1.8 Combinations of Functions: Composite Functions

What you should learn

- Add, subtract, multiply, and divide functions.
- Find the composition of one function with another function.
- Use combinations and compositions of functions to model and solve real-life problems.

Why you should learn it

Compositions of functions can be used to model and solve real-life problems. For instance, in Exercise 68 on page 92, compositions of functions are used to determine the price of a new hybrid car.

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Arithmetic Combinations of Functions

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two *functions* can be combined to create new functions. For example, the functions given by $f(x) = 2x - 3$ and $g(x) = x^2 - 1$ can be combined to form the sum, difference, product, and quotient of f and g.

$$f(x) + g(x) = (2x - 3) + (x^2 - 1)$$

$$= x^2 + 2x - 4 \qquad \text{Sum}$$

$$f(x) - g(x) = (2x - 3) - (x^2 - 1)$$

$$= -x^2 + 2x - 2 \qquad \text{Difference}$$

$$f(x)g(x) = (2x - 3)(x^2 - 1)$$

$$= 2x^3 - 3x^2 - 2x + 3 \qquad \text{Product}$$

$$\frac{f(x)}{g(x)} = \frac{2x - 3}{x^2 - 1}, \quad x \neq \pm 1 \qquad \text{Quotient}$$

The domain of an **arithmetic combination** of functions f and g consists of all real numbers that are common to the domains of f and g. In the case of the quotient $f(x)/g(x)$, there is the further restriction that $g(x) \neq 0$.

Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the *sum*, *difference*, *product*, and *quotient* of f and g are defined as follows.

1. *Sum:* $(f + g)(x) = f(x) + g(x)$
2. *Difference:* $(f - g)(x) = f(x) - g(x)$
3. *Product:* $(fg)(x) = f(x) \cdot g(x)$
4. *Quotient:* $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Example 1 Finding the Sum of Two Functions

Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$, find $(f + g)(x)$.

Solution

$$(f + g)(x) = f(x) + g(x) = (2x + 1) + (x^2 + 2x - 1) = x^2 + 4x$$

CHECKPOINT Now try Exercise 5(a).

Additional Examples

a. Given $f(x) = x + 5$ and $g(x) = 3x$, find $(fg)(x)$.

Solution

$$(fg)(x) = f(x) \cdot g(x) = (x + 5)(3x) = 3x^2 + 15x$$

b. Given $f(x) = \frac{1}{x}$ and $g(x) = \frac{x}{x+1}$, find $(gf)(x)$.

Solution

$$(gf)(x) = g(x) \cdot f(x) = \left(\frac{x}{x+1}\right)\left(\frac{1}{x}\right) = \frac{1}{x+1}, \quad x \neq 0$$

Example 2 Finding the Difference of Two Functions

Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$, find $(f - g)(x)$. Then evaluate the difference when $x = 2$.

Solution

The difference of f and g is

$$\begin{aligned}(f - g)(x) &= f(x) - g(x)\\ &= (2x + 1) - (x^2 + 2x - 1)\\ &= -x^2 + 2.\end{aligned}$$

When $x = 2$, the value of this difference is

$$\begin{aligned}(f - g)(2) &= -(2)^2 + 2\\ &= -2.\end{aligned}$$

CHECKPOINT Now try Exercise 5(b).

In Examples 1 and 2, both f and g have domains that consist of all real numbers. So, the domains of $(f + g)$ and $(f - g)$ are also the set of all real numbers. Remember that any restrictions on the domains of f and g must be considered when forming the sum, difference, product, or quotient of f and g.

Example 3 Finding the Domains of Quotients of Functions

Find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{g}{f}\right)(x)$ for the functions given by

$$f(x) = \sqrt{x} \qquad \text{and} \qquad g(x) = \sqrt{4 - x^2}.$$

Then find the domains of f/g and g/f.

Solution

The quotient of f and g is

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{4 - x^2}}$$

and the quotient of g and f is

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{4 - x^2}}{\sqrt{x}}.$$

The domain of f is $[0, \infty)$ and the domain of g is $[-2, 2]$. The intersection of these domains is $[0, 2]$. So, the domains of $\left(\frac{f}{g}\right)$ and $\left(\frac{g}{f}\right)$ are as follows.

$$\text{Domain of } \left(\frac{f}{g}\right): [0, 2) \qquad \text{Domain of } \left(\frac{g}{f}\right): (0, 2]$$

Note that the domain of (f/g) includes $x = 0$, but not $x = 2$, because $x = 2$ yields a zero in the denominator, whereas the domain of (g/f) includes $x = 2$, but not $x = 0$, because $x = 0$ yields a zero in the denominator.

