Calculus I
UNIT 5: Integration – PART 2 REVIEW

Name: KEY
Date: ____________

PART 2: DEFINITE INTEGRALS & APPROXIMATIONS – STUDY GUIDE

SECTION 6: Definite Integrals

SECTION 7: Properties of Definite Integrals

SECTION 8: Summation/Sigma Notation

SECTION 9: Rectangular Approximation Methods (RAM) for Area under a Curve

SECTION 10: Trapezoidal Approximation for Area under a Curve

SECTION 11: Riemann Sums using a Table of Values

PART 2: DEFINITE INTEGRALS & APPROXIMATIONS – REVIEW

Directions: Write out the terms for the given sequence and find the sum of each.

1.) $\sum_{n=1}^{6}(2n+3)$ $= 60$

2.) $\sum_{n=2}^{5}(3n^2+1)$ $= 164$

Directions: Write the following sum using sigma notation.

3.) $5+7+9+11+13+15+17$

$\sum_{n=1}^{7} 2n+3$ or $\sum_{n=0}^{6} 2n+5$

4.) $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots+\frac{1}{2187}$

$\sum_{n=1}^{7} \frac{1}{3^k}$

Directions: Evaluate the definite integrals below. $\boxed{\text{MATH}} \rightarrow \boxed{9}$

5.) $\int_{-2}^{4}(6x^2-10x+3)dx$ $= 102$

6.) $\int_{\frac{\pi}{2}}^{\pi}\cos x dx$ $= -1$

7.) $\int_0^1 (4x+1)^5\,dx$ $= 651$

8.) $\int_0^3 \frac{t}{(t^2+3)^2}\,dt$ $= \frac{1}{8} \approx 0.125$

9.) $\int_1^2 \frac{x^3-16}{x^2}\,dx$ $= -6.5$ or $-\frac{13}{2}$

Directions: Apply the definite integral properties to the following to solve.

GIVEN		
$\int_0^3 f(x)dx = 7$	$\int_3^7 f(x)dx = 17$	$\int_0^3 g(x)dx = 4$

10.) $\int_7^0 f(x)dx =$ $\boxed{-24}$

11.) $\int_0^3 [6g(x)-3f(x)+1]\,dx =$

$= 6\int_0^3 g(x)\,dx - 3\int_0^3 f(x)\,dx + \int_0^3 1\,dx$

$= 6(4) - 3(7) + [x]_0^3$

$= \boxed{6}$

Directions: Use your calculator to find the following sums.

12.) $\sum_{k=20}^{42} (3k^2+4k-5)$ $= \boxed{72082}$

13.) $\sum_{k=2}^{15} (-1)^k (k^2-4k)$ $= \boxed{-91}$

Directions: Use your calculator to approximate the area under the curve using the indicated method over the given interval for n rectangles.

14.) LRAM: $f(x) = x^3+2x+7$ $\Delta x: 0.2$ $[-5,5]$ $n = 50$ sum(seq((f(x))·0.2, x, -5, 4.8, 0.2))

15.) RRAM: $f(x) = \sqrt{x}+4$ $\Delta x: 0.1$ $[0,6]$ $n = 60$ sum(seq((f(x))·0.1, x, 0.1, 6, 0.1))

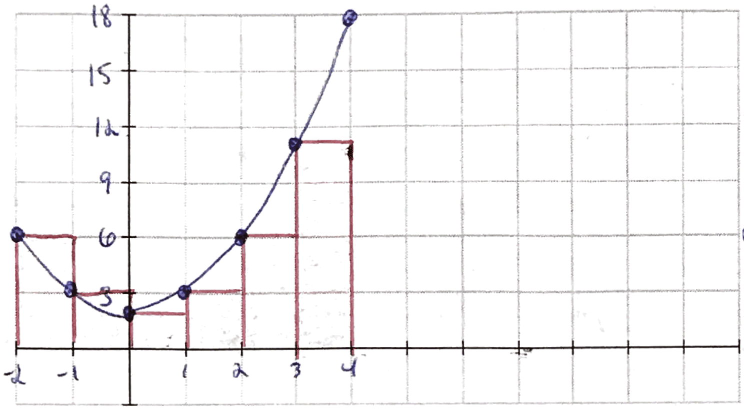
16.) MMRAM: $f(x) = 5+x+2x^2$ $\Delta x: 0.04$ $[0,4]$ $n = 100$ sum(seq((f(x))·0.04, x, 0.02, 3.98, 0.04))

Directions: Draw the graph for the function and use LRAM, RRAM, MRAM, and the Trapezoid Rule to approximate the area under the curve in the given interval for n rectangles/trapezoids.

