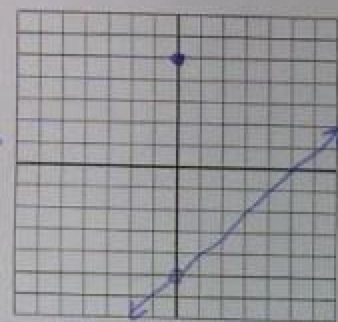
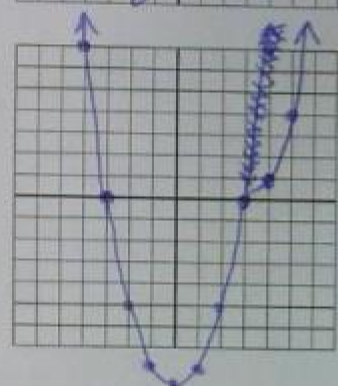
3. *Graph each of the following piecewise function and complete each part below.*
a) *Determine the limit from the left-side and right-side limit?*
b) *Determine if the limit exists at the indicated x-value.*
c) *State whether the function is continuous or discontinuous at the indicated x-value.*
d) *If it is discontinuous, state what type of discontinuity occurs at the x-value.*

$\frac{x(x-5)}{x}$ hole at zero but it's not in domain anyway



$g(x) = \begin{cases} \frac{x^2-5x}{x}, & x \neq 0 \\ 5, & x = 0 \end{cases}$

a) LEFT = −5
R = −5
b) yes it's −5
c) discontinuous since $f(0)=5 \neq \lim_{x\to 0} = -5$
d) removeable discontinuity (hole)



$k(x) = \begin{cases} x^2-9, & x \leq 3 \\ (3-x)^2, & x > 3 \end{cases}$

a) LEFT = 0
R = 0
b) yes it's 0
c) continuous since $f(3)=0=\lim_{x\to 3} f(x)$
d) ______

4. *Determine if the following functions are continuous. If not, state the x-value(s) of the discontinuity and what type of discontinuity is present.*

a) $h(x)=(x-3)(x+4)$ — Cont.

b) $f(x) = \begin{cases} 2x^2+3 & \text{if } x \leq 0 \\ -x+3 & \text{if } x > 0 \end{cases}$ — $f(0)=3$ and $\lim_{x\to 0}=3$ — Cont.

c) $f(x)=\frac{x+1}{|x+1|}$ — jump

x	y
−2	1
−1	∅
0	−1
1	−1
2	−1
−3	1

note similar structure

d) $f(x) = \begin{cases} -x+2 & \text{if } x < -1 \\ 2x+3 & \text{if } -1 \leq x < 3 \\ 6x-9 & \text{if } x > 3 \end{cases}$

$\lim_{x\to -1^-} f(x) = 3 \neq \lim_{x\to -1^+} f(x) = 1$

Jump at $x=-1$

Cont at $x=3$ since $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 9 = f(3)$

e) $p(x)=\frac{1-x}{x^2-1}$ — infinite disc (VA) at $x=-1$; removeable disc (hole) at $x=1$

f) $j(x)=\frac{1}{\sqrt{x}}$ — nonexistence discontinuity at $(-\infty, 0]$ since in the denom

5. *Evaluate each of the limits below using the appropriate technique. You may NOT use a graphing calculator to find the following.*

a) $\lim_{x\to\infty} \frac{15x^4+20x^3}{5x^3}$ = ∞ SA HAS + SLOPE

b) $\lim_{x\to -9} \frac{2x+18}{x^2+5x-36} = \frac{2(x+9)}{(x-4)(x+9)} = \frac{2}{(-13)}$

c) $\lim_{x\to\infty} \frac{15x^3+10x^2-5x}{27x^3} = \frac{5}{9}$

d) $\lim_{x\to \frac{1}{2}} \frac{10x-5}{2x^2-3x+1} = \frac{5(2x-1)}{(2x-1)(x-1)} = \frac{5}{x-1} = \frac{5}{\frac{1}{2}-1} = -10$

e) $\lim_{x\to 3} \frac{3x+9}{x^2-9} = \frac{3(x+3)}{(x+3)(x-3)}$ DNE recip funct.

f) $\lim_{x\to 2} \frac{-x^2}{2x-4} = \frac{-4}{-8} = \frac{1}{2}$

g) $\lim_{x\to\infty} \frac{x^2+4x+3}{3x^3-27x}$ HA $y=0$ → 0

h) $\lim_{x\to \frac{1}{3}} \frac{3x^2-13x+4}{3x^2-16x+5} = \frac{(3x-1)(x-4)}{(3x-1)(x-5)} = \frac{11}{14}$

i) $\lim_{x\to 1} \frac{15x-15}{5x-5} = \frac{15(x-1)}{5(x-1)} = 3$ (a hole at $x=1$)

j) $\lim_{x\to 1^+} \frac{\frac{3}{x+2}-1}{2x-2} = \frac{\frac{3-x-2}{x+2}}{2(x-1)} = \frac{-(x-1)}{x+2}\cdot\frac{1}{2(x-1)} = \frac{-1}{2(x+2)} = \frac{-1}{2(3)} = -\frac{1}{6}$

k) $\lim_{x\to -\infty} 1^{-x}+5 = 1^{\infty}+5 = 1+5 = 6$

l) $\lim_{x\to \frac{\pi}{2}} \sec x =$ DNE

m) $\lim_{x\to -2} \frac{-\sqrt{x+2}}{2x-4}$ = 0 LoL!

n) $\lim_{x\to \frac{\pi}{2}} \frac{\sin x}{x} = \frac{1}{\pi/2} = \frac{2}{\pi}$

o) $\lim_{x\to 3} \frac{-\sqrt{x+1}+2}{x-3} \cdot \frac{-\sqrt{x+1}-2}{-\sqrt{x+1}-2} = \frac{1}{-2-2} = -\frac{1}{4}$

p) $\lim_{x\to -\pi} \cot x =$ DNE

q) $\lim_{x\to 5^-} \frac{|x-5|}{x-5} =$ −1, hole at $x=5$

r) $\lim_{x\to -4} \frac{x^3+64}{-x-4}$ = $\frac{(x+4)(x^2-4x+16)}{-1(x+4)}$ = $-x^2+4x-16$, do direct sub → −48

6. *You may use a graphing calculator:* $\lim_{x\to 0} \frac{\cos x}{x} =$ DNE

7. *Find the values for a and b that make the functions continuous.*

a) $f(x) = \begin{cases} 2x^2 & \text{if } x \leq 3 \\ -x+a & \text{if } x > 3 \end{cases}$

$2(3)^2 = 18 = -3+a$
$21 = a$

b) $f(x) = \begin{cases} -x+3 & \text{if } x < -1 \\ ax+b & \text{if } -1 \leq x < 3 \\ 6x & \text{if } x > 3 \end{cases}$

$-(-1)+3 = 1+3 = 4 = a(-1)+b$
$4 = -a+b$
$4 = b-a$

$6(3) = 18 = a(3)+b$
$18 = 3a+b$
$-(4 = -a+b)$
$14 = 4a$

$a = \frac{14}{4} = \frac{7}{2}$
$4 = b - \frac{7}{2}$
$\frac{15}{2} = b$