CHECKPOINT Now try Exercise 5(d).

Composition of Functions

Another way of combining two functions is to form the **composition** of one with the other. For instance, if $f(x) = x^2$ and $g(x) = x + 1$, the composition of f with g is

$$\begin{aligned} f(g(x)) &= f(x + 1) \\ &= (x + 1)^2. \end{aligned}$$

This composition is denoted as $(f \circ g)$ and reads as "f composed with g."

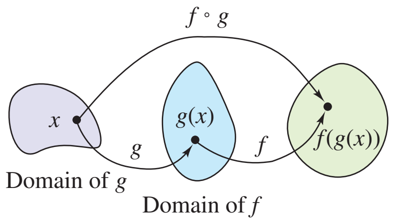


FIGURE 1.90

Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. (See Figure 1.90.)

Example 4 Composition of Functions

Given $f(x) = x + 2$ and $g(x) = 4 - x^2$, find the following.

a. $(f \circ g)(x)$ **b.** $(g \circ f)(x)$ **c.** $(g \circ f)(-2)$

Solution

a. The composition of f with g is as follows.

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) && \text{Definition of } f \circ g \\ &= f(4 - x^2) && \text{Definition of } g(x) \\ &= (4 - x^2) + 2 && \text{Definition of } f(x) \\ &= -x^2 + 6 && \text{Simplify.} \end{aligned}$$

b. The composition of g with f is as follows.

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) && \text{Definition of } g \circ f \\ &= g(x + 2) && \text{Definition of } f(x) \\ &= 4 - (x + 2)^2 && \text{Definition of } g(x) \\ &= 4 - (x^2 + 4x + 4) && \text{Expand.} \\ &= -x^2 - 4x && \text{Simplify.} \end{aligned}$$

Note that, in this case, $(f \circ g)(x) \neq (g \circ f)(x)$.

c. Using the result of part (b), you can write the following.

$$\begin{aligned} (g \circ f)(-2) &= -(-2)^2 - 4(-2) && \text{Substitute.} \\ &= -4 + 8 && \text{Simplify.} \\ &= 4 && \text{Simplify.} \end{aligned}$$

CHECKPOINT Now try Exercise 31.

STUDY TIP

The following tables of values help illustrate the composition $(f \circ g)(x)$ given in Example 4.

x	0	1	2	3
$g(x)$	4	3	0	-5

$g(x)$	4	3	0	-5
$f(g(x))$	6	5	2	-3

x	0	1	2	3
$f(g(x))$	6	5	2	-3

Note that the first two tables can be combined (or "composed") to produce the values given in the third table.

Example 5 Finding the Domain of a Composite Function

Given $f(x) = x^2 - 9$ and $g(x) = \sqrt{9 - x^2}$, find the composition $(f \circ g)(x)$. Then find the domain of $(f \circ g)$.

Solution

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\sqrt{9 - x^2}\right) \\ &= \left(\sqrt{9 - x^2}\right)^2 - 9 \\ &= 9 - x^2 - 9 \\ &= -x^2\end{aligned}$$

From this, it might appear that the domain of the composition is the set of all real numbers. This, however is not true. Because the domain of f is the set of all real numbers and the domain of g is $-3 \le x \le 3$, the domain of $(f \circ g)$ is $-3 \le x \le 3$.

✓CHECKPOINT Now try Exercise 35.

In Examples 4 and 5, you formed the composition of two given functions. In calculus, it is also important to be able to identify two functions that make up a given composite function. For instance, the function h given by

$$h(x) = (3x - 5)^3$$

is the composition of f with g, where $f(x) = x^3$ and $g(x) = 3x - 5$. That is,

$$h(x) = (3x - 5)^3 = [g(x)]^3 = f(g(x)).$$

Basically, to "decompose" a composite function, look for an "inner" function and an "outer" function. In the function h above, $g(x) = 3x - 5$ is the inner function and $f(x) = x^3$ is the outer function.

Example 6 Decomposing a Composite Function

Write the function given by $h(x) = \dfrac{1}{(x - 2)^2}$ as a composition of two functions.

