $\int_{e^2}^{e^4} \frac{1}{x\ln x}\,dx$ $\ln|\ln x|\Big|_{e^2}^{e^4}$

$u = \ln x$

$du = \frac{1}{x}dx$

$\frac{1}{u}$

$\ln 4 - \ln 2$

$\ln 2$

28. $\int_{1/2}^{1} \frac{e^{1/x^2}}{x^3}\,dx$ x^{-2} $-\frac{1}{2}e^{1/x^2}\Big|_{1/2}^{1}$

$-2e^4$

$-\frac{1}{2}e^1 - -\frac{1}{2}e^4 = -\frac{1}{2}e + \frac{1}{2}e^4$

29. Find the area enclosed by $y = e^x$, $x = 0$, $y = 0$ and $x = \ln 4$.

$\int_0^{\ln 4} e^x\,dx = e^x\Big|_0^{\ln 4} = 4 - 1 = 3$

30. Find the volume formed by rotating the area enclosed by $y = \frac{1}{\sqrt{x-1}}$, $x = 2$, $x = 5$ and $y = 0$ about the x-axis.

$\pi\int_2^5 \left(\frac{1}{\sqrt{x-1}}\right)^2 dx = \pi\int_2^5 \frac{1}{x-1}\,dx = \ln|x-1|\Big|_2^5$

$= \ln 4 - \ln 1 = \pi\ln 4$

31. The half life of a particular radioactive substance is 876 years. If the initial size was 13 grams, what will be the size in 500 years?

$\frac{1}{2} = e^{876r}$

$r = -.000791$

$y = 13e^{-.000791 \cdot 500}$

$= 8.75$ grams

32. The population of a certain organism tripled between 1920 and 1980. In what year will the population have quadrupled?

$3 = 1e^{60r}$

$r = .0183$

$4 = 1e^{.0183t}$

$t = 75.75$ yrs

1995