

$$1) \int x e^{2x} dx$$

$$u = x \quad dv = e^{2x} dx$$

$$du = dx \quad v = \frac{1}{2} e^{2x}$$

$$\int x e^{2x} dx = x \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} dx$$

$$u = 2x$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \int \frac{1}{4} e^u du$$

$$\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$$

$$4) \int x \ln x dx$$

$$u = \ln x \quad dv = x dx$$

$$du = \frac{1}{x} dx \quad v = \frac{1}{2} x^2$$

$$\int x \ln x dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx$$

$$\int x \ln x dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{\frac{1}{2} x^2}{2} + C$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

$$7) \int (\ln x)^2 dx$$

$$u = (\ln x)^2 \quad dv = dx$$

$$du = 2 \ln x \cdot \frac{1}{x} dx \quad v = x$$

$$du = \frac{2}{x} \ln x dx$$

$$\int (\ln x)^2 dx = (\ln x)^2 \cdot x - \int x \cdot \frac{2}{x} \ln x dx$$

$$= x(\ln x)^2 - \int 2 \ln x dx$$

$$= x(\ln x)^2 - 2 \int \ln x dx$$

$$u = \ln x \quad dv = dx$$

$$du = \frac{1}{x} dx \quad v = x$$

$$= x(\ln x)^2 - 2 \left[ x \ln x - \int x \cdot \frac{1}{x} dx \right]$$

$$= x(\ln x)^2 - 2x \ln x + 2 \int dx$$

$$= x(\ln x)^2 - 2x \ln x + 2x + C$$

$$10) \int \theta \sec^2 \theta d\theta$$

$$u = \theta \quad dv = \sec^2 \theta d\theta$$

$$du = d\theta \quad v = \tan \theta$$

$$\int \theta \sec^2 \theta d\theta = \theta \tan \theta - \int \tan \theta d\theta$$

$$= \theta \tan \theta - \int \frac{\sin \theta}{\cos \theta} d\theta$$

$$u = \cos \theta$$

$$du = -\sin \theta d\theta$$

$$-du = \sin \theta d\theta$$

$$= \theta \tan \theta - \int -\frac{du}{u}$$

$$= \theta \tan \theta + \ln |u| + C$$

$$= \theta \tan \theta + \ln |\cos \theta| + C$$

13)  $\int e^{2\theta} \sin 3\theta d\theta$

$u = \sin 3\theta \quad dv = e^{2\theta} d\theta$   
 $du = 3\cos 3\theta d\theta \quad v = \frac{1}{2}e^{2\theta}$

$\int e^{2\theta} \sin 3\theta d\theta = \frac{1}{2}e^{2\theta} \sin 3\theta - \int \frac{1}{2}e^{2\theta} \cdot 3\cos 3\theta d\theta$   
 $= \frac{1}{2}e^{2\theta} \sin 3\theta - \frac{3}{2} \int e^{2\theta} \cos 3\theta d\theta$

$u = \cos 3\theta \quad dv = e^{2\theta} d\theta$   
 $du = -3\sin 3\theta d\theta \quad v = \frac{1}{2}e^{2\theta}$

$= \frac{1}{2}e^{2\theta} \sin 3\theta - \frac{3}{2} \left[ \frac{1}{2}e^{2\theta} \cos 3\theta - \int \frac{1}{2}e^{2\theta} (-3\sin 3\theta) d\theta \right]$   
 $= \frac{1}{2}e^{2\theta} \sin 3\theta - \frac{3}{2} \left[ \frac{1}{2}e^{2\theta} \cos 3\theta + \frac{3}{2} \int e^{2\theta} \sin 3\theta d\theta \right]$

$\int e^{2\theta} \sin 3\theta d\theta = \frac{1}{2}e^{2\theta} \sin 3\theta - \frac{3}{4}e^{2\theta} \cos 3\theta - \frac{9}{4} \int e^{2\theta} \sin 3\theta d\theta$