Solution

One way to write h as a composition of two functions is to take the inner function to be $g(x) = x - 2$ and the outer function to be

$$f(x) = \frac{1}{x^2} = x^{-2}.$$

Then you can write

$$h(x) = \frac{1}{(x - 2)^2} = (x - 2)^{-2} = f(x - 2) = f(g(x)).$$

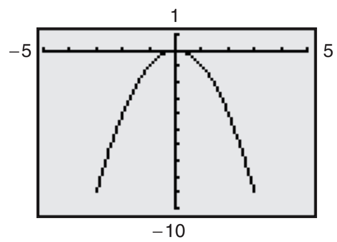
✓CHECKPOINT Now try Exercise 47.

Technology

You can use a graphing utility to determine the domain of a composition of functions. For the composition in Example 5, enter the function composition as

$$y = \left(\sqrt{9 - x^2}\right)^2 - 9.$$

You should obtain the graph shown below. Use the *trace* feature to determine that the x-coordinates of points on the graph extend from -3 to 3. So, the domain of $(f \circ g)(x)$ is $-3 \le x \le 3$.



Activities

1. Given $f(x) = 3x^2 + 2$ and $g(x) = 2x$, find $f \circ g$.
   Answer: $(f \circ g)(x) = 12x^2 + 2$
2. Given the functions
   $f(x) = \dfrac{1}{x - 2}$ and $g(x) = \sqrt{x}$,
   find the composition of f with g. Then find the domain of the composition.
   Answer: $(f \circ g)(x) = \dfrac{1}{\sqrt{x} - 2}$. The domain of $(f \circ g)$ is the set of all nonnegative real numbers except $x = 4$.
3. Find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)
   a. $h(x) = \dfrac{1}{\sqrt{3x + 1}}$
      Answer: $f(x) = \dfrac{1}{\sqrt{x}}$ and $g(x) = 3x + 1$
   b. $h(x) = (2x + 3)^4$
      Answer: $f(x) = x^4$ and $g(x) = 2x + 3$

Application

Example 7 **Bacteria Count**

The number N of bacteria in a refrigerated food is given by

$$N(T) = 20T^2 - 80T + 500, \qquad 2 \le T \le 14$$

where T is the temperature of the food in degrees Celsius. When the food is removed from refrigeration, the temperature of the food is given by

$$T(t) = 4t + 2, \qquad 0 \le t \le 3$$

where t is the time in hours. (a) Find the composition $N(T(t))$ and interpret its meaning in context. (b) Find the time when the bacterial count reaches 2000.

Solution

a.
$$\begin{aligned} N(T(t)) &= 20(4t + 2)^2 - 80(4t + 2) + 500 \\ &= 20(16t^2 + 16t + 4) - 320t - 160 + 500 \\ &= 320t^2 + 320t + 80 - 320t - 160 + 500 \\ &= 320t^2 + 420 \end{aligned}$$

The composite function $N(T(t))$ represents the number of bacteria in the food as a function of the amount of time the food has been out of refrigeration.

b. The bacterial count will reach 2000 when $320t^2 + 420 = 2000$. Solve this equation to find that the count will reach 2000 when $t \approx 2.2$ hours. When you solve this equation, note that the negative value is rejected because it is not in the domain of the composite function.

CHECKPOINT Now try Exercise 65.

Writing About Mathematics

To expand on this activity, you might consider asking your students to use the tables they created in parts (a) and (b), along with a table of values for x and $f(x)$, to demonstrate and explain how the tables can be manipulated to yield tables of values for $h(x)$ and $g(x)$.

WRITING ABOUT MATHEMATICS

Analyzing Arithmetic Combinations of Functions

a. Use the graphs of f and $(f + g)$ in Figure 1.91 to make a table showing the values of $g(x)$ when $x = 1, 2, 3, 4, 5,$ and 6. Explain your reasoning.

b. Use the graphs of f and $(f - h)$ in Figure 1.91 to make a table showing the values of $h(x)$ when $x = 1, 2, 3, 4, 5,$ and 6. Explain your reasoning.

