PRE-CALCULUS/TRIG 3 Name: KEY Date:

3.2 Logarithmic Functions

Warm-Up

Solving for answer.	Solving for base.	Solving for exponent.
1) $3^4 = x$ 81	3) $\sqrt{x^2} = \sqrt{49}$ $x = \pm 7$	5) $3^x = 729$ $x = 6$
2) $2^5 = x$ 32	4) $x^3 = 125$ $x = 5$	6) $5^x = 3125$ $x = 5$

A logarithm is an Exponent.

Reminder: e ≈ 2.718

- exponential function: $y = a^x$
- inverse of an exponential function: $x = a^y$ } If given an x value how do you solve for "y"

★ Use logs to solve for missing variables → that are EXPONENTS! Examples: $10^x = 150$, $4^x = 23$

y is the **logarithm** of x: $y = \log_a x$ "y = log base a of x"

Exponential **Logarithmic Form:** $a^y = x \iff y = \log_a x$ Logarithmic **:Exponential Form**

**When the base of the exponent is 10, the subscript is not written, they are Common Log

Ex: $\log_{10} 1 = 0$ b/c $10^0 = 1$

$\log_{10} 10{,}000 = 4$ b/c $10^4 = 10{,}000$

$\log_{10} 1 = 0 \rightarrow 10^0 = 1$
$\log_{10} 10 = 1 \rightarrow 10^1 = 10$
$\log_{10} 100 = 2 \rightarrow 10^2 = 100$

**When the base of the exponent is e, the logarithm is called Natural Log and written ln

Ex: $\ln_e 1 = 0$ b/c $e^0 = 1$

$\ln_e e^3 = 3$ b/c $e^3 = e^3$

$\ln_e 1 = 0$ b/c $e^0 = 1$
$\ln_e e = 1$ b/c $e^1 = e$
$\ln_e e^2 = 2$ b/c $e^2 = e^2$

Examples:

Write the following in **exponential** form.

1. $\log_7 2401 = 4$ $7^4 = 2401$
2. $\log_{81} 27 = \frac{3}{4}$ $81^{3/4} = 27$
3. $\ln 10 \approx 2.302$ $e^{2.302} \approx 10$
4. $\ln 20.085 \approx 3$ $e^3 \approx 20.085$
5. $\log_5 \frac{1}{125} = -3$ $5^{-3} = \frac{1}{125}$

Write the following in **logarithmic** form.

1. $2^7 = 128$ $\log_2 128 = 7$
2. $10^6 = 1{,}000{,}000$ $\log_{10} 1{,}000{,}000 = 6$ ✗ base 10 not needed
3. $7^{-2} = \frac{1}{49}$ $\log_7 \frac{1}{49} = -2$
4. $e^4 \approx 54.598$ $\log_e 54.598 = 4$ ↳ base e not needed
5. $5^0 = 1$ $\log_5 1 = 0$

Properties of Logarithms

1. $\log_a 1 = 0$ because $a^0 = 1$
2. $\log_a a = 1$ because $a^1 = a$
3. $\log_a a^x = x$ because $a^x = a^x$

   $a^{\log_a x} = x$ ← Inverse Properties →
4. If $\log_a x = \log_a y$ then $x = y$ ← 1 to 1 Property →

Properties of Natural Logarithms

1. ln 1 = 0 because $e^0 = 1$
2. ln e = 1 because $e^1 = e$
3. ln e^x = x because $e^x = x$

   $e^{\ln x} = x$
4. If ln x = ln y then x = y

Evaluate each expression. Set ea expression = x & solve for x

1) $\log_3 27 = x$

$3^x = 27$

$x = 3$

2) $\log_4 \frac{1}{16} = x$

$4^x = \frac{1}{16}$

$x = -2$

3) $\log_8 8 = x$

$8^x = 8$

$x = 1$

4) $\log_3 3^5 = x$

$3^x = 3^5$

$x = 5$

5) $\log_{10} \frac{1}{10{,}000} = x$

$10^x = 10{,}000$

$x = 4$

6) (solve for b) $\log_b \sqrt{3} = \frac{1}{4}$

$(b^{1/4})^4 = (\sqrt{3})^4$ $(3^{1/2})^4$

$b = 9$

7) $\log_7 \sqrt[4]{7} = x$

$7^x = \sqrt[4]{7}$

$x = 1/4$

Prop #3

8) $4^{\log_4 7} = x$

Exp Form

$\log_4 x = \log_4 7$

$x = 7$

Prop #3

9) $\ln e^{-2} = x$ (e)

$e^x = e^{-2}$

$x = -2$

Prop #3

10) $\ln e^4 = x$ (e)

$e^x = e^4$

$x = 4$

11) $3^{\log_3 (2x-1)} = x$

$\log_3 x = \log_3 (2x-1)$

$2x - 1$

Evaluate with a calculator: log / ln * CANNOT log or ln a neg #

The log button on the calculator will do base 10 logs (common logs).

1. log 13 ≈ 1.11
2. $\log \frac{1}{3} \approx -.48$
3. log -22 ≈ ∅

$10^x = -22$

never = -22!

The ln button on the calculator will do base e log (natural logs).

1. ln 16 ≈ 2.77
2. $\ln \frac{3}{4} \approx -.29$