4.5 Graphs of Sine and Cosine Functions

What you should learn
• Sketch the graphs of basic sine and cosine functions.
• Use amplitude and period to help sketch the graphs of sine and cosine functions.
• Sketch translations of the graphs of sine and cosine functions.
• Use sine and cosine functions to model real-life data.

Why you should learn it
Sine and cosine functions are often used in scientific calculations. For instance, in Exercise 73 on page 330, you can use a trigonometric function to model the airflow of your respiratory cycle.

Basic Sine and Cosine Curves
In this section, you will study techniques for sketching the graphs of the sine and cosine functions. The graph of the sine function is a **sine curve**. In Figure 4.47, the black portion of the graph represents one period of the function and is called **one cycle** of the sine curve. The gray portion of the graph indicates that the basic sine curve repeats indefinitely in the positive and negative directions. The graph of the cosine function is shown in Figure 4.48.

Recall from Section 4.2 that the domain of the sine and cosine functions is the set of all real numbers. Moreover, the range of each function is the interval $[-1, 1]$, and each function has a period of $2\pi$. Do you see how this information is consistent with the basic graphs shown in Figures 4.47 and 4.48?

Note in Figures 4.47 and 4.48 that the sine curve is symmetric with respect to the **origin**, whereas the cosine curve is symmetric with respect to the **$y$-axis**. These properties of symmetry follow from the fact that the sine function is odd and the cosine function is even.
To sketch the graphs of the basic sine and cosine functions by hand, it helps to note five key points in one period of each graph: the intercepts, maximum points, and minimum points (see Figure 4.49).

Example 1 Using Key Points to Sketch a Sine Curve

Sketch the graph of \( y = 2 \sin x \) on the interval \([−\pi, 4\pi]\).

**Solution**

Note that

\[
y = 2 \sin x = 2(\sin x)
\]

indicates that the \(y\)-values for the key points will have twice the magnitude of those on the graph of \( y = \sin x \). Divide the period \(2\pi\) into four equal parts to get the key points for \( y = 2 \sin x \).

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Maximum</th>
<th>Intercept</th>
<th>Minimum</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0))</td>
<td>(\left(\frac{\pi}{2}, 2\right))</td>
<td>(\pi, 0)</td>
<td>(\left(\frac{3\pi}{2}, -2\right))</td>
<td>(2\pi, 0)</td>
</tr>
</tbody>
</table>

By connecting these key points with a smooth curve and extending the curve in both directions over the interval \([−\pi, 4\pi]\), you obtain the graph shown in Figure 4.50.

**Technology**

When using a graphing utility to graph trigonometric functions, pay special attention to the viewing window you use. For instance, try graphing \( y = [\sin(10x)]/10 \) in the standard viewing window in radian mode. What do you observe? Use the zoom feature to find a viewing window that displays a good view of the graph.

**Checkpoint**

Now try Exercise 35.
Amplitude and Period

In the remainder of this section you will study the graphic effect of each of the constants \( a, b, c, \) and \( d \) in equations of the forms

\[
y = d + a \sin(bx - c)
\]

and

\[
y = d + a \cos(bx - c).
\]

A quick review of the transformations you studied in Section 1.7 should help in this investigation.

The constant factor \( a \) in \( y = a \sin x \) acts as a scaling factor—a vertical stretch or vertical shrink of the basic sine curve. If \( |a| > 1 \), the basic sine curve is stretched, and if \( |a| < 1 \), the basic sine curve is shrunk. The result is that the graph of \( y = a \sin x \) ranges between \( -a \) and \( a \) instead of between \(-1\) and \(1\). The absolute value of \( a \) is the amplitude of the function \( y = a \sin x \). The range of the function \( y = a \sin x \) for \( a > 0 \) is \(-a \leq y \leq a\).

**Definition of Amplitude of Sine and Cosine Curves**

The amplitude of \( y = a \sin x \) and \( y = a \cos x \) represents half the distance between the maximum and minimum values of the function and is given by

\[
\text{Amplitude} = |a|.
\]

**Example 2**  Scaling: Vertical Shrinking and Stretching

On the same coordinate axes, sketch the graph of each function.