$\frac{13}{4} \int e^{2\theta} \sin 3\theta d\theta = \frac{1}{2}e^{2\theta} \sin 3\theta - \frac{3}{4}e^{2\theta} \cos 3\theta + C$

$\int e^{2\theta} \sin 3\theta d\theta = \frac{2}{13}e^{2\theta} \sin 3\theta - \frac{3}{13}e^{2\theta} \cos 3\theta + C$

$\int e^{2\theta} \sin 3\theta d\theta = \frac{\text{or}}{13}e^{2\theta} [2\sin 3\theta - 3\cos 3\theta] + C$

16)  $\int_1^4 \sqrt{t} \ln t dt$

$u = \ln t \quad dv = t^{1/2} dt$   
 $du = \frac{1}{t} dt \quad v = \frac{t^{3/2}}{3/2} = \frac{2}{3}t^{3/2}$

$\int_1^4 t^{1/2} \ln t dt = \left[ \frac{2}{3}t^{3/2} \ln t \right]_1^4 - \int_1^4 \frac{2}{3}t^{3/2} \cdot t^{-1} dt$

$= \left[ \frac{2}{3}t^{3/2} \ln t \right]_1^4 - \frac{2}{3} \int_1^4 t^{1/2} dt$

$= \left[ \frac{2}{3}t^{3/2} \ln t \right]_1^4 - \left[ \frac{2}{3} \cdot \frac{2}{3}t^{3/2} \right]_1^4$

$= \left[ \frac{2}{3}t^{3/2} \ln t \right]_1^4 - \left[ \frac{4}{9}t^{3/2} \right]_1^4$

$= \left[ \frac{16}{3} \ln 4 - \frac{2}{3} \ln 1 \right] - \left[ \frac{32}{9} - \frac{4}{9} \right]$

$= \frac{16}{3} \ln 4 - \frac{28}{9}$

$\approx 4.282$

$$19) \int_0^{1/2} \sin^{-1} x \, dx$$

$$u = \sin^{-1} x \quad dv = dx$$

$$du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$$

$$\int_0^{1/2} \sin^{-1} x \, dx = \left[ x \sin^{-1} x \right]_0^{1/2} - \int_0^{1/2} x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$\int_0^{1/2} \sin^{-1} x \, dx = \left[ x \sin^{-1} x \right]_0^{1/2} - \int_0^{1/2} x (1-x^2)^{-1/2} dx$$

$$u = 1-x^2$$

$$x=0 \Rightarrow u=1$$

$$du = -2x dx$$

$$x=1/2 \Rightarrow u=3/4$$

$$-\frac{1}{2} du = x dx$$

$$\int_0^{1/2} \sin^{-1} x \, dx = \left[ x \sin^{-1} x \right]_0^{1/2} - \int_1^{3/4} -\frac{1}{2} u^{-1/2} du$$

$$\int_0^{1/2} \sin^{-1} x \, dx = \left[ x \sin^{-1} x \right]_0^{1/2} - \left[ -\frac{1}{2} \frac{u^{1/2}}{1/2} \right]_1^{3/4}$$

$$\int_0^{1/2} \sin^{-1} x \, dx = \left[ x \sin^{-1} x \right]_0^{1/2} - \left[ -\sqrt{u} \right]_1^{3/4}$$

$$= \left[ \frac{1}{2} \cdot \frac{\pi}{6} - 0 \cdot 0 \right] - \left[ -\frac{\sqrt{3}}{2} - (-1) \right]$$

$$= \frac{\pi}{12} - \left[ 1 - \frac{\sqrt{3}}{2} \right]$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

$$= \frac{\pi}{12} + \frac{6\sqrt{3}}{12} - \frac{12}{12}$$

$$= \frac{1}{12} (\pi + 6\sqrt{3} - 12)$$

$$\approx 0.128$$

$$22) \int_0^1 x \operatorname{TAN}^{-1} x \, dx$$

$$u = \operatorname{TAN}^{-1} x \quad dv = x \, dx$$

$$du = \frac{1}{1+x^2} dx \quad v = \frac{1}{2} x^2$$

$$\int_0^1 x \operatorname{TAN}^{-1} x \, dx = \left[ \frac{1}{2} x^2 \operatorname{TAN}^{-1} x \right]_0^1 - \int_0^1 \frac{x^2}{2} \cdot \frac{1}{1+x^2} dx$$