$f(x) = x^2 + 2$ $[-2, 4]$ $n = 6$

17.) LRAM = 31 units^2

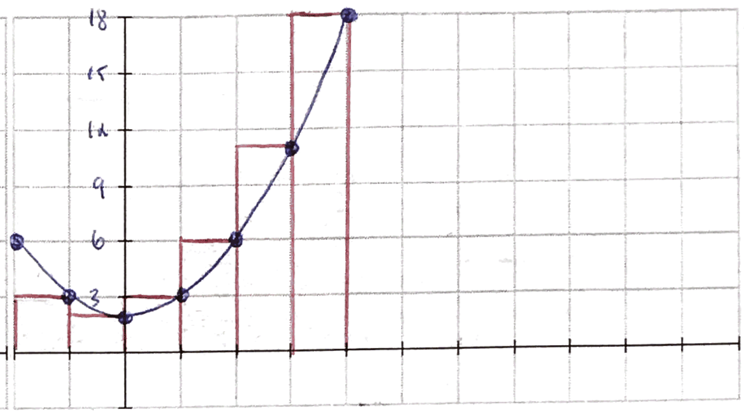
$A = (6 + 3 + 2 + 3 + 6 + 11) \cdot 1$



X	-2	-1	0	1	2	3	4
Y	6	3	2	3	6	11	18

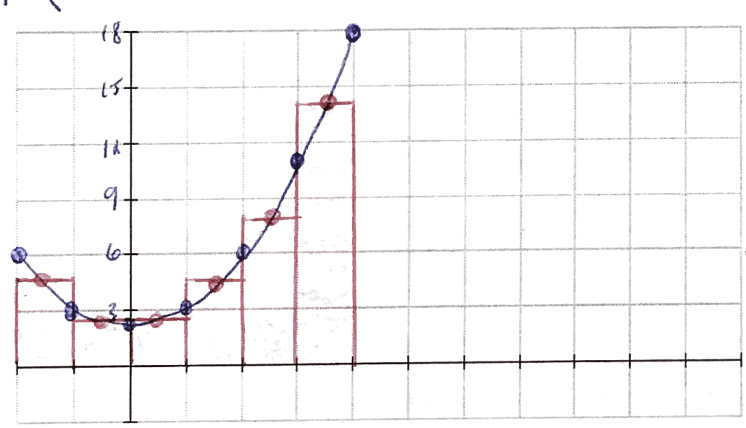
18.) RRAM = 43 units^2

$A = (3 + 2 + 3 + 6 + 11 + 18) \cdot 1$



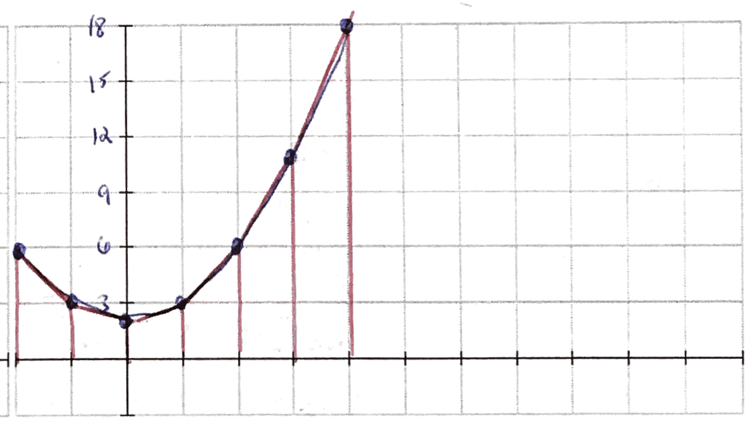
19.) MRAM = 35.5 units^2

$A = (4.25 + 2.25 + 2.25 + 4.25 + 8.25 + 14.25) \cdot 1$



X	-1.5	-0.5	0.5	1.5	2.5	3.5
Y	4.25	2.25	2.25	4.25	8.25	14.25

20.) Trapezoidal Rule = 37 units^2

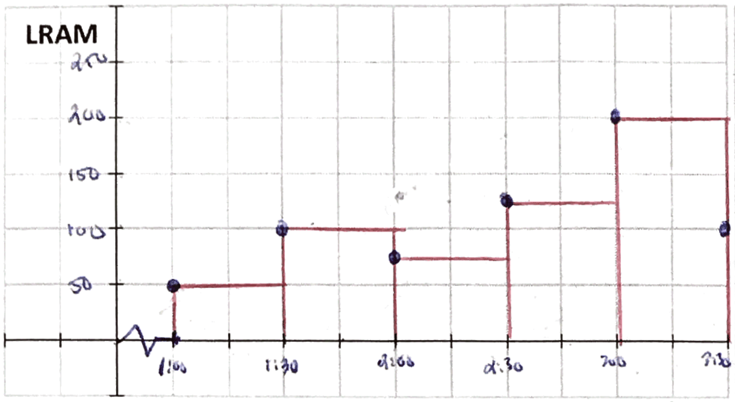


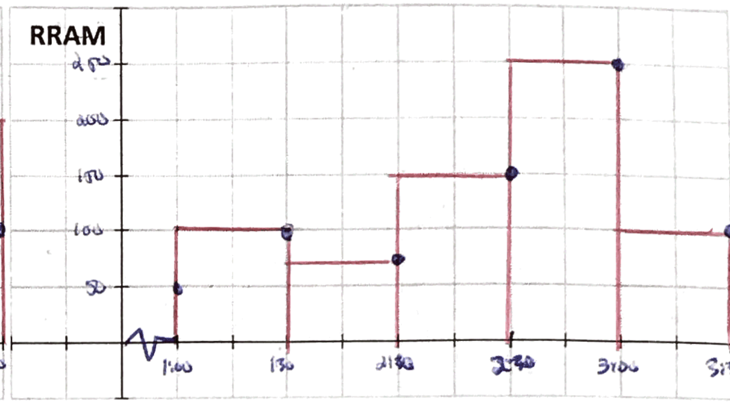
$A = \frac{1}{2}[6 + 2(3) + 2(2) + 2(3) + 2(6) + 2(11) + 18]$

21.) The table below shows the rate in ounces per minute in which ice cream is melting on a hot summer day that is being monitored every 30 minutes.