- **a.** \( y = \frac{1}{2} \cos x \)
- **b.** \( y = 3 \cos x \)

**Solution**

**a.** Because the amplitude of \( y = \frac{1}{2} \cos x \) is \( \frac{1}{2} \), the maximum value is \( \frac{1}{2} \) and the minimum value is \( -\frac{1}{2} \). Divide one cycle, \( 0 \leq x \leq 2\pi \), into four equal parts to get the key points

\[
\begin{align*}
\text{Maximum} & : \left(0, \frac{1}{2}\right), \quad \left(\frac{\pi}{2}, 0\right), \quad \left(\pi, -\frac{1}{2}\right), \quad \left(\frac{3\pi}{2}, 0\right), \quad \text{and} \quad \left(2\pi, \frac{1}{2}\right)
\end{align*}
\]

**b.** A similar analysis shows that the amplitude of \( y = 3 \cos x \) is 3, and the key points are

\[
\begin{align*}
\text{Maximum} & : \left(0, 3\right), \quad \left(\frac{\pi}{2}, 0\right), \quad \left(\pi, -3\right), \quad \left(\frac{3\pi}{2}, 0\right), \quad \text{and} \quad \left(2\pi, 3\right).
\end{align*}
\]

The graphs of these two functions are shown in Figure 4.51. Notice that the graph of \( y = \frac{1}{2} \cos x \) is a vertical shrink of the graph of \( y = \cos x \) and the graph of \( y = 3 \cos x \) is a vertical stretch of the graph of \( y = \cos x \).

**Checkpoint**  Now try Exercise 37.
You know from Section 1.7 that the graph of \( y = -f(x) \) is a reflection in the \( x \)-axis of the graph of \( y = f(x) \). For instance, the graph of \( y = -3 \cos x \) is a reflection of the graph of \( y = 3 \cos x \), as shown in Figure 4.52.

Because \( y = a \sin x \) completes one cycle from \( x = 0 \) to \( x = 2\pi \), it follows that \( y = a \sin bx \) completes one cycle from \( x = 0 \) to \( x = 2\pi/b \).

**Period of Sine and Cosine Functions**

Let \( b \) be a positive real number. The period of \( y = a \sin bx \) and \( y = a \cos bx \) is given by

\[
\text{Period} = \frac{2\pi}{b}.
\]

Note that if \( 0 < b < 1 \), the period of \( y = a \sin bx \) is greater than \( 2\pi \) and represents a horizontal stretching of the graph of \( y = a \sin x \). Similarly, if \( b > 1 \), the period of \( y = a \sin bx \) is less than \( 2\pi \) and represents a horizontal shrinking of the graph of \( y = a \sin x \). If \( b \) is negative, the identities \( \sin(-x) = -\sin x \) and \( \cos(-x) = \cos x \) are used to rewrite the function.

**Example 3** Scaling: Horizontal Stretching

Sketch the graph of \( y = \sin \frac{x}{2} \).

**Solution**

The amplitude is 1. Moreover, because \( b = \frac{1}{2} \), the period is

\[
\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi.
\]

Substitute for \( b \).

Now, divide the period-interval \([0, 4\pi]\) into four equal parts with the values \( \pi, 2\pi, \) and \( 3\pi \) to obtain the key points on the graph.

- **Intercept**: \( (0, 0) \)
- **Maximum**: \((\pi, 1)\)
- **Intercept**: \((2\pi, 0)\)
- **Minimum**: \((3\pi, -1)\)
- **Intercept**: \((4\pi, 0)\)

The graph is shown in Figure 4.53.

**STUDY TIP**

In general, to divide a period-interval into four equal parts, successively add “period/4,” starting with the left endpoint of the interval. For instance, for the period-interval \([-\pi/6, \pi/2]\) of length \(2\pi/3\), you would successively add

\[
\frac{2\pi/3}{4} = \frac{\pi}{6}
\]

to get \(-\pi/6, 0, \pi/6, \pi/3,\) and \(\pi/2\) as the \(x\)-values for the key points on the graph.

