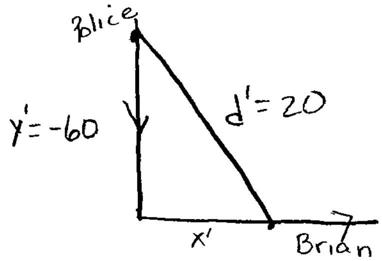
AP Calc AB

Mrs. Smith

# Review Sheet Solutions - Chapter 4

1)




$y = .6$ miles

$x = .8$

$d' = 20$

$y' = -60$

$x' = ?$

$(.8)^2 + (.6)^2 = d^2$

$d = 1$

$x^2 + y^2 = d^2$

$xx' + yy' = dd'$

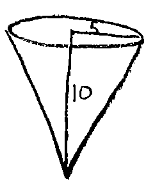
$(.8)x' + (.6)(-60) = 1(20)$

$.8x' = 56$

$x' = 70$

**Brian's speed is 70mph**

2)




$V' = 9 \text{ ft}^3/\text{min}$

$\frac{h}{r} = \frac{10}{5} = 10r = 5h$

$r = \frac{h}{2}$

$h' = ?$ when $h = 6$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{4}$

$V = \frac{\pi h^3}{12}$

$V' = \frac{\pi}{4} h^2 h'$

$9 = \frac{\pi}{4}(6)^2 h'$

$\boxed{h' = 0.3183 \text{ ft/min}}$

*3) $V = \frac{4}{3}\pi r^3$  Surface Area = $S = 4\pi r^2$

$V' = 100\pi\ ft^3/min$

a) $r' = ?$ when $r = 5$ ft

$V = \frac{4}{3}\pi r^3$

$V' = 4\pi r^2 r'$

$100\pi = 4\pi(5)^2 r'$

$\boxed{r' = 1\ ft/min}$

b) $S'$ when $r = 5$

$S = 4\pi r^2$

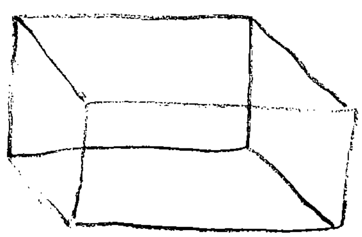
$S' = 8\pi r r'$

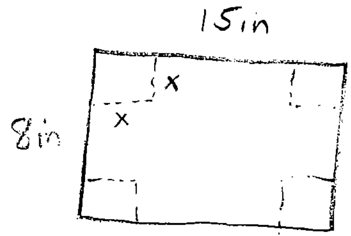
$S' = 8\pi(5)(1)$

$S' = 40\pi\ ft^2/min$

$\boxed{\approx 125.66\ ft^2/min}$

4)






$V = \underset{L}{(15-2x)}\underset{W}{(8-2x)}\underset{h}{(x)}$  $(0, 4)$

$V = (120 - 30x - 16x + 4x^2)(x)$

$V = 4x^3 - 46x^2 + 120x$

$V' = 12x^2 - 92x + 120$

$(0 = 12x^2 - 92x + 120)\ 1/4$

$0 = 3x^2 - 23x + 30$  30 (3,1) 15 2 (6,5)

$0 = (3x - 5)(x - 6)$

$\boxed{x = 5/3}$  $x = 6$ ↳ NOT IN INTERVAL ∴ NOT POSSIBLE

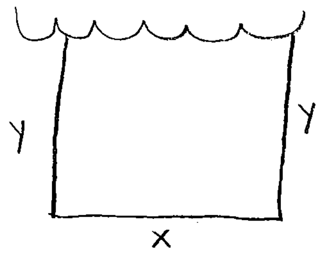
Dimensions

$V = \underset{L}{35/3} \times \underset{W}{14/3} \times \underset{h}{5/3}$

$11.67 \times 4.67 \times 1.67$

$\boxed{V = 91.013\ in^3}$

5)




$P = 800\text{m}$

$P = x + 2y$

$800 = x + 2y$

$800 - 2y = x$

$A = xy$

$A = (800-2y)(y)$

$A = 800y - 2y^2$

$A' = 800 - 4y$

$0 = 800 - 4y$

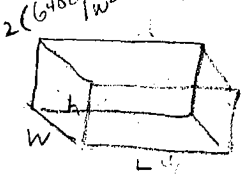
$4y = 800$

$y = 200$

$x = 800 - 2(200) = 400$

$$\boxed{A = (200)(400) = 80{,}000\text{m}^2}$$

6) $C = .75(\overbrace{6400/w^2 \cdot w}^{\text{base} = L \cdot w}) + .25\,(2(w^2) + 2(6400/w^2 \cdot w))$




