CALCULUS 1-NOTES Name: ______________________________

<u>OPTIMIZATION 4.5-1</u> Date: ________________ Block: _____

1. Find two positive numbers whose sum is 20 and whose product is as large as possible.

Two positive numbers whose sum is 20

$x + y = 20$ (write an equation)

$y = 20 - x$ (solve for x or y) $\longrightarrow$

whose product is as large as possible

$f(x) = xy$ (define a function)

$f(x) = x(20 - x)$ (replace so have one variable)

$f(x) = 20x - x^2$ (simplify equation)

Maximize (derivative)

$f'(x) = 20 - 2x$ (take derivative)

$0 = 20 - 2x$ (set derivative = 0)

$2x = 20$

$x = 10$ (find critical points)

$+ \mid -$ at 10

(check to make sure critical point is a max)

$y = 20 - x$ (find other number)

$= 20 - 10 = 10$

2. The product of two numbers is 210 and the sum of the first plus three times the second yields a minimum sum.

The product of two numbers is 210

$x\,y = 210$ (write an equation)

$y = \frac{210}{x}$ (solve for x or y) $\longrightarrow$

the sum of the first plus three times the second yields a minimum sum

$f(x) = x+3y$ (define a function)

$f(x) = x+3\left(\frac{210}{x}\right)$ (replace so have one variable)

$f(x) = x + 630x^{-1}$ (simplify equation)

Minimize (derivative)

$f'(x) = 1 - 630x^{-2}$ (take derivative)

$0 = 1 - 630x^{-2}$ (set derivative = 0)

$\frac{630}{x^2} = 1$

$x^2 = 630$ (find critical points)

$x = \sqrt{630} = 3\sqrt{70}$

$- \mid +$ at $3\sqrt{70}$

(check to make sure critical point is a min)

$y = \frac{210}{x}$ (find other number)

$= \frac{210}{3\sqrt{70}} = \frac{70}{\sqrt{70}} = \frac{70\sqrt{70}}{70} = \sqrt{70}$

3. The area of a rectangle is 100 m². Find the dimensions if the perimeter is to be a minimum.

$A = \text{length} \bullet \text{width}$

$100 = xy$

$y = \frac{100}{x}$

$P = 2(\text{length}) + 2(\text{width})$

$P = 2x + 2y$

$P = 2x + 2\left(\frac{100}{x}\right)$

$P = 2x + 200x^{-1}$

$P' = 2 - 200x^{-2}$

$0 = 2 - 200x^{-2}$

$\frac{200}{x^2} = 2$

$x^2 = 100$

$x = \sqrt{100} = 10$

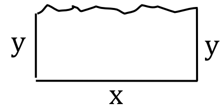
$- \mid +$ at 10

(check to make sure critical point is a min)

$y = \frac{100}{x}$ (find other number)

$= \frac{100}{10} = 10$

4. A family has 2400 feet of fencing to close off a rectangular field that borders a river. What dimensions would yield the maximum area?



$P = x + 2y$

$2400 = x + 2y$

$2400 - 2y = x$

$A(x) = xy$

$A(x) = (2400 - 2y)y$

$A(x) = 2400y - 2y^2$

$A'(x) = 2400 - 4y$

$0 = 2400 - 4y$

$4y = 2400$

$y = 600$

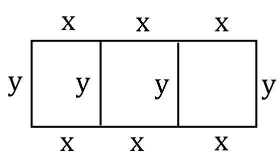
$+ \quad -$

600

$x = 2400 - 2(600) = 1200$

The dimensions are 600 ft by 1200 ft. The maximum area would be 720,000 ft^2.

5. Fence off 3 adjoining rectangular pens with equal areas. There is 500 feet of fencing. Find the dimensions of the entire enclosure to maximize the area.



$P = 6x + 4y$

$500 = 6x + 4y$

$500 - 6x = 4y$

$125 - \frac{3}{2}x = y$

$A(x) = xy$

$A(x) = x(125 - \frac{3}{2}x)$

$A(x) = 125x - \frac{3}{2}x^2$

$A'(x) = 125 - 3x$

$0 = 125 - 3x$

$3x = 125$

$x = \frac{125}{3}$

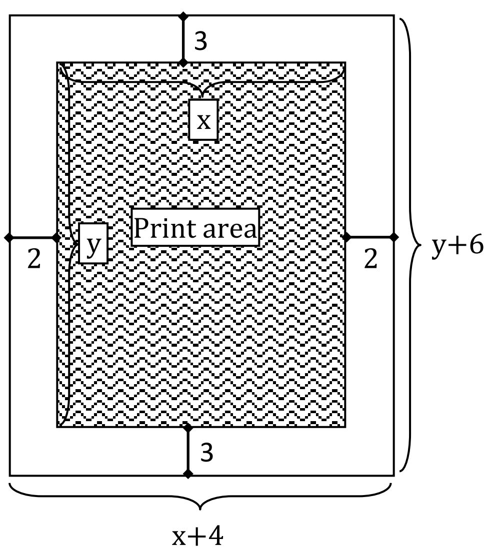
$+ \quad -$

$\frac{125}{3}$

$y = 125 - \frac{3}{2}\left(\frac{125}{3}\right) = \frac{125}{2} = 62.5$ ft.

The dimensions of each pen is $\frac{125}{3}$ ft by $\frac{125}{2}$ ft. The dimension of the <u>entire enclosure</u> would be 125 ft. by 62.5 ft (3x by y). The maximum area would be 7812.5 ft^2.

7. A poster is to contain a printed area of 150 in^2, with clear margins of 3 inches on the top and bottom and 2 inches on each side. What overall dimensions would minimize the paper used for the poster?



$- \quad +$

15

Printed Area

$150 = xy$

$x = \frac{150}{y}$

Paper Area

$A(x) = (x + 4)(y + 6)$

$A(x) = (\frac{150}{y} + 4)(y + 6)$

$A(x) = (150y^{-1} + 4)(y + 6)$

$A(x) = 150 + 900y^{-1} + 4y + 24$

$A'(x) = -900y^{-2} + 4$

$0 = -\frac{900}{y^2} + 4$

$-4 = -\frac{900}{y^2}$

$-4y^2 = -900$

$y^2 = \frac{900}{4}$

$y = 15$

$x = \frac{150}{15} = 10$

Entire dimensions:

$y + 6 = 15 + 6 = 21$ in.

$x + 4 = 10 + 4 = 14$ in