**CHECKPOINT** Now try Exercise 39.
Translations of Sine and Cosine Curves

The constant $c$ in the general equations

$$y = a \sin(bx - c) \quad \text{and} \quad y = a \cos(bx - c)$$

creates a horizontal translation (shift) of the basic sine and cosine curves. Comparing $y = a \sin bx$ with $y = a \sin(bx - c)$, you find that the graph of $y = a \sin(bx - c)$ completes one cycle from $bx - c = 0$ to $bx - c = 2\pi$. By solving for $x$, you can find the interval for one cycle to be

$$\frac{c}{b} \leq x \leq \frac{c + 2\pi}{b}.$$  

This implies that the period of $y = a \sin(bx - c)$ is $2\pi/b$, and the graph of $y = a \sin bx$ is shifted by an amount $c/b$. The number $c/b$ is the phase shift.

Graphs of Sine and Cosine Functions

The graphs of $y = a \sin(bx - c)$ and $y = a \cos(bx - c)$ have the following characteristics. (Assume $b > 0$.)

- **Amplitude** = $|a|
- **Period** = $\frac{2\pi}{b}$

The left and right endpoints of a one-cycle interval can be determined by solving the equations $bx - c = 0$ and $bx - c = 2\pi$.

Example 4

**Horizontal Translation**

Sketch the graph of $y = \frac{1}{2} \sin \left(x - \frac{\pi}{3}\right)$.

**Solution**

The amplitude is $\frac{1}{2}$ and the period is $2\pi$. By solving the equations

$$x - \frac{\pi}{3} = 0 \quad \text{and} \quad x = \frac{\pi}{3}$$

and

$$x - \frac{\pi}{3} = 2\pi \quad \text{and} \quad x = \frac{7\pi}{3}$$

you see that the interval $[\pi/3, 7\pi/3]$ corresponds to one cycle of the graph. Dividing this interval into four equal parts produces the key points

- **Intercept** $\left(\frac{\pi}{3}, 0\right)$
- **Maximum** $\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
- **Intercept** $\left(\frac{4\pi}{3}, 0\right)$
- **Minimum** $\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$

The graph is shown in Figure 4.54.

Now try Exercise 45.
Sketch the graph of
\[ y = -3 \cos(2\pi x + 4\pi). \]

**Solution**
The amplitude is 3 and the period is \( \frac{2\pi}{2\pi} = 1 \). By solving the equations
\[
2\pi x + 4\pi = 0 \\
2\pi x = -4\pi \\
x = -2
\]
and
\[
2\pi x + 4\pi = 2\pi \\
2\pi x = -2\pi \\
x = -1
\]
you see that the interval \([-2, -1]\) corresponds to one cycle of the graph. Dividing this interval into four equal parts produces the key points

\[
\begin{array}{c|c|c|c|c}
\text{Minimum} & \text{Intercept} & \text{Maximum} & \text{Intercept} & \text{Minimum} \\
(-2, -3) & \left(-\frac{7}{4}, 0\right) & \left(-\frac{3}{2}, 3\right) & \left(-\frac{5}{4}, 0\right) & (-1, -3).
\end{array}
\]

The graph is shown in Figure 4.55.

**Example 6**  **Vertical Translation**

Sketch the graph of
\[ y = 2 + 3 \cos 2x. \]

**Solution**
The amplitude is 3 and the period is \( \pi \). The key points over the interval \([0, \pi]\) are
\[
(0, 5), \left(\frac{\pi}{4}, 2\right), \left(\frac{\pi}{2}, -1\right), \left(\frac{3\pi}{4}, 2\right), \text{ and } (\pi, 5).
\]
The graph is shown in Figure 4.56. Compared with the graph of \( f(x) = 3 \cos 2x \), the graph of \( y = 2 + 3 \cos 2x \) is shifted upward two units.

**Activities**
1. Describe the relationship between the graphs of \( f(x) = \sin x \) and \( g(x) = 3 \sin(2x + 1) \).
   **Answer:** The amplitude of the basic sine curve is 1, whereas the amplitude of \( g \) is 3. The period of the basic sine curve is \( 2\pi \), whereas the period of \( g \) is \( \pi \). Lastly, the graph of \( g \) has a phase shift \( \frac{1}{2} \) unit to the left of the graph of \( f(x) = \sin x \).
2. Determine the amplitude, period, and phase shift of
   \[ y = \frac{1}{2} \cos(\pi x - 1). \]
   **Answer:** Amplitude: \( \frac{1}{2} \); period: 2; phase shift: \( \frac{1}{\pi} \).
Mathematical Modeling

Sine and cosine functions can be used to model many real-life situations, including electric currents, musical tones, radio waves, tides, and weather patterns.

