Unit 5 Worksheet 3 **Name** ____________________

AP Calculus AB **Date** ____________________

Use substitution to evaluate the following.

1. $\int \sqrt{3x+2}\ dx$
2. $\int \cos(3x+2)\ dx$
3. $\int x\sqrt{x^2+4}\ dx$
4. $\int x\sin\left(x^2+4\right)\ dx$
5. $\int \left(\dfrac{x\sin\sqrt{x^2+4}}{\sqrt{x^2+4}}\right)dx$
6. $\int x\cos\left(x^2+4\right)\sqrt{\sin\left(x^2+4\right)}\ dx$
7. $\int \left(\dfrac{\left(\sqrt{t}+4\right)^3}{\sqrt{t}}\right)dt$
8. $\int_0^1 (3x+1)^3\ dx$
9. $\int_0^2 \left(\dfrac{t}{\left(t^2+9\right)^2}\right) dt$
10. $\int_0^1 \left(\dfrac{x+2}{\left(x^2+4x+1\right)^2}\right) dx$
11. $\int_0^{\pi/6} \sin^3\theta\cos\theta\ d\theta$
12. $\int_0^1 \cos(3x-3)\ dx$
13. $\int_0^1 x\sin\left(\pi x^2\right)\ dx$
14. $\int_0^{\pi/2} \sin x\sin(\cos x)\ dx$
15. $\int_1^4 \left(\dfrac{1}{\sqrt{t}\left(\sqrt{t}+1\right)^3}\right) dt$

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① $\int \sqrt{3x+2}\,dx$

$\int (3x+2)^{1/2}dx$

$u=3x+2$

$du=3\,dx$

$\frac{1}{3}du=dx$

$\int \frac{1}{3}u^{1/2}du$

$\frac{\frac{1}{3}u^{3/2}}{3/2}+C$

$F(u)=\frac{2}{9}u^{3/2}+C$

$\boxed{F(x)=\frac{2}{9}(3x+2)^{3/2}+C}$

③ $\int x\sqrt{x^2+4}\,dx$

$(x^2+4)^{1/2}x\,dx$

$u=x^2+4$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

$\int \frac{1}{2}u^{1/2}du$

$F(u)=\frac{\frac{1}{2}u^{3/2}}{3/2}+C$

$\boxed{F(x)=\frac{1}{3}(x^2+4)^{3/2}+C}$

⑤ $\int \left(\frac{x\sin\sqrt{x^2+4}}{\sqrt{x^2+4}}\right)dx$

$u=(x^2+4)^{1/2}$

$du=\frac{1}{2}(x^2+4)^{-1/2}2x\,dx$

$\int \sin u\,du$

$F(u)=-\cos u+C$

$\boxed{F(x)=-\cos\sqrt{x^2+4}+C}$

OR

$\boxed{F(x)=-\cos(x^2+4)^{1/2}+C}$

⑦ $\int \left(\frac{(\sqrt{t}+4)^3}{\sqrt{t}}\right)dt$

$u=t^{1/2}+4$

$du=\frac{1}{2}t^{-1/2}dt$

$2du=t^{-1/2}dt$

$\int u^3\cdot 2du=\int 2u^3du$

$F(u)=\frac{2u^4}{4}+C$

$\boxed{F(x)=\frac{1}{2}(\sqrt{t}+4)^4+C}$

⑨ $\int_0^2 \left(\frac{t}{(t^2+9)^2}\right)dt$

$u=t^2+9$

$du=2t\,dt$

$\frac{1}{2}du=t\,dt$

$\int \frac{1}{2}u^{-2}du$

$\frac{\frac{1}{2}u^{-1}}{-1}+C$

$\left.\frac{-1}{2(t^2+9)}\right]_0^2$

$\frac{-1}{2(2^2+9)}-\frac{-1}{2(0^2+9)}$

$-\frac{1}{26}+\frac{1}{18}=\boxed{\frac{2}{117}}\approx\boxed{0.017}$

⑪ $\int_0^{\pi/6}\sin^3\theta\cos\theta\,d\theta$

$u=\sin\theta$

$du=\cos\theta$

$\int u^3du$

$F(u)=\frac{u^4}{4}+C$

$\left.\frac{\sin^4\theta}{4}\right]_0^{\pi/6}$

$\frac{\sin^4(\pi/6)}{4}-\frac{\sin^4 0}{4}$

$\frac{\frac{1}{4}\left(\frac{1}{2}\right)^4}{4}=\frac{1}{64}\approx 0.016$

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(13) $\int_0^1 x\sin(\pi x^2)dx$

