Theorem

NOTE: (also called the Sandwich Theorem OR Pinching Theorem)

If \( f(x) \leq g(x) \leq h(x) \) when \( x \) is near \( a \) (except possibly at \( a \))

\[ \lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L, \]

then \( \lim_{x \to a} g(x) = L. \)

Examples:

1. Show that, \( \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0 \)

Can’t use \( \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0 \) because the \( \sin \frac{1}{x} \) does not exist as \( x \to 0 \).

We know \(-1 \leq \sin \frac{1}{x} \leq 1\)

So, \(-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2\)

\[ \lim_{x \to 0} (-x^2) = 0 \quad \lim_{x \to 0} x^2 = 0 \]

\[ \therefore \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0 \]

Apply the Squeeze Theorem to the following to determine the limit.

2. If \( 2x + 2 \leq f(x) \leq x^2 + 3 \) for all values of \( 0 \leq x \leq 2 \)

then \( \lim_{x \to 1} f(x) = \)

3. If \( -x^2 - 2 \leq g(x) \leq 2x - 1 \) for \( -2 \leq x \leq 0 \)

then \( \lim_{x \to -1} g(x) = \)