Example 7  Finding a Trigonometric Model

Throughout the day, the depth of water at the end of a dock in Bar Harbor, Maine varies with the tides. The table shows the depths (in feet) at various times during the morning. (Source: Nautical Software, Inc.)

a. Use a trigonometric function to model the data.

b. Find the depths at 9 A.M. and 3 P.M.

c. A boat needs at least 10 feet of water to moor at the dock. During what times in the afternoon can it safely dock?

Solution

a. Begin by graphing the data, as shown in Figure 4.57. You can use either a sine or cosine model. Suppose you use a cosine model of the form

\[ y = a \cos(bt - c) + d. \]

The difference between the maximum height and the minimum height of the graph is twice the amplitude of the function. So, the amplitude is

\[ a = \frac{1}{2}[(\text{maximum depth}) - (\text{minimum depth})] = \frac{1}{2}(11.3 - 0.1) = 5.6. \]

The cosine function completes one half of a cycle between the times at which the maximum and minimum depths occur. So, the period is

\[ p = 2[(\text{time of min. depth}) - (\text{time of max. depth})] = 2(10 - 4) = 12 \]

which implies that \( b = \frac{2\pi}{p} \approx 0.524. \) Because high tide occurs 4 hours after midnight, consider the left endpoint to be \( c/b = 4, \) so \( c \approx 2.094. \) Moreover, because the average depth is \( \frac{1}{2}((11.3 + 0.1) = 5.7), \) it follows that \( d = 5.7. \) So, you can model the depth with the function given by

\[ y = 5.6 \cos(0.524t - 2.094) + 5.7. \]

b. The depths at 9 A.M. and 3 P.M. are as follows.

\[ y = 5.6 \cos(0.524 \cdot 9 - 2.094) + 5.7 \approx 0.84 \text{ foot} \quad (9 \text{ A.M.}) \]

\[ y = 5.6 \cos(0.524 \cdot 15 - 2.094) + 5.7 \approx 10.57 \text{ feet} \quad (3 \text{ P.M.}) \]

c. To find out when the depth \( y \) is at least 10 feet, you can graph the model with the line \( y = 10 \) using a graphing utility, as shown in Figure 4.58. Using the intersect feature, you can determine that the depth is at least 10 feet between 2:42 P.M. \( (t \approx 14.7) \) and 5:18 P.M. \( (t \approx 17.3) \).

Now try Exercise 77.
4.5 Exercises

VOCABULARY CHECK: Fill in the blanks.
1. One period of a sine or cosine function function is called one _______ of the sine curve or cosine curve.
2. The _______ of a sine or cosine curve represents half the distance between the maximum and minimum values of the function.
3. The period of a sine or cosine function is given by _______.
4. For the function given by \( y = a \sin(bx - c) \), \( \frac{c}{b} \) represents the _______ _______ of the graph of the function.
5. For the function given by \( y = d + a \cos(bx - c) \), \( d \) represents a _______ _______ of the graph of the function.


In Exercises 1–14, find the period and amplitude.
1. \( y = 3 \sin 2x \)
2. \( y = 2 \cos 3x \)
3. \( y = \frac{5}{2} \cos \frac{x}{2} \)
4. \( y = -3 \sin \frac{x}{3} \)
5. \( y = \frac{1}{2} \sin \frac{\pi x}{3} \)
6. \( y = \frac{3}{2} \cos \frac{\pi x}{2} \)
7. \( y = -2 \sin x \)
8. \( y = -\cos \frac{2x}{3} \)
9. \( y = 3 \sin 10x \)
10. \( y = \frac{1}{5} \sin 8x \)
11. \( y = \frac{1}{2} \cos \frac{2x}{3} \)
12. \( y = \frac{5}{2} \cos \frac{x}{4} \)
13. \( y = \frac{1}{4} \sin 2\pi x \)
14. \( y = \frac{2}{3} \cos \frac{\pi x}{10} \)

