

Key

Unit 5 Worksheet 2

Calculus 1

Find the antiderivative using U-substitution.

1. $\int x(x^2 + 1)^2 dx$

2. $\int (2x^2 + 3)^3 4x dx$

3. $\int \sqrt{2x-1} dx$

4. $\int 3(3x-1)^4 dx$

5. $\int (x^2 + x)^7 (4x+2) dx$

6. $\int x^2 \sqrt{x^3 - 2} dx$

7. $\int \frac{-4x}{(1-2x^2)^2} dx$

8. $\int (5x^2 + 1)^2 20x dx$

9. $\int x^2 \sqrt{x^3 + 1} dx$

10. $\int \frac{4x}{\sqrt{x^2 + 1}} dx$

④ $\int 3(3x-1)^4 dx$

$u = 3x - 1$

$du = 3 dx$

$\int u^4 du$

$\frac{u^5}{5} + C = \frac{(3x-1)^5}{5} + C$

⑤ $\int (x^2 + x)^7 (4x+2) dx$

$u = x^2 + x$

$du = 2x + 1 dx$

$2 du = 4x + 2 dx$

$2 \int u^7 du$

$2 \left(\frac{u^8}{8} \right) + C$

$\frac{(x^2 + x)^8}{4} + C$

① $\int x(x^2 + 1)^2 dx$

$u = x^2 + 1$

$du = 2x dx$

$\frac{1}{2} du = x dx$

$\frac{1}{2} \int u^2 du$

$\frac{1}{2} \left(\frac{u^3}{3} \right) + C$

$\frac{(x^2 + 1)^3}{6} + C$

② $\int (2x^2 + 3)^3 4x dx$

$u = 2x^2 + 3$

$du = 4x dx$

$\int u^3 du$

$\frac{u^4}{4} + C$

$\frac{(2x^2 + 3)^4}{4} + C$

③ $\int \sqrt{2x-1} dx$

$u = 2x - 1$

$du = 2 dx$

$\frac{1}{2} du = dx$

$\frac{1}{2} \int u^{1/2} du$

$\frac{1}{2} \left(\frac{2}{3} u^{3/2} \right) + C$

$\frac{(2x-1)\sqrt{2x-1}}{3} + C$

$$(6) \int x \sqrt{x^3 - 2} dx$$

$$u = x^3 - 2$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{3} \int u^{1/2} du$$

$$\frac{1}{3} \left(\frac{2}{3} u^{3/2} \right) + C$$

$$\frac{2u^{3/2}}{9} + C = \frac{2(x^3 - 2)\sqrt{x^3 - 2}}{9} + C$$

$$\frac{2\sqrt{(x^3 - 2)^3}}{9} + C$$

$$(7) \int \frac{-4x}{(1 + 2x^2)^2} dx$$

$$u = 1 + 2x^2$$

$$du = 4x dx$$

$$\int \frac{1}{u^2} du = \int u^{-2} du$$

$$\frac{u^{-1}}{-1} + C$$

$$\frac{-1}{1 - 2x^2} + C$$

$$(9) \int x^2 \sqrt{x^3 + 1} dx$$

$$u = x^3 + 1$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{3} \int u^{1/2} du$$

$$\frac{1}{3} \left(\frac{2}{3} u^{3/2} \right) + C$$

$$\frac{2}{9} (x^3 + 1)^{3/2} + C$$

$$\frac{2\sqrt{(x^3 + 1)^3}}{9} + C$$

$$\frac{2(x^3 + 1)\sqrt{x^3 + 1}}{9} + C$$

$$(8) \int (5x^2 + 1)^2 20x dx$$

$$u = 5x^2 + 1$$

$$du = 10x dx$$

$$2du = 20x dx$$

$$2 \int u^2 du$$

$$2 \left(\frac{u^3}{3} \right) + C$$

$$\frac{2(5x^2 + 1)^3}{3} + C$$

$$(10) \int \frac{4x}{\sqrt{x^2 + 1}} dx$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$2du = 4x dx$$

$$2 \int \frac{1}{u^{1/2}} du$$

$$2 \int u^{-1/2} du$$

$$2 \cdot 2 u^{1/2} + C = 4\sqrt{x^2 + 1} + C$$