$$\int_0^1 x \operatorname{TAN}^{-1} x \, dx = \left[ \frac{1}{2} x^2 \operatorname{TAN}^{-1} x \right]_0^1 - \int_0^1 \frac{1}{2} \cdot \frac{x^2}{1+x^2} dx$$

$$\int_0^1 x \operatorname{TAN}^{-1} x \, dx = \left[ \frac{1}{2} x^2 \operatorname{TAN}^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 \frac{(1+x^2) - 1}{1+x^2} dx$$

$$= \left[ \frac{1}{2} x^2 \operatorname{TAN}^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 \left( \frac{1+x^2}{1+x^2} - \frac{1}{1+x^2} \right) dx$$

$$= \left[ \frac{1}{2} x^2 \operatorname{TAN}^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 1 \, dx + \frac{1}{2} \int_0^1 \frac{1}{1+x^2} dx$$

$$= \left[ \frac{1}{2} x^2 \operatorname{TAN}^{-1} x \right]_0^1 - \left[ \frac{1}{2} x \right]_0^1 + \left[ \frac{1}{2} \operatorname{TAN}^{-1} x \right]_0^1$$

$$= \left[ \frac{1}{2} \cdot \frac{\pi}{4} - 0 \right] - \left[ \frac{1}{2} - 0 \right] + \left[ \frac{1}{2} \cdot \frac{\pi}{4} - 0 \right]$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{2\pi}{8} - \frac{1}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$\approx 0.285$$

2)  $\int x \cos x \, dx$

$u = x \quad dv = \cos x \, dx$   
 $du = dx \quad v = \sin x$

$$\begin{aligned} \int x \cos x \, dx &= x \sin x - \int \sin x \, dx \\ &= x \sin x - (-\cos x) + C \\ &= x \sin x + \cos x + C \end{aligned}$$

5)  $\int x^2 \cos 3x \, dx$

$u = x^2 \quad dv = \cos 3x \, dx$   
 $du = 2x \, dx \quad v = \frac{1}{3} \sin 3x$

$$\begin{aligned} \int x^2 \cos 3x \, dx &= x^2 \cdot \frac{1}{3} \sin 3x - \int \frac{1}{3} \sin 3x \cdot 2x \, dx \\ &= \frac{1}{3} x^2 \sin 3x - \int \frac{2}{3} x \sin 3x \, dx \\ &= \frac{1}{3} x^2 \sin 3x - \frac{2}{3} \int x \sin 3x \, dx \end{aligned}$$

$u = x \quad dv = \sin 3x \, dx$   
 $du = dx \quad v = -\frac{1}{3} \cos 3x$

$$\begin{aligned} &= \frac{1}{3} x^2 \sin 3x - \frac{2}{3} \left[ x \cdot -\frac{1}{3} \cos 3x - \int -\frac{1}{3} \cos 3x \, dx \right] \\ &= \frac{1}{3} x^2 \sin 3x - \frac{2}{3} \left[ -\frac{1}{3} x \cos 3x + \frac{1}{3} \int \cos 3x \, dx \right] \\ &= \frac{1}{3} x^2 \sin 3x + \frac{2}{9} x \cos 3x - \frac{2}{9} \int \cos 3x \, dx \\ &= \frac{1}{3} x^2 \sin 3x + \frac{2}{9} x \cos 3x - \frac{2}{9} \cdot \frac{1}{3} \sin 3x + C \\ &= \frac{1}{3} x^2 \sin 3x + \frac{2}{9} x \cos 3x - \frac{2}{27} \sin 3x + C \end{aligned}$$