In Exercises 15–22, describe the relationship between the graphs of \( f \) and \( g \). Consider amplitude, period, and shifts.
15. \( f(x) = \sin x \)
   \( g(x) = \sin(x - \pi) \)
16. \( f(x) = \cos x \)
   \( g(x) = \cos(x + \pi) \)
17. \( f(x) = \cos 2x \)
   \( g(x) = -\cos 2x \)
18. \( f(x) = \sin 3x \)
   \( g(x) = \sin(3x) \)
19. \( f(x) = \cos x \)
   \( g(x) = \cos 2x \)
20. \( f(x) = \sin x \)
   \( g(x) = 3x \)
21. \( f(x) = \sin 2x \)
   \( g(x) = 3 + \sin 2x \)
22. \( f(x) = \cos 4x \)
   \( g(x) = -2 + \cos 4x \)

In Exercises 23–26, describe the relationship between the graphs of \( f \) and \( g \). Consider amplitude, period, and shifts.

23.
24.
25.
26.
In Exercises 27–34, graph \( f \) and \( g \) on the same set of coordinate axes. (Include two full periods.)

27. \( f(x) = \sin x \)  
   \( g(x) = 4 \sin x \)

28. \( f(x) = \cos x \)  
   \( g(x) = \cos 2x \)

29. \( f(x) = 1 + \sin x \)  
   \( g(x) = \sin x \)

30. \( f(x) = 2 \cos 2x \)  
   \( g(x) = \cos x \)

31. \( f(x) = 3 \cos \frac{x}{2} \)  
   \( g(x) = 3 \cos 2x \)

32. \( f(x) = \sin x \)  
   \( g(x) = \sin 2x \)

33. \( f(x) = \cos x \)  
   \( g(x) = \cos x \)

34. \( f(x) = \sin x \)  
   \( g(x) = \cos x \)

In Exercises 35–56, sketch the graph of the function. (Include two full periods.)

35. \( y = 3 \sin x \)

36. \( y = \frac{1}{2} \sin x \)

37. \( y = \frac{1}{3} \cos x \)

38. \( y = 4 \cos x \)

39. \( y = \cos \frac{x}{2} \)

40. \( y = \cos 4x \)

41. \( y = \cos 2x \)

42. \( y = \sin \frac{x}{4} \)

43. \( y = -\sin \frac{2x}{3} \)

44. \( y = -10 \cos \frac{\pi x}{6} \)

45. \( y = \sin \left( x - \frac{\pi}{4} \right) \)

46. \( y = \sin(x - \pi) \)

47. \( y = 3 \cos(x + \pi) \)

48. \( y = 4 \cos \left( x + \frac{\pi}{4} \right) \)

49. \( y = 2 - \sin \frac{2x}{3} \)

50. \( y = -3 + 5 \cos \frac{\pi x}{12} \)

51. \( y = 2 + \frac{1}{10} \cos 60\pi x \)

52. \( y = 2 \cos x - 3 \)

53. \( y = 3 \cos(x + \pi) - 3 \)

54. \( y = 4 \cos \left( x + \frac{\pi}{4} \right) + 4 \)

55. \( y = \frac{2}{3} \cos \left( x - \frac{\pi}{4} \right) \)

56. \( y = -3 \cos(6x + \pi) \)

Graphical Reasoning In Exercises 63–66, find \( a \) and \( d \) for the function \( f(x) = a \cos x + d \) such that the graph of \( f \) matches the figure.

63. \[
\begin{array}{cc}
\text{Graph 1} & \text{Graph 2}
\end{array}
\]

64. \[
\begin{array}{cc}
\text{Graph 1} & \text{Graph 2}
\end{array}
\]

Graphical Reasoning In Exercises 67–70, find \( a, b, \) and \( c \) for the function \( f(x) = a \sin bx - c \) such that the graph of \( f \) matches the figure.

67. \[
\begin{array}{cc}
\text{Graph 1} & \text{Graph 2}
\end{array}
\]

68. \[
\begin{array}{cc}
\text{Graph 1} & \text{Graph 2}
\end{array}
\]

69. \[
\begin{array}{cc}
\text{Graph 1} & \text{Graph 2}
\end{array}
\]

70. \[
\begin{array}{cc}
\text{Graph 1} & \text{Graph 2}
\end{array}
\]

In Exercises 71 and 72, use a graphing utility to graph \( y_1 \) and \( y_2 \) in the interval \([-2\pi, 2\pi]\). Use the graphs to find real numbers \( x \) such that \( y_1 = y_2 \).