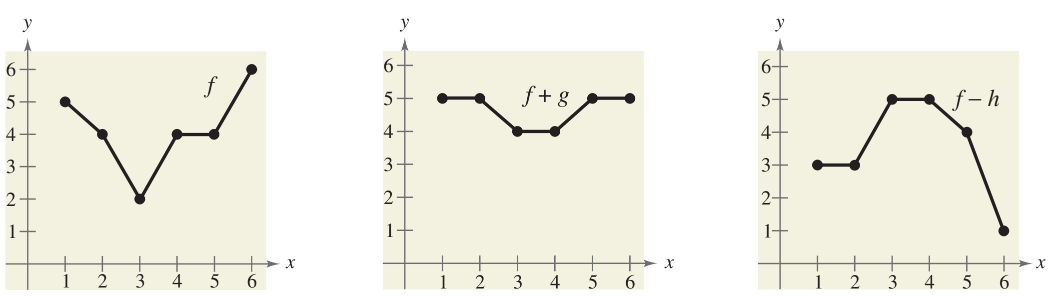


FIGURE 1.91

1.8 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. Two functions f and g can be combined by the arithmetic operations of ________, ________, ________, and ________ to create new functions.

2. The ________ of the function f with g is $(f \circ g)(x) = f(g(x))$.

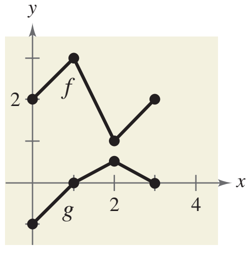
3. The domain of $(f \circ g)$ is all x in the domain of g such that _______ is in the domain of f.

4. To decompose a composite function, look for an ________ function and an ________ function.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, use the graphs of f and g to graph $h(x) = (f + g)(x)$. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

1.



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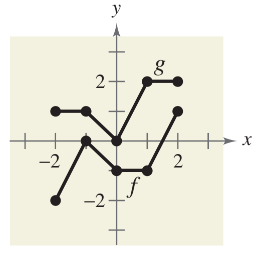

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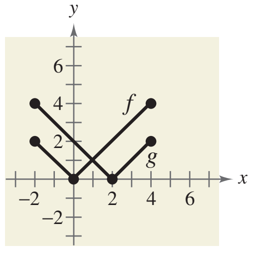

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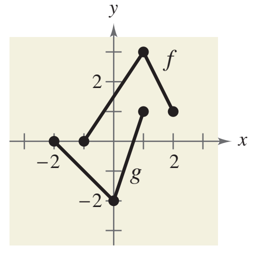


In Exercises 5–12, find (a) $(f + g)(x)$, (b) $(f - g)(x)$, (c) $(fg)(x)$, and (d) $(f/g)(x)$. What is the domain of f/g?

5. $f(x) = x + 2,\quad g(x) = x - 2$

6. $f(x) = 2x - 5,\quad g(x) = 2 - x$

7. $f(x) = x^2,\quad g(x) = 4x - 5$

8. $f(x) = 2x - 5,\quad g(x) = 4$

9. $f(x) = x^2 + 6,\quad g(x) = \sqrt{1 - x}$

10. $f(x) = \sqrt{x^2 - 4},\quad g(x) = \dfrac{x^2}{x^2 + 1}$

11. $f(x) = \dfrac{1}{x},\quad g(x) = \dfrac{1}{x^2}$

12. $f(x) = \dfrac{x}{x + 1},\quad g(x) = x^3$

In Exercises 13–24, evaluate the indicated function for $f(x) = x^2 + 1$ and $g(x) = x - 4$.

13. $(f + g)(2)$ **14.** $(f - g)(-1)$

15. $(f - g)(0)$ **16.** $(f + g)(1)$

17. $(f - g)(3t)$ **18.** $(f + g)(t - 2)$

19. $(fg)(6)$ **20.** $(fg)(-6)$

21. $\left(\dfrac{f}{g}\right)(5)$ **22.** $\left(\dfrac{f}{g}\right)(0)$

23. $\left(\dfrac{f}{g}\right)(-1) - g(3)$ **24.** $(fg)(5) + f(4)$

In Exercises 25–28, graph the functions f, g, and $f + g$ on the same set of coordinate axes.

25. $f(x) = \frac{1}{2}x,\quad g(x) = x - 1$

26. $f(x) = \frac{1}{3}x,\quad g(x) = -x + 4$

27. $f(x) = x^2,\quad g(x) = -2x$

28. $f(x) = 4 - x^2,\quad g(x) = x$

Graphical Reasoning **In Exercises 29 and 30, use a graphing utility to graph f, g, and $f + g$ in the same viewing window. Which function contributes most to the magnitude of the sum when $0 \le x \le 2$? Which function contributes most to the magnitude of the sum when $x > 6$?**

29. $f(x) = 3x,\quad g(x) = -\dfrac{x^3}{10}$

30. $f(x) = \dfrac{x}{2},\quad g(x) = \sqrt{x}$

In Exercises 31–34, find (a) $f \circ g$, (b) $g \circ f$, and (c) $f \circ f$.