SQUARE ENDS $\Rightarrow w = h$

$V = Lwh$

$V = 6400 \text{ ft}^3$

$V = Lw^2$

$6400 = Lw^2$

$L = 6400/w^2$

$C = .75\left(\frac{6400}{w}\right) + .25\left(\underbrace{2w^2 + \frac{12800}{w}}_{}\right)$ $\quad \frac{2w^3 + 12800}{w}$

$C = \frac{4800}{w} + \frac{.5w^3 + 3200}{w}$

$C = \frac{.5w^3 + 8000}{w}$

$C = \frac{1}{2}w^2 + 8000/w$

$C' = w - 8000/w^2$

$0 = w - 8000/w^2$

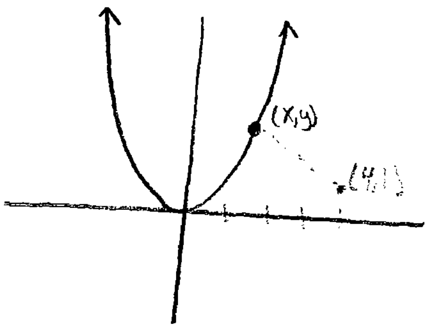
$\frac{8000}{w^2} = w$

$8000 = w^3$

$w = 20$

$$\boxed{\text{Dimensions: } \begin{matrix} L & w & h \\ 16 \times & 20 \times & 20 \end{matrix}}$$

7) $y = x^2$ $(4,1)$




$d = \sqrt{(x-4)^2 + (y-1)^2}$

$d^2 = (x-4)^2 + (x^2-1)^2$

$S = x^2 - 8x + 16 + x^4 - 2x^2 + 1$

$S = x^4 - x^2 - 8x + 17$

$S' = 4x^3 - 2x - 8$

$0 = 2(2x^3 - x - 4)$

Calc $x = 1.3917688$

$\boxed{(1.3, 1.93)}$

8) $f(x) = \frac{x^3}{3}$ $[-3, 3]$

MVT applies.

$f'(x) = x^2$

$\frac{f(3) - f(-3)}{3 - -3} = \frac{9 - (-9)}{6} = \frac{18}{6} = 3$

$x^2 = 3$

$\boxed{x = \pm\sqrt{3}}$

9) $f(x) = x^{3/5} + 1$ $[-1, 1]$

MVT Does Not Apply since $f(x)$ is NOT Differentiable at $x = 0$ which is in the interval.

10) $f(x) = \frac{x+1}{x-1}$ $[2, 3]$

MVT Applies.

$f'(x) = \frac{\overbrace{(x-1)(1)}^{x-1} - \overbrace{(x+1)(1)}^{-x-1}}{(x-1)^2}$

$f'(x) = \frac{-2}{(x-1)^2}$ $\qquad \frac{f(3) - f(2)}{3 - 2} = \frac{2 - 3}{1} = \frac{-1}{1} = -1$

$-1 = \frac{-2}{(x-1)^2} = +(x-1)^2 = +2$

$x - 1 = \sqrt{2}$

$\boxed{x = 1 + \sqrt{2} \approx 2.41}$

11) $f(x) = 3x^4 - 4x^3 \quad [-2,3]$

$f'(x) = 12x^3 - 12x^2$

$0 = 12x^2(x-1)$

| CP | f(x) |
|---|---|
| 0 | $\to$ 0 |
| 1 | $\to$ -1 |
| -2 | $\to$ 80 |
| 3 | $\to$ 135 |