$$a) \int \sin^{-1} x \, dx$$

$$u = \sin^{-1} x \quad dv = dx$$

$$du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$$

$$du = (1-x^2)^{-\frac{1}{2}} dx$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int x (1-x^2)^{-\frac{1}{2}} dx$$

$$u = 1-x^2$$

$$du = -2x dx$$

$$-\frac{1}{2} du = x dx$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int -\frac{1}{2} u^{-\frac{1}{2}} du$$

$$= x \sin^{-1} x + \frac{1}{2} \int u^{-\frac{1}{2}} du$$

$$= x \sin^{-1} x + \frac{1}{2} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= x \sin^{-1} x + (1-x^2)^{\frac{1}{2}} + C$$

$$= x \sin^{-1} x + \sqrt{1-x^2} + C$$

$$ii) \int t^2 \ln t \, dt$$

$$u = \ln t \quad dv = t^2 dt$$

$$du = \frac{1}{t} dt \quad v = \frac{1}{3} t^3$$

$$\int t^2 \ln t \, dt = \frac{1}{3} t^3 \ln t - \int \frac{1}{3} t^3 \cdot \frac{1}{t} dt$$

$$= \frac{1}{3} t^3 \ln t - \frac{1}{3} \int t^2 dt$$

$$= \frac{1}{3} t^3 \ln t - \frac{1}{3} \cdot \frac{1}{3} t^3 + C$$

$$= \frac{1}{3} t^3 \ln t - \frac{1}{9} t^3 + C$$

or

$$= \frac{3}{9} t^3 \ln t - \frac{1}{9} t^3 + C$$

$$= \frac{1}{9} t^3 (3 \ln t - 1) + C$$

$$14) \int e^{-\theta} \cos 3\theta \, d\theta$$

$$u = \cos 3\theta \quad dv = e^{-\theta} \, d\theta$$

$$du = -3 \sin 3\theta \, d\theta \quad v = -e^{-\theta}$$

$$\int e^{-\theta} \cos 3\theta \, d\theta = -e^{-\theta} \cos 3\theta - \int 3e^{-\theta} \sin 3\theta \, d\theta$$

$$= -e^{-\theta} \cos 3\theta - 3 \int e^{-\theta} \sin 3\theta \, d\theta$$

$$u = \sin 3\theta \quad dv = e^{-\theta} \, d\theta$$

$$du = 3 \cos 3\theta \, d\theta \quad v = -e^{-\theta}$$

$$= -e^{-\theta} \cos 3\theta - 3 \left[ -e^{-\theta} \sin 3\theta - \int -3e^{-\theta} \cos 3\theta \, d\theta \right]$$

$$= -e^{-\theta} \cos 3\theta + 3e^{-\theta} \sin 3\theta + 3 \int -3e^{-\theta} \cos 3\theta \, d\theta$$

$$\int e^{-\theta} \cos 3\theta \, d\theta = -e^{-\theta} \cos 3\theta + 3e^{-\theta} \sin 3\theta - 9 \int e^{-\theta} \cos 3\theta \, d\theta$$

$$\int e^{-\theta} \cos 3\theta \, d\theta + 9 \int e^{-\theta} \cos 3\theta \, d\theta = -e^{-\theta} \cos 3\theta + 3e^{-\theta} \sin 3\theta$$

$$10 \int e^{-\theta} \cos 3\theta \, d\theta = 3e^{-\theta} \sin 3\theta - e^{-\theta} \cos 3\theta$$

$$\int e^{-\theta} \cos 3\theta \, d\theta = \frac{3}{10} e^{-\theta} \sin 3\theta - \frac{1}{10} e^{-\theta} \cos 3\theta + C$$

$$= \frac{1}{10} e^{-\theta} (3 \sin 3\theta - \cos 3\theta) + C$$



$$17) \int_0^{\pi/2} x \cos 2x dx$$

$$u = x \quad dv = \cos 2x dx$$

$$du = dx \quad v = \frac{1}{2} \sin 2x$$

$$\int_0^{\pi/2} x \cos 2x dx = \left[ \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x dx \right]_0^{\pi/2}$$