$u=\pi x^2$

$du = 2\pi x\,dx$

$\frac{1}{2\pi}du = x\,dx$

$\int \frac{1}{2\pi}\sin u\,du$

$F(u) = \frac{1}{2\pi}(-\cos u) + C$

$\left[-\frac{1}{2\pi}\cos(\pi x^2)\right]_0^1$

$-\frac{1}{2\pi}\cos\pi(1)^2 - -\frac{1}{2\pi}\cos\pi(0)$

$-\frac{1}{2\pi}(-1) + \frac{1}{2\pi}$

$\frac{1}{2\pi} + \frac{1}{2\pi}$

$\frac{1}{\pi} \approx 0.318$

(15) $\int_1^4 \left(\frac{1}{\sqrt{t}(\sqrt{t}+1)^3}\right)dt$

$u=\sqrt{t}+1$

$du = \frac{1}{2}t^{-1/2}\,dt$

$2du = t^{-1/2}dt$

$\int 2u^{-3}du$

$F(u) = \frac{2u^{-2}}{-2} + C$

$\left[-\frac{1}{(\sqrt{t}+1)^2}\right]_1^4$

$\frac{-1}{(\sqrt{4}+1)^2} - \frac{-1}{(\sqrt{1}+1)^2}$

$\frac{-1}{9} + \frac{1}{4}$

$\boxed{=\frac{5}{36} \approx .139}$

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(2) $\int \cos(3x+2)\,dx$

$u = 3x+2$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$\int \frac{1}{3}\cos u\,du$

$F(u) = \frac{1}{3}\sin u + C$

$\boxed{F(x) = \frac{1}{3}\sin(3x+2) + C}$

(4) $\int x\sin(x^2+4)\,dx$

$u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int \frac{1}{2}\sin u\,du$

$F(u) = -\frac{1}{2}\cos u + C$

$\boxed{F(x) = -\frac{1}{2}\cos(x^2+4) + C}$

(6) $\int x\cos(x^2+4)\sqrt{\sin(x^2+4)}\,dx$

$u = \sin(x^2+4)$

$du = \cos(x^2+4)(2x)\,dx$

$\frac{1}{2}du = x\cos(x^2+4)\,dx$

$\int \frac{1}{2}u^{1/2}\,du$

$F(u) = \frac{\frac{1}{2}u^{3/2}}{3/2} + C$

$\boxed{F(x) = \frac{1}{3}\sin^{3/2}(x^2+4) + C}$

(8) $\int_0^1 (3x+1)^3\,dx$

$u = 3x+1$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$\int \frac{1}{3}u^3\,du$

$F(u) = \frac{\frac{1}{3}u^4}{4} + C$

$\left[\frac{1}{12}(3x+1)^4\right]_0^1$

$\frac{1}{12}(3(1)+1)^4 - \frac{1}{12}(3(0)+1)^4$

$\frac{256}{12} - \frac{1}{12} = \frac{255}{12} \approx 21.250$

(10) $\int_0^1 \left(\frac{x+2}{(x^2+4x+1)^2}\right)dx$

$u = x^2+4x+1$

$du = (2x+4)\,dx$

$\frac{1}{2}du = (x+2)\,dx$

$\int \frac{1}{2}u^{-2}\,du$

$F(u) = -\frac{1}{2}u^{-1} + C$

$\left[-\frac{1}{2(x^2+4x+1)}\right]_0^1$

$-\frac{1}{2(6)} - -\frac{1}{2(1)}$

$-\frac{1}{12} + \frac{1}{2}$

$= 5/12 \approx 0.416$

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(12) $\int_0^1 \cos(3x-3)\,dx$

$u = 3x-3$

$du = 3\,dx$

$\frac{1}{3}\,du = dx$

$\int \frac{1}{3}\cos u\,du$

$F(u) = \frac{1}{3}\sin u + C$

$\left[\frac{1}{3}\sin(3x-3)\right]_0^1$

$\frac{1}{3}\sin(3(1)-3) - \frac{1}{3}\sin(3(0)-3)$

$\frac{1}{3}\sin(0) - \frac{1}{3}\sin(-3)$

$0 - (-0.0470)$

≈ 0.047

(14) $\int_0^{\pi/2} \sin x \sin(\cos x)\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$\int -\sin u\,du$

$F(u) = \cos u + C$

$\left[\cos(\cos x)\right]_0^{\pi/2}$

$\cos(\cos \frac{\pi}{2}) - \cos(\cos(0))$

$\cos(0) - \cos(1)$

$1 - \cos(1)$

$1 - 0.5403$

0.4597

$.460$