Derivative Applications Highlights

Projectile Motion Applications /Higher Order Derivatives

- ✓ Position-Velocity-Acceleration Formulas

$$s(t) \to Position$$
$$v(t) = s'(t) \to Velocity$$
$$a(t) = v'(t) = s''(t) \to Acceleration$$

Graphing Characteristics:

- ✓ Increasing / Decreasing Intervals & 1st Derivative Test
- ✓ Identifying Local Maximum(s) / Local Minimum(s) & 2nd Derivative Test

Increasing / Decreasing Intervals: (Use 1st Derivative)

1. Find $f'(x)$.
2. Find critical numbers for $f'(x)$.
3. Place the critical numbers on a number line and pick test values on each interval.
4. Replace these test values in $f'(x)$ to determine (+ or -) .
   + → Increasing   - → Decreasing
5. Write your results using interval notation.

Local Max/ Local Min: ***(Be sure to write in point form!)***
(Apply the 1st Derivative Test)
These are found when a function changes from:
-increasing to decreasing (Local Max) OR -decreasing to increasing (Local Min)

- ✓ Finding Intervals of Concavity (Up / Down)
- ✓ Identifying Point(s) of Inflection

Intervals of Concavity (Up/Down): (Use 2nd Derivative)

1. Find $f''(x)$.
2. Find critical points for $f''(x)$.
3. Place critical points on a number line and pick test values.
4. Replace these test values in $f''(x)$ to determine (+ or -) .
   + → Concave UP   - → Concave DOWN
5. Write your results using interval notation.

Points of Inflection: ***(Be sure to write in point form!)***
These are found when a continuous function changes from
-(concave up to concave down) OR (concave down to concave up).

Theorems:

- ✓ **Extreme Value Theorem** (EVT → Absolute Max-Min Existence)
  If f *is continuous* on a closed interval [a,b], then f attains both a maximum and minimum value there.

- ✓ **Mean Value Theorem** (MVT → Slope of the Tangent Line = Slope of the Secant Line)
  If f is *continuous* on a closed interval [a,b] and differentiable on its interior (a,b),
  then there is at least one number c in (a,b) where: $\frac{f(b)-f(a)}{b-a}=f'(c)$

- ✓ **Rolle's Theorem**
  If f is continuous on a closed interval [a,b] and differentiable on its interior (a,b) and *f(a)=f(b)*,
  then there is at least one number c in (a,b) such that $f'(c)=0.$

Real-World APPLICATIONS:

Related Rates: Refers to problems in the situation where there are two or more variables that are closely related which are changing with respect to *time*.

Procedure:

1. Make a drawing. Assign variables to the different quantities and label the appropriate parts of the figure with these variables.
2. State what is GIVEN about the variables.
   Determine any rates of change given in the conditions of the problem.
   (*The derivative of each variable is with respect to time).*
3. Determine an equation, which is appropriate for the drawing and the conditions of the problem.
   Examples: (Pythagorean Theorem/ Area Formulas/ Surface Area & Volume Formulas)
4. If necessary, find values of missing parts in the figure for the moment of in time being discussed.
5. Find the derivative of the equation implicitly with respect to time.
6. Substitute all known information into the derivative and solve for the desired rate of change.

Optimization: The derivative may be used to find maximum and minimum values in applied scenarios. These are referred to as problems of optimization.

Procedure:

1. State the given information and make a drawing if necessary.
2. Write a formula for the maximized or minimized value.
3. Rewrite the formula in terms of a single variable.
4. Find the derivative and its critical values.
5. Find the maximum or minimum value.

Linear Approximation of a Value through use of the Tangent Line:

Procedure:

1. Find the equation of the line tangent to the curve at a value of x near the value you are seeking the approximation of.
2. Place the value you want the approximation for into the equation of the tangent line.