$$\int_0^{\pi/2} x \cos 2x dx = \left[ \frac{1}{2} x \sin 2x - \frac{1}{2} \int \sin 2x dx \right]_0^{\pi/2}$$

$$= \left[ \frac{1}{2} x \sin 2x - \frac{1}{2} \cdot -\frac{1}{2} \cos 2x \right]_0^{\pi/2}$$

$$= \left[ \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x \right]_0^{\pi/2}$$

$$= \left[ (-1/4) - (+1/4) \right]$$

$$= -1/2$$

Pg. 407 # 2-23 (x3)

$$\#20) \int_{\pi/4}^{\pi/2} x \csc^2 x \, dx =$$

$$u = x \quad dv = \csc^2 x \, dx$$

$$du = dx \quad v = -\cot x$$

$$= \left[ -x \cot x - \int -\cot x \, dx \right]_{\pi/4}^{\pi/2}$$

$$= \left[ -x \cot x + \int \frac{\cos x}{\sin x} \, dx \right]_{\pi/4}^{\pi/2}$$

$$= \left[ -x \cot x + \ln |\sin x| \right]_{\pi/4}^{\pi/2}$$

$$= \left[ \left( -\frac{\pi}{2} \cot \frac{\pi}{2} + \ln |\sin \frac{\pi}{2}| \right) - \left( -\frac{\pi}{4} \cot \frac{\pi}{4} + \ln |\sin \frac{\pi}{4}| \right) \right]$$

$$= \left[ (0 + 0) - \left( -\frac{\pi}{4} + \ln \frac{1}{\sqrt{2}} \right) \right]$$

$$= \frac{\pi}{4} - \ln \frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{4} - \ln 2^{-1/2}$$

$$= \frac{\pi}{4} + \frac{1}{2} \ln 2$$

$$\approx 1.132$$

Pr. 407 # 2-23 (x3)

$$23) \int_0^1 (x^2-1)e^x dx =$$

$$u = x^2 - 1 \quad du = 2x dx$$

$$du = 2x dx \quad v = e^x$$

$$= \left[ (x^2-1)e^x - \int 2x e^x dx \right]_0^1$$

$$= \left[ (x^2-1)e^x - 2 \int x e^x dx \right]_0^1$$

$$u = x \quad dv = e^x dx$$

$$du = dx \quad v = e^x$$

$$= \left[ (x^2-1)e^x - 2(xe^x - \int e^x dx) \right]_0^1$$

$$= \left[ (x^2-1)e^x - 2xe^x + 2e^x \right]_0^1$$

$$= \left[ e^x(x^2-1-2x+2) \right]_0^1$$

$$= \left[ e^x(x^2-2x+1) \right]_0^1$$

$$= \left[ e^x(x-1)^2 \right]_0^1$$

$$= [0 - 1]$$

$$= -1$$

3)  $\int x \sin 4x \, dx$

$u = x \quad dv = \sin 4x \, dx$

$du = dx \quad v = -\frac{1}{4} \cos 4x$

$$\begin{aligned} \int x \sin 4x \, dx &= -\frac{1}{4} x \cos 4x - \int -\frac{1}{4} \cos 4x \, dx \\ &= -\frac{1}{4} x \cos 4x + \frac{1}{4} \int \cos 4x \, dx \\ &= -\frac{1}{4} x \cos 4x + \frac{1}{4} \cdot \frac{1}{4} \sin 4x + C \\ &= -\frac{1}{4} x \cos 4x + \frac{1}{16} \sin 4x + C \end{aligned}$$

6)  $\int x^2 \sin ax \, dx$

$u = x^2 \quad dv = \sin ax \, dx$

$du = 2x \, dx \quad v = -\frac{1}{a} \cos ax$

$$\begin{aligned} \int x^2 \sin ax \, dx &= -\frac{1}{a} x^2 \cos ax - \int -x \cos ax \, dx \\ &= -\frac{1}{a} x^2 \cos ax + \int x \cos ax \, dx \end{aligned}$$