31. $f(x) = x^2,\quad g(x) = x - 1$

32. $f(x) = 3x + 5,\quad g(x) = 5 - x$

33. $f(x) = \sqrt[3]{x - 1},\quad g(x) = x^3 + 1$

34. $f(x) = x^3,\quad g(x) = \dfrac{1}{x}$

In Exercises 35–42, find (a) $f \circ g$ and (b) $g \circ f$. Find the domain of each function and each composite function.

35. $f(x) = \sqrt{x + 4}$, $g(x) = x^2$

36. $f(x) = \sqrt[3]{x - 5}$, $g(x) = x^3 + 1$

37. $f(x) = x^2 + 1$, $g(x) = \sqrt{x}$

38. $f(x) = x^{2/3}$, $g(x) = x^6$

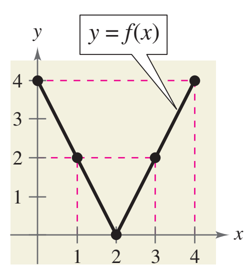
39. $f(x) = |x|$, $g(x) = x + 6$

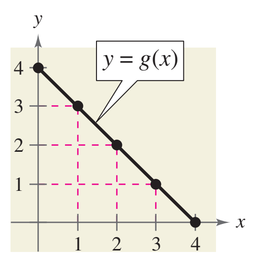
40. $f(x) = |x - 4|$, $g(x) = 3 - x$

41. $f(x) = \dfrac{1}{x}$, $g(x) = x + 3$

42. $f(x) = \dfrac{3}{x^2 - 1}$, $g(x) = x + 1$

In Exercises 43–46, use the graphs of f and g to evaluate the functions.





43. (a) $(f + g)(3)$ (b) $(f/g)(2)$

44. (a) $(f - g)(1)$ (b) $(fg)(4)$

45. (a) $(f \circ g)(2)$ (b) $(g \circ f)(2)$

46. (a) $(f \circ g)(1)$ (b) $(g \circ f)(3)$

In Exercises 47–54, find two functions f and g such that $(f \circ g)(x) = h(x)$. (There are many correct answers.)

47. $h(x) = (2x + 1)^2$

48. $h(x) = (1 - x)^3$

49. $h(x) = \sqrt[3]{x^2 - 4}$

50. $h(x) = \sqrt{9 - x}$

51. $h(x) = \dfrac{1}{x + 2}$

52. $h(x) = \dfrac{4}{(5x + 2)^2}$

53. $h(x) = \dfrac{-x^2 + 3}{4 - x^2}$

54. $h(x) = \dfrac{27x^3 + 6x}{10 - 27x^3}$

55. ***Stopping Distance*** The research and development department of an automobile manufacturer has determined that when a driver is required to stop quickly to avoid an accident, the distance (in feet) the car travels during the driver's reaction time is given by $R(x) = \frac{3}{4}x$, where x is the speed of the car in miles per hour. The distance (in feet) traveled while the driver is braking is given by $B(x) = \frac{1}{15}x^2$. Find the function that represents the total stopping distance T. Graph the functions R, B, and T on the same set of coordinate axes for $0 \le x \le 60$.

56. ***Sales*** From 2000 to 2005, the sales R_1 (in thousands of dollars) for one of two restaurants owned by the same parent company can be modeled by

$$R_1 = 480 - 8t - 0.8t^2, \qquad t = 0, 1, 2, 3, 4, 5$$

where $t = 0$ represents 2000. During the same six-year period, the sales R_2 (in thousands of dollars) for the second restaurant can be modeled by

$$R_2 = 254 + 0.78t, \qquad t = 0, 1, 2, 3, 4, 5.$$

(a) Write a function R_3 that represents the total sales of the two restaurants owned by the same parent company.

(b) Use a graphing utility to graph R_1, R_2, and R_3 in the same viewing window.

57. ***Vital Statistics*** Let $b(t)$ be the number of births in the United States in year t, and let $d(t)$ represent the number of deaths in the United States in year t, where $t = 0$ corresponds to 2000.