$\boxed{\text{Minimum } (1,-1) \quad \text{Maximum } (3,135)}$

12) $f(x) = 2x^5 - 5x^4 + 7 \quad [-1,3]$

$f'(x) = 10x^4 - 20x^3$

$0 = 10x^3(x-2)$

CP

$0 \to 7$

$2 \to -9$

$-1 \to 0$

$3 \to 88$

$\boxed{\text{Minimum } (2,-9) \quad \text{Maximum } (3,88)}$

13) $f(x) = (x-1)^3(x+2)^2 \quad [-2,2]$

$f'(x) = (x-1)^3(2)(x+2)(1) + (x+2)^2(3)(x-1)^2(1)$

$f'(x) = (x-1)^2(x+2)\left[\overset{2x-2+3x+6}{(x-1)(2)+(x+2)(3)}\right]$

$f'(x) = (x-1)^2(x+2)(5x+4)$

CP

$1 \to 0$

$-2 \to 0$

$-\frac{4}{5} \to -8.40$

$2 \to 16$

$\boxed{\text{Minimum } (-0.8,-8.4) \quad \text{Maximum } (2,16)}$

14) $y = 4x^3 + 21x^2 + 36x - 20$

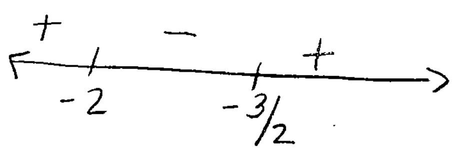
$y' = 12x^2 + 42x + 36$

$0 = 3(4x^2 + 14x + 12)$

$0 = 6(2x^2 + 7x + 6)$

$0 = 6(2x + 3)(x + 2)$

$x = -3/2 \quad x = -2$




Inc $(-\infty, -2) \cup (-3/2, \infty)$

Dec $(-2, -3/2)$

local max $(-2, -40)$

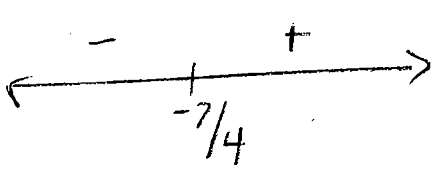
local min $(-3/2, -40.25)$

$y'' = 24x + 42$

$0 = 2(12x + 21)$

$0 = 6(4x + 7)$

$x = -7/4$




Inflection pt. $(-7/4, -40.13)$

CD $(-\infty, -7/4)$

CU $(-7/4, \infty)$

15) $y = 3x^4 - 4x^3$

$y' = 12x^3 - 12x^2$

$0 = 12x^2(x-1)$

CP

0

1

– – +

0 1

Dec $(-\infty, 1)$

Inc $(1, \infty)$

local min $(1, -1)$

local max None

$y'' = 36x^2 - 24x$

$0 = 12x(3x-2)$

+ – +

0 2/3

CU $(-\infty, 0)$ $(2/3, \infty)$

CD $(0, 2/3)$

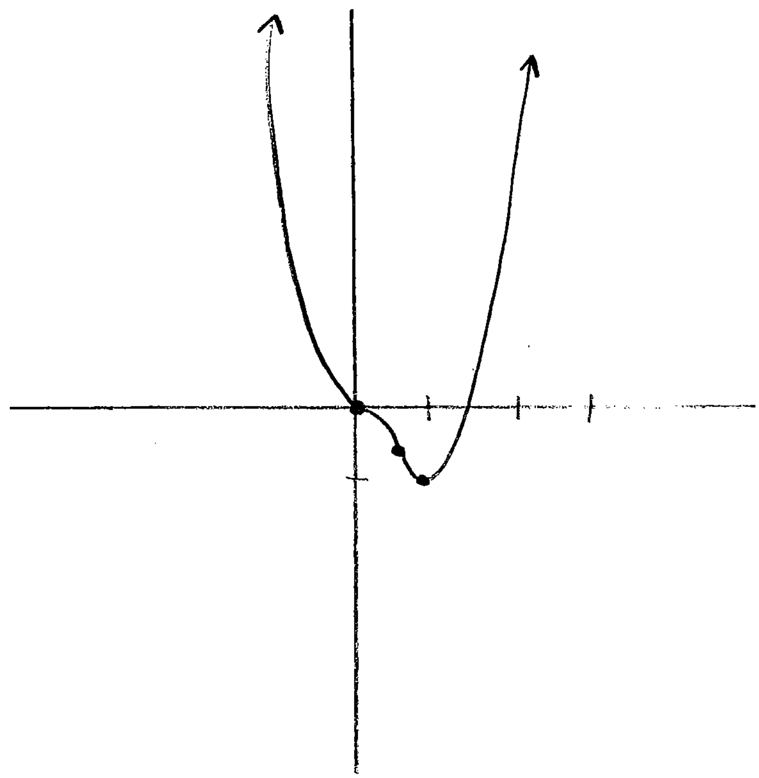
Points of Inflection: $(0,0)$

$(2/3, -.5926)$

x-int: $0 = 3x^4 - 4x^3$

$0 = x^3(3x-4)$

$x = 0 \quad x = 4/3$



16) $f(x) = \frac{x^2-1}{x}$

Domain: $\mathbb{R}$ except $x=0$

y-int: none

x-int: $0 = \frac{x^2-1}{x}$

$0 = x^2-1$

$\boxed{x = 1, -1}$

Asymptotes

$x=0$ vertical

Slant:

$$\begin{array}{r} x \\ x \overline{) x^2+0x-1} \\ \underline{x^2 \phantom{+0x-1}} \\ 0x \phantom{-1} \end{array}$$

$\boxed{y=x}$

$$f'(x) = \frac{x(2x)-(x^2-1)}{x^2} = \frac{2x^2-x^2+1}{x^2} = \frac{x^2+1}{x^2}$$

$+ \quad \cup \quad +$

$0$

Inc $(-\infty, \infty)$

Dec Never

local max & local min : NONE

$$f''(x) = \frac{x^2(2x)-(x^2+1)(2x)}{x^4} = \frac{2x(x^2-x^2-1)}{x^4} = \frac{-2}{x^3}$$

$+ \quad \cup \quad -$

$0$

CU $(-\infty, 0)$

CD $(0, \infty)$

Inflection point: NONE Since it's NOT defined at $x=0$.

16) Graph