71. \[
\begin{array}{cc}
\text{Graph 1} & \text{Graph 2}
\end{array}
\]

72. \[
\begin{array}{cc}
\text{Graph 1} & \text{Graph 2}
\end{array}
\]
73. Respiratory Cycle For a person at rest, the velocity \((v)\) (in liters per second) of air flow during a respiratory cycle (the time from the beginning of one breath to the beginning of the next) is given by \(v = 0.85 \sin \left( \frac{\pi t}{3} \right)\), where \(t\) is the time (in seconds). (Inhalation occurs when \(v > 0\), and exhalation occurs when \(v < 0\).)

(a) Find the time for one full respiratory cycle.
(b) Find the number of cycles per minute.
(c) Sketch the graph of the velocity function.

74. Respiratory Cycle After exercising for a few minutes, a person has a respiratory cycle for which the velocity of air flow is approximated by \(v = 1.75 \sin \left( \frac{\pi t}{2} \right)\), where \(t\) is the time (in seconds). (Inhalation occurs when \(v > 0\), and exhalation occurs when \(v < 0\).)

(a) Find the time for one full respiratory cycle.
(b) Find the number of cycles per minute.
(c) Sketch the graph of the velocity function.

75. Data Analysis: Meteorology The table shows the maximum daily high temperatures for Tallahassee \(T\) and Chicago \(C\) (in degrees Fahrenheit) for month \(t\), with \(t = 1\) corresponding to January. (Source: National Climatic Data Center)

<table>
<thead>
<tr>
<th>Month, (t)</th>
<th>Tallahassee, (T)</th>
<th>Chicago, (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63.8</td>
<td>29.6</td>
</tr>
<tr>
<td>2</td>
<td>67.4</td>
<td>34.7</td>
</tr>
<tr>
<td>3</td>
<td>74.0</td>
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</tr>
<tr>
<td>4</td>
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<td>58.0</td>
</tr>
<tr>
<td>5</td>
<td>86.5</td>
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</tr>
<tr>
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<td>90.9</td>
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</tr>
<tr>
<td>7</td>
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<td>47.1</td>
</tr>
<tr>
<td>12</td>
<td>65.8</td>
<td>34.4</td>
</tr>
</tbody>
</table>

(a) A model for the temperature in Tallahassee is given by
\[ T(t) = 77.90 + 14.10 \cos \left( \frac{\pi t}{6} - 3.67 \right). \]

Find a trigonometric model for Chicago.

(b) Use a graphing utility to graph the data points and the model for the temperatures in Tallahassee. How well does the model fit the data?

76. Health The function given by \(P = 100 - 20 \cos \left( \frac{5\pi t}{3} \right)\) approximates the blood pressure \(P\) (in millimeters) of mercury at time \(t\) (in seconds) for a person at rest.

(a) Find the period of the function.
(b) Find the number of heartbeats per minute.

77. Piano Tuning When tuning a piano, a technician strikes a tuning fork for the A above middle C and sets up a wave motion that can be approximated by \(y = 0.001 \sin 880 \pi t\), where \(t\) is the time (in seconds).

(a) What is the period of the function?
(b) The frequency \(f\) is given by \(f = 1/p\). What is the frequency of the note?

78. Data Analysis: Astronomy The percent \(y\) of the moon’s face that is illuminated on day \(x\) of the year 2007, where \(x = 1\) represents January 1, is shown in the table. (Source: U.S. Naval Observatory)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.0</td>
</tr>
<tr>
<td>11</td>
<td>0.5</td>
</tr>
<tr>
<td>19</td>
<td>0.0</td>
</tr>
<tr>
<td>26</td>
<td>0.5</td>
</tr>
<tr>
<td>32</td>
<td>1.0</td>
</tr>
<tr>
<td>40</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(a) Create a scatter plot of the data.
(b) Find a trigonometric model that fits the data.
(c) Add the graph of your model in part (b) to the scatter plot. How well does the model fit the data?
(d) What is the period of the model?
(e) Estimate the moon’s percent illumination for March 12, 2007.
97. Fuel Consumption The daily consumption \( C \) (in gallons) of diesel fuel on a farm is modeled by

\[
C = 30.3 + 21.6 \sin \left( \frac{2\pi t}{365} + 10.9 \right)
\]

where \( t \) is the time (in days), with \( t = 1 \) corresponding to January 1.

