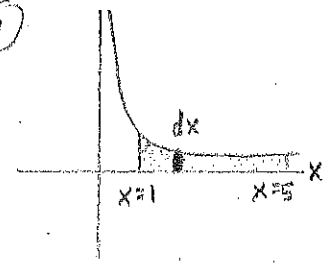
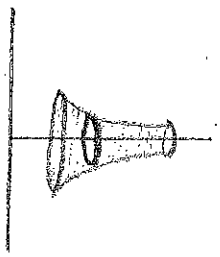
Volume Review

Disk

(1)



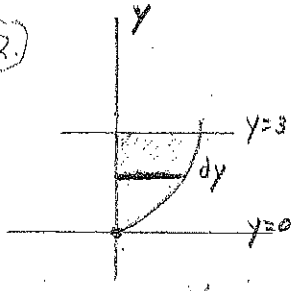


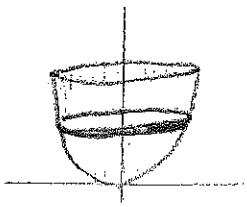
$$V = \int_1^5 \pi\left[\frac{1}{x}\right]^2 dx = \pi \int_1^5 \frac{1}{x^2}\,dx$$

$$= \pi\left[-\frac{1}{x}\right]_1^5$$

$$= \frac{4\pi}{5} \approx 2.513$$

(2.)





$$V = \int_0^3 \pi\left[y^{3/4}\right]^2 dy = \pi \int_0^3 y^{3/2}\,dy$$

$$= \pi\left[\frac{2y^{5/2}}{5}\right]_0^3$$

$$= \frac{18\sqrt{3}\,\pi}{5} \approx 19.589$$

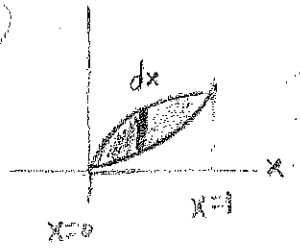
$y = x^{4/3}$

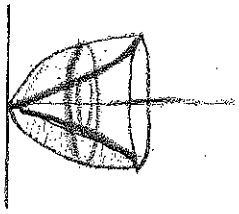
$y^{3/4} = \left(x^{4/3}\right)^{3/4}$

$y^{3/4} = x$

WASHER

(3)



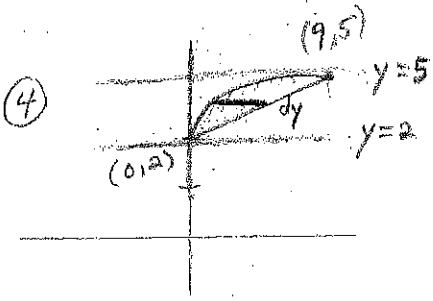


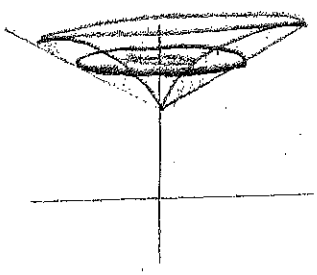
$$V = \int_0^1 \pi\left[(\sqrt{x})^2 - (x^3)^2\right] dx$$

$$= \pi \int_0^1 (x - x^6)\,dx = \pi\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \frac{3\pi}{10} \approx .942$$

(4)





$$V = \int_2^5 \pi\left[(3y-6)^2 - (y^2-4y+4)^2\right] dy$$

(see below)

$$\pi \int_2^5 \left[9y^2 - 36y + 36 - (y^4 - 8y^3 + 24y^2 - 32y + 16)\right] dy$$

$$\pi \int_2^5 \left[-y^4 + 8y^3 - 15y^2 - 4y + 20\right] dy$$

$$\pi\left[-\frac{y^5}{5} + 2y^4 - 5y^3 - 2y^2 + 20y\right]_2^5$$

$$= 162\pi$$

$y = \sqrt{x} + 2$

$y - 2 = \sqrt{x}$

$(y-2)^2 = x$

$y = \frac{x}{3} + 2$

$y - 2 = \frac{x}{3}$

$3y - 6 = x$

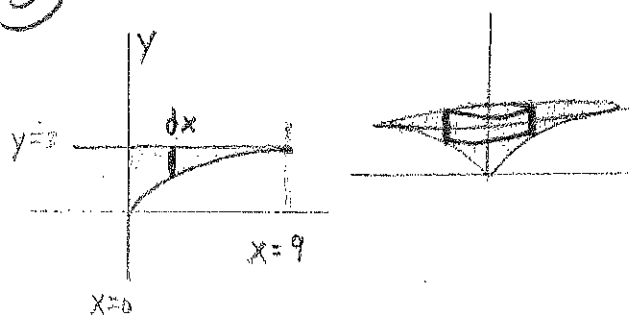
$(y^2-4y+4)(y^2-4y+4)$

$y^4 - 4y^3 + 4y^2 - 4y^3 + 16y^2 - 16y + 4y^2 - 16y + 16$

$y^4 - 8y^3 + 24y^2 - 32y + 16$

SHELL

(5)



$\sqrt{x} = 3$

$x = 9$

$$V = \int_0^9 2\pi x \left[3 - \sqrt{x}\right] dx$$

$$2\pi \int_0^9 (3x - x^{3/2})\, dx$$

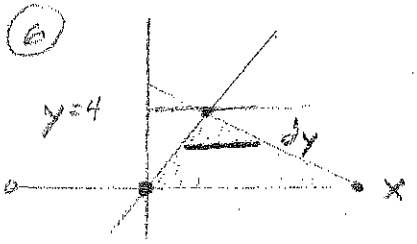
$$2\pi \left[\frac{3x^2}{2} - \frac{2x^{5/2}}{5}\right]_0^9 = \frac{243\pi}{5} \approx 152.681$$