$u = x \quad dv = \cos ax \, dx$

$du = dx \quad v = \frac{1}{a} \sin ax$

$$\begin{aligned} &= -\frac{1}{a} x^2 \cos ax + \left[ \frac{1}{a} x \sin ax - \int \frac{1}{a} \sin ax \, dx \right] \\ &= -\frac{1}{a} x^2 \cos ax + \frac{1}{a} x \sin ax - \frac{1}{a} \int \sin ax \, dx \\ &= -\frac{1}{a} x^2 \cos ax + \frac{1}{a} x \sin ax - \frac{1}{a} \cdot -\frac{1}{a} \cos ax + C \\ &= -\frac{1}{a} x^2 \cos ax + \frac{1}{a} x \sin ax + \frac{1}{a^2} \cos ax + C \end{aligned}$$

9)  $\int \theta \sin \theta \cos 2\theta d\theta$

$\int \theta \cdot \frac{1}{2} \cdot 2 \sin \theta \cos 2\theta d\theta$

$\int \frac{1}{2} \theta \sin 2\theta d\theta$

$u = \frac{1}{2} \theta \quad dv = \sin 2\theta d\theta$

$du = \frac{1}{2} d\theta \quad v = -\frac{1}{2} \cos 2\theta$

$\int \frac{1}{2} \theta \sin 2\theta d\theta = -\frac{1}{4} \theta \cos 2\theta - \int -\frac{1}{4} \cos 2\theta d\theta$

$= -\frac{1}{4} \theta \cos 2\theta + \frac{1}{4} \int \cos 2\theta d\theta$

$= -\frac{1}{4} \theta \cos 2\theta + \frac{1}{4} \cdot \frac{1}{2} \sin 2\theta + C$

$= -\frac{1}{4} \theta \cos 2\theta + \frac{1}{8} \sin 2\theta + C$

or

$= -\frac{2}{8} \theta \cos 2\theta + \frac{1}{8} \sin 2\theta + C$

$= \frac{1}{8} (\sin 2\theta - 2\theta \cos 2\theta) + C$

12)  $\int t^3 e^t dt$

$u = t^3 \quad dv = e^t dt$

$du = 3t^2 dt \quad v = e^t$

$\int t^3 e^t dt = t^3 e^t - \int 3t^2 e^t dt$

$u = 3t^2 \quad dv = e^t dt$

$du = 6t dt \quad v = e^t$

$= t^3 e^t - [3t^2 e^t - \int 6t e^t dt]$

$= t^3 e^t - 3t^2 e^t + 6 \int t e^t dt$

$u = t \quad dv = e^t dt$

$du = dt \quad v = e^t$

$= t^3 e^t - 3t^2 e^t + 6 [t e^t - \int e^t dt]$

$= t^3 e^t - 3t^2 e^t + 6t e^t - 6 \int e^t dt$

$= t^3 e^t - 3t^2 e^t + 6t e^t - 6e^t + C$

or

$= e^t (t^3 - 3t^2 + 6t - 6) + C$

$$15) \int_0^1 t e^{-t} dt$$

$$u = t \quad dv = e^{-t} dt$$

$$du = dt \quad v = -e^{-t}$$

$$\begin{aligned} \int_0^1 t e^{-t} dt &= -t e^{-t} - \int -e^{-t} dt \\ &= -t e^{-t} + \int e^{-t} dt \\ &= -t e^{-t} - e^{-t} + C \end{aligned}$$

$$\begin{aligned} [-t e^{-t} - e^{-t}]_0^1 &= [-e^{-1} - e^{-1}] - [0 - e^0] \\ &= [-2e^{-1}] - [-1] \\ &= -\frac{2}{e} + 1 \quad \text{or} \quad 1 - \frac{2}{e} \\ &\approx 0.264 \end{aligned}$$