(a) What is the period of the model? Is it what you expected? Explain.

(b) What is the average daily fuel consumption? Which term of the model did you use? Explain.

(c) Use a graphing utility to graph the model. Use the graph to approximate the time of the year when consumption exceeds 40 gallons per day.

80. Ferris Wheel A Ferris wheel is built such that the height \( h \) (in feet) above ground of a seat on the wheel at time \( t \) (in seconds) can be modeled by

\[
h(t) = 53 + 50 \sin \left( \frac{\pi}{10} t - \frac{\pi}{2} \right)
\]

(a) Find the period of the model. What does the period tell you about the ride?

(b) Find the amplitude of the model. What does the amplitude tell you about the ride?

(c) Use a graphing utility to graph one cycle of the model.

**Synthesis**

**True or False?** In Exercises 81–83, determine whether the statement is true or false. Justify your answer.

81. The graph of the function given by \( f(x) = \sin(x + 2\pi) \) translates the graph of \( f(x) = \sin x \) exactly one period to the right so that the two graphs look identical.

82. The function given by \( y = \frac{1}{2} \cos 2x \) has an amplitude that is twice that of the function given by \( y = \cos x \).

83. The graph of \( y = \cos x \) is a reflection of the graph of \( y = \sin(x + \pi/2) \) in the x-axis.

84. Writing Use a graphing utility to graph the function given by \( y = d + a \sin(bx - c) \), for several different values of \( a \), \( b \), \( c \), and \( d \). Write a paragraph describing the changes in the graph corresponding to changes in each constant.

**Conjecture** In Exercises 85 and 86, graph \( f \) and \( g \) on the same set of coordinate axes. Include two full periods. Make a conjecture about the functions.

85. \( f(x) = \sin x \), \( g(x) = \cos \left( x - \frac{\pi}{2} \right) \)

86. \( f(x) = \sin x \), \( g(x) = -\cos \left( x + \frac{\pi}{2} \right) \)

87. Exploration Using calculus, it can be shown that the sine and cosine functions can be approximated by the polynomials

\[
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \quad \text{and} \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}
\]

where \( x \) is in radians.

(a) Use a graphing utility to graph the sine function and its polynomial approximation in the same viewing window. How do the graphs compare?

(b) Use a graphing utility to graph the cosine function and its polynomial approximation in the same viewing window. How do the graphs compare?

(c) Study the patterns in the polynomial approximations of the sine and cosine functions and predict the next term in each. Then repeat parts (a) and (b). How did the accuracy of the approximations change when an additional term was added?

88. Exploration Use the polynomial approximations for the sine and cosine functions in Exercise 87 to approximate the following function values. Compare the results with those given by a calculator. Is the error in the approximation the same in each case? Explain.

(a) \( \sin \frac{1}{2} \) \quad (b) \( \sin 1 \) \quad (c) \( \sin \frac{\pi}{6} \)

(d) \( \cos(-0.5) \) \quad (e) \( \cos 1 \) \quad (f) \( \cos \frac{\pi}{4} \)

**Skills Review**

In Exercises 89–92, use the properties of logarithms to write the expression as a sum, difference, and/or constant multiple of a logarithm.

89. \( \log_{10} \sqrt{x - \frac{2}{5}} \) \quad 90. \( \log_{10} (x^2(x - 3)) \)

91. \( \ln \frac{t^3}{t - 1} \) \quad 92. \( \ln \frac{t}{t+1} \)

In Exercises 93–96, write the expression as the logarithm of a single quantity.

93. \( \frac{1}{3} (\log_{10} x + \log_{10} y) \) \quad 94. \( 2 \log_2 x + \log_2 (xy) \)

95. \( \ln 3x - 4 \ln y \) \quad 96. \( \frac{1}{3} (2x - 2 \ln x) + 3 \ln x \)

97. Make a Decision To work an extended application analyzing the normal daily maximum temperature and normal precipitation in Honolulu, Hawaii, visit this text’s website at college.hmco.com. (Data Source: NOAA)