(a) If $p(t)$ is the population of the United States in year t, find the function $c(t)$ that represents the percent change in the population of the United States.

(b) Interpret the value of $c(5)$.

58. ***Pets*** Let $d(t)$ be the number of dogs in the United States in year t, and let $c(t)$ be the number of cats in the United States in year t, where $t = 0$ corresponds to 2000.

(a) Find the function $p(t)$ that represents the total number of dogs and cats in the United States.

(b) Interpret the value of $p(5)$.

(c) Let $n(t)$ represent the population of the United States in year t, where $t = 0$ corresponds to 2000. Find and interpret

$$h(t) = \frac{p(t)}{n(t)}.$$

59. ***Military Personnel*** The total numbers of Army personnel (in thousands) A and Navy personnel (in thousands) N from 1990 to 2002 can be approximated by the models

$$A(t) = 3.36t^2 - 59.8t + 735$$

and

$$N(t) = 1.95t^2 - 42.2t + 603$$

where t represents the year, with $t = 0$ corresponding to 1990. (Source: Department of Defense)

(a) Find and interpret $(A + N)(t)$. Evaluate this function for $t = 4$, 8, and 12.

(b) Find and interpret $(A - N)(t)$. Evaluate this function for $t = 4$, 8, and 12.

60. ***Sales*** The sales of exercise equipment E (in millions of dollars) in the United States from 1997 to 2003 can be approximated by the function

$E(t) = 25.95t^2 - 231.2t + 3356$

and the U.S. population P (in millions) from 1997 to 2003 can be approximated by the function

$P(t) = 3.02t + 252.0$

where t represents the year, with $t = 7$ corresponding to 1997. (Source: National Sporting Goods Association, U.S. Census Bureau)

(a) Find and interpret $h(t) = \dfrac{E(t)}{P(t)}$.

(b) Evaluate the function in part (a) for $t = 7$, 10, and 12.

Model It

61. ***Health Care Costs*** The table shows the total amounts (in billions of dollars) spent on health services and supplies in the United States (including Puerto Rico) for the years 1995 through 2001. The variables y_1, y_2, and y_3 represent out-of-pocket payments, insurance premiums, and other types of payments, respectively. (Source: Centers for Medicare and Medicaid Services)

Year	y_1	y_2	y_3
1995	146.2	329.1	44.8
1996	152.0	344.1	48.1
1997	162.2	359.9	52.1
1998	175.2	382.0	55.6
1999	184.4	412.1	57.8
2000	194.7	449.0	57.4
2001	205.5	496.1	57.8

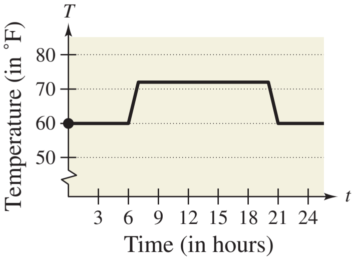
(a) Use the *regression* feature of a graphing utility to find a linear model for y_1 and quadratic models for y_2 and y_3. Let $t = 5$ represent 1995.

(b) Find $y_1 + y_2 + y_3$. What does this sum represent?

(c) Use a graphing utility to graph y_1, y_2, y_3, and $y_1 + y_2 + y_3$ in the same viewing window.

(d) Use the model from part (b) to estimate the total amounts spent on health services and supplies in the years 2008 and 2010.

62. ***Graphical Reasoning*** An electronically controlled thermostat in a home is programmed to lower the temperature automatically during the night. The temperature in the house T (in degrees Fahrenheit) is given in terms of t, the time in hours on a 24-hour clock (see figure).



(a) Explain why T is a function of t.

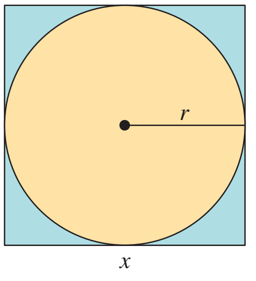
(b) Approximate $T(4)$ and $T(15)$.

(c) The thermostat is reprogrammed to produce a temperature H for which $H(t) = T(t - 1)$. How does this change the temperature?

(d) The thermostat is reprogrammed to produce a temperature H for which $H(t) = T(t) - 1$. How does this change the temperature?

(e) Write a piecewise-defined function that represents the graph.