Time (min)	1:00	1:30	2:00	2:30	3:00	3:30
Rate (oz./min)	50	100	75	125	200	100

a.) Draw a graph that could represent the ice cream's melting rate during 2 ½ hours.





b.) What would be the meaning of the area underneath the graph?

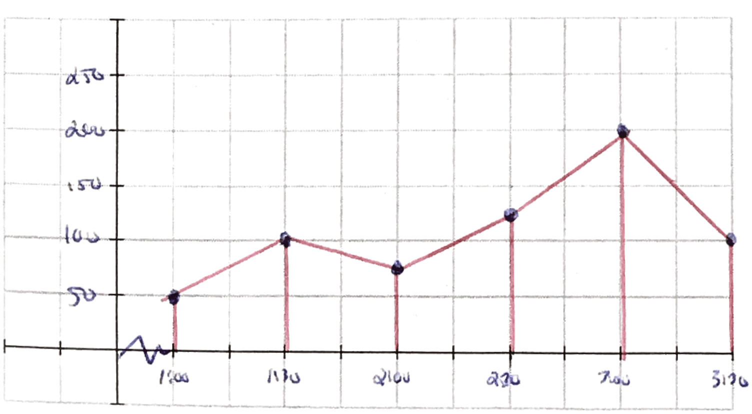
OUNCES OF ICE CREAM MELTED BETWEEN 1:00 – 3:30

c.) Sketch and approximate the volume of ice cream that has melted after 2 ½ hours using the Left-Endpoint & Right-Endpoint Approximation methods.

LRAM: $30(50 + 100 + 75 + 125 + 200) = \boxed{16{,}500}$

RRAM: $30(100 + 75 + 125 + 200 + 100) = \boxed{18{,}000}$

d.) Approximate the volume of ice cream that has melted after 2 ½ hours using the Trapezoidal Rule.



$A = \frac{30}{2}\left[50 + 2(100) + 2(75) + 2(125) + 2(200) + 100\right]$

$A = \boxed{17250}$