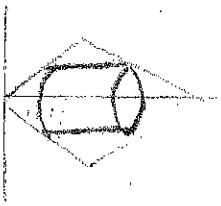
$\frac{243}{2} - \frac{486}{5}$

$\frac{1215}{10} - \frac{972}{10}$

$\frac{243}{10}$

(6)





$y = 6 - x$ $\quad y = 2x$

$x = 6 - y$ $\quad x = \frac{y}{2}$

$$V = \int_0^4 2\pi y \left[(6 - y) - \left(\tfrac{y}{2}\right)\right] dy$$

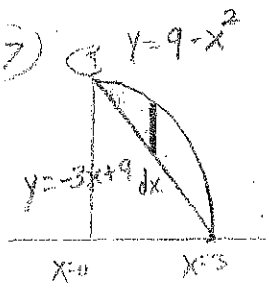
$(6 - 3y/2)$

$$2\pi \int_0^4 \left(6y - \frac{3y^2}{2}\right) dy$$

$$2\pi \left[3y^2 - \frac{y^3}{2}\right]_0^4 \quad (= 32\pi \approx 100.530)$$

Method of Choice

(7)



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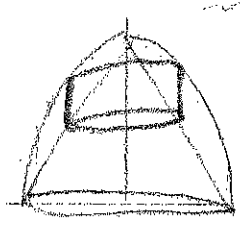


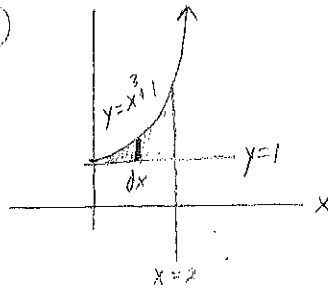
$$V = \int_0^3 2\pi x \left[(9 - x^2) - (-3x + 9)\right] dx$$

$-x^2 + 3x$

$$2\pi \int (-x^3 + 3x^2)\, dx = 2\pi \left[\frac{-x^4}{4} + x^3\right]_0^3$$

$$\left(= \frac{27\pi}{2} \approx 42.412\right)$$

(8)



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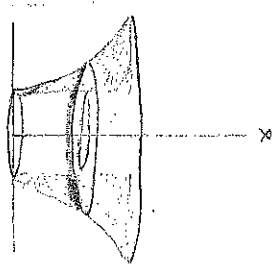


$$V = \int_0^2 \pi \left[(x^3 + 1)^2 - (1 - 0)^2\right] dx$$

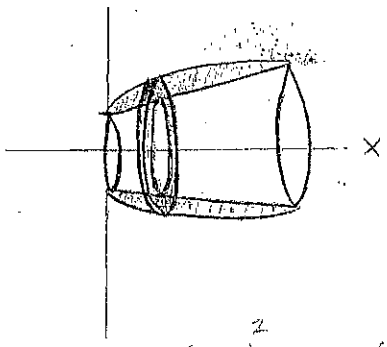
$$\pi \int \left[(x^6 + 2x^3 + 1) - 1\right] dx$$

$$\pi \int_0^2 (x^6 + 2x^3)\, dx = \pi \left[\frac{x^7}{7} + \frac{x^4}{2}\right]_0^2 = \frac{184\pi}{7}$$

$$\approx 82.599$$

9. $y=\sqrt{x}+1$, $y=x+1$ and $x=0$ rotated about x-axis

WASHER



$$V=\int_{x=0}^{x=1}\pi\left((\sqrt{x}+1)^2-(x+1)^2\right)dx=\int_0^1\pi\left(-x^2+2\sqrt{x}-x\right)dx$$

$$=\left[\pi\left(-\frac{x^3}{3}+\frac{4x^{3/2}}{3}-\frac{x^2}{2}\right)\right]_0^1$$

$V=1.57u^3$

$*\ (\sqrt{x}+1)^2=x+2\sqrt{x}+1$

$(x+1)^2=x^2+2x+1$

outer^2 − inner^2

$x+2\sqrt{x}+1-(x^2+2x+1)$

$x+2\sqrt{x}+1-x^2-2x-1$

$-x^2+2\sqrt{x}-x$

$(\sqrt{x}+1)^2=(x)^2+1$

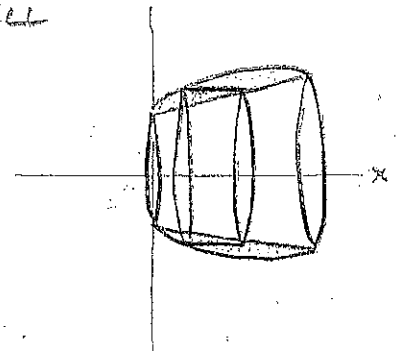
$x=x^2$

$0=x^2-x$

$0=x(x-1)$

$x=0,1$

SHELL



$r=y$

$l=2\pi y$

$w=dy$

$h=y-1-(y^2-2y+1)$

$-y^2+3y-2$

$$V=\int_{y=1}^{y=2}2\pi y\left(-y^2+3y-2\right)dy$$

$$=\int_1^2 2\pi\left(-y^3+3y^2-2y\right)dy$$

$$=\left[2\pi\left(-\frac{y^4}{4}+y^3-y^2\right)\right]_1^2$$

$=1.57u^3$

$y=\sqrt{x}+1$

$y-1=\sqrt{x}$

$y^2-2y+1=x$

$y=x+1$

$y-1=x$

$y^2-2y+1=y-1$

$y^2-3y+2=0$

$(y-1)(y-2)=0$

$y=1,2$