UNIT 5: Rules Integration / Integration Concepts Highlights

As a Concept: Integration is referred to as the "anti-derivative" or the inverse process of the finding the derivative. It provides us a way to get from a function calculating change back to the original.

An indefinite integral refers to an integral in its general form that represents a family of functions.

$$\int f(x)dx = F(x) + C$$

Basic Integration Rules

$\int x^n dx \rightarrow y = \frac{x^{n+1}}{n+1} + C$

$\int \frac{1}{x} dx \rightarrow y = \ln|x| + C$

$\int e^x dx \rightarrow y = e^x + C$

$\int v(t)dt \rightarrow s(t) + C$

$\int a(t)dt \rightarrow v(t) + C$

$\int a^x dx \rightarrow y = \frac{a^x}{\ln a} + C$

Trigonometric Integration Rules

- ✓ Basic Trig Integrals

Function	Antiderivative
$f(x) = \sin x$ or $\int \sin x\, dx$	$F(x) = -\cos x + C$
$f(x) = \cos x$ or $\int \cos x\, dx$	$F(x) = \sin x + C$
$f(x) = \sec^2 x$ or $\int \sec^2 x\, dx$	$F(x) = \tan x + C$
$f(x) = \csc^2 x$ or $\int \csc^2 x\, dx$	$F(x) = -\cot x + C$
$f(x) = \sec x \tan x$ or $\int \sec x \tan x\, dx$	$F(x) = \sec x + C$
$f(x) = \csc x \cot x$ or $\int \csc x \cot x\, dx$	$F(x) = -\csc x + C$

Eliminating the Constant (+C)

When given initial condition, the value of the constant (+C) can be determined after the integration by substituting the given into the result.

- ✓ Working backwards from f'' with initial conditions
- ✓ Working backwards from $a(t)$ with initial conditions

Ex: $f''(x) = 24x^2 - 18x + 2,\quad f'(1) = 6 \quad and \quad f(1) = 3$

$$f'(x) = 8x^3 - 9x^2 + 2x + 5$$
$$f(x) = 2x^4 - 3x^3 + x^2 + 5x - 2$$

Integrating with U-Substitution

✓ Algebraic (Quantity raised to power)

Procedure:

1. Let "u" equal the algebraic quantity on the inside.
2. Find $\frac{du}{dx}$.
3. Match $\frac{du}{dx}$ to the original function.
4. Substitute.
5. Find the antiderivative in terms of "u".
6. Re-substitute for u in terms of x.

$$\int x^3(x^4+3)^2\,dx = \frac{(x^4+3)^3}{12}+C$$

✓ Trigonometric (Quantity raised to a Power)

Procedure:

1. Let "u" equal the trigonometric ratio being raised to the power (inside quantity).
2. Find $\frac{du}{dx}$.
3. Match $\frac{du}{dx}$ to the original function.
4. Substitute.
5. Find the antiderivative in terms of "u".
6. Re-substitute for u in terms of x.

$$\int \sin^6 x \cos x dx = \frac{\sin^7 x}{7}+C$$

✓ Trigonometric (Unusual Angle)

Procedure:

1. Let "u" equal the unusual angle.
2. Find $\frac{du}{dx}$.
3. Match $\frac{du}{dx}$ to the original function.
4. Substitute.
5. Find the antiderivative in terms for trig(u).
6. Re-substitute for u in terms of x.

$$\int 2x\cos(x^2+3)dx = \sin(x^2+3)+C$$

Evaluating Definite Integrals

A definite integral refers to an integral that has a lower limit and an upper limit which can be evaluated to a specific value.

$$\int_a^b f(x)dx = [F(b)+C]-[F(a)+C] = F(b)-F(a)$$

*[Upper – Lower]!

Properties of Definite Integrals

I. Addition Property:

If $a<b<c$, then $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

II. Coefficient Property:

For any Real Number c, $\int_a^b cf(x)dx = c\int_a^b f(x)dx$

III. Bounds Property:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

IV. Integral Sum/Difference Property:

$$\int_a^b (f(x)\pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Separation of Variables (Differential Equations)

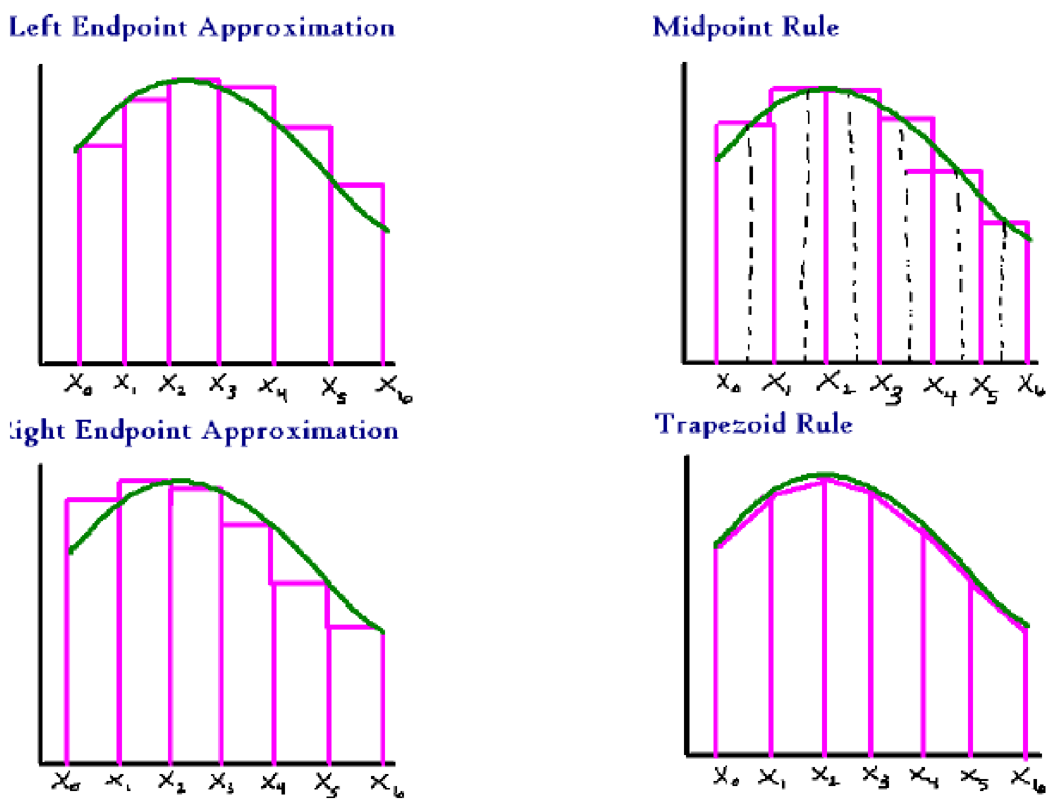
Procedure:

1. Use differentials to get like variables on the same side.
2. Find the antiderivative of each side.
3. Put into *y* – form, if possible.

$$\frac{dy}{dx} = \frac{-2x}{y^2} \quad \rightarrow \quad y = \sqrt[3]{-3x^2 + C}$$

Approximation Methods involving Summation

Approximation Summary



NOTE: Most commonly will be applied to draw conclusions from **a table OR graph of data** where the function is unknown.