63. ***Geometry*** A square concrete foundation is prepared as a base for a cylindrical tank (see figure).

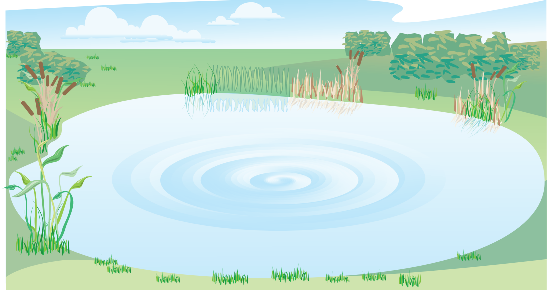


(a) Write the radius r of the tank as a function of the length x of the sides of the square.

(b) Write the area A of the circular base of the tank as a function of the radius r.

(c) Find and interpret $(A \circ r)(x)$.

64. ***Physics*** A pebble is dropped into a calm pond, causing ripples in the form of concentric circles (see figure). The radius r (in feet) of the outer ripple is $r(t) = 0.6t$, where t is the time in seconds after the pebble strikes the water. The area A of the circle is given by the function $A(r) = \pi r^2$. Find and interpret $(A \circ r)(t)$.



65. ***Bacteria Count*** The number N of bacteria in a refrigerated food is given by

$N(T) = 10T^2 - 20T + 600, \quad 1 \le T \le 20$

where T is the temperature of the food in degrees Celsius. When the food is removed from refrigeration, the temperature of the food is given by

$T(t) = 3t + 2, \quad 0 \le t \le 6$

where t is the time in hours.

(a) Find the composition $N(T(t))$ and interpret its meaning in context.

(b) Find the time when the bacterial count reaches 1500.

66. ***Cost*** The weekly cost C of producing x units in a manufacturing process is given by

$C(x) = 60x + 750.$

The number of units x produced in t hours is given by

$x(t) = 50t.$

(a) Find and interpret $(C \circ x)(t)$.

(b) Find the time that must elapse in order for the cost to increase to $15,000.

67. ***Salary*** You are a sales representative for a clothing manufacturer. You are paid an annual salary, plus a bonus of 3% of your sales over $500,000. Consider the two functions given by

$f(x) = x - 500{,}000 \qquad \text{and} \qquad g(x) = 0.03x.$

If x is greater than $500,000, which of the following represents your bonus? Explain your reasoning.

(a) $f(g(x))$ (b) $g(f(x))$

68. ***Consumer Awareness*** The suggested retail price of a new hybrid car is p dollars. The dealership advertises a factory rebate of $2000 and a 10% discount.

(a) Write a function R in terms of p giving the cost of the hybrid car after receiving the rebate from the factory.

(b) Write a function S in terms of p giving the cost of the hybrid car after receiving the dealership discount.

(c) Form the composite functions $(R \circ S)(p)$ and $(S \circ R)(p)$ and interpret each.

(d) Find $(R \circ S)(20{,}500)$ and $(S \circ R)(20{,}500)$. Which yields the lower cost for the hybrid car? Explain.

Synthesis

True or False? **In Exercises 69 and 70, determine whether the statement is true or false. Justify your answer.**

69. If $f(x) = x + 1$ and $g(x) = 6x$, then

$(f \circ g)(x) = (g \circ f)(x).$

70. If you are given two functions $f(x)$ and $g(x)$, you can calculate $(f \circ g)(x)$ if and only if the range of g is a subset of the domain of f.

71. ***Proof*** Prove that the product of two odd functions is an even function, and that the product of two even functions is an even function.

72. ***Conjecture*** Use examples to hypothesize whether the product of an odd function and an even function is even or odd. Then prove your hypothesis.

Skills Review

Average Rate of Change **In Exercises 73–76, find the difference quotient**

$$\frac{f(x + h) - f(x)}{h}$$

and simplify your answer.

73. $f(x) = 3x - 4$

74. $f(x) = 1 - x^2$

75. $f(x) = \dfrac{4}{x}$

76. $f(x) = \sqrt{2x + 1}$

In Exercises 77–80, find an equation of the line that passes through the given point and has the indicated slope. Sketch the line.

77. $(2, -4), m = 3$

78. $(-6, 3), m = -1$

79. $(8, -1), m = -\frac{3}{2}$

80. $(7, 0), m = \frac{5}{7}$