Holes and Vertical Asymptotes

The graph of a rational function will have a hole in the location of any \( x \) value that causes the denominator to equal zero and is in a factor that can be cancelled.

The graph of a rational function will have a vertical asymptote in the location of any value of \( x \) which causes the denominator to equal zero and it is not in a factor that can be cancelled.

1) \( f(x) = \frac{3x+12}{x^2-16} \), \( x \neq \pm 4 \)

\[ f(x) = \frac{3(x+4)}{(x-4)(x+4)} \] The \((x+4)\) factor cancels.

There is a hole in the graph at \( x = -4 \) because \(-4\) causes the denominator to equal zero and that factor can be cancelled.

There is a vertical asymptote in the graph at \( x = 4 \) because \( 4 \) causes the denominator to equal zero and that factor cannot be cancelled.

Horizontal Asymptotes

The graph of a rational function will have horizontal asymptotes whenever the leading power in the numerator is less than or equal to the leading power in the denominator.

Rules:
1) If the numerator power is less than the denominator power, the horizontal asymptote is \( y = 0 \) (the x-axis)

Examples:
1) \( f(x) = \frac{x+7}{x^2+3} \) H.A. \( y = 0 \)

2) If the numerator power is equal to the denominator power, the horizontal asymptote is \( y = \) the fraction formed by the leading coefficients.

Examples:
2) \( f(x) = \frac{4x^3 - 7x + 1}{6x^3 + 5} \) H.A. \( y = \frac{4}{6} \Rightarrow y = \frac{2}{3} \)

3) \( f(x) = \frac{8x^2 + 3}{5 - 2x^2} = \frac{8x^2 + 3}{-2x^2 + 5} \) H.A. \( y = \frac{8}{-2} \Rightarrow y = -4 \)

Slant Asymptotes: The graph of a rational function will have a slant asymptote whenever the leading numerator power is exactly one more than the leading denominator power.
Finding the slant asymptote

Procedure:
1) Divide the numerator by denominator.
2) Ignore the remainder.
3) Set the answer equal to y.

Examples:
1) \[ f(x) = \frac{4x^3 - 6x^2 + 3x + 2}{2x^2 + 5x - 1} \]

2x^2 + 5x - 1 \[ \underline{4x^3 - 6x^2 + 3x + 2} \]

\[ -(4x^3 + 10x^2 - 2x) \]

\[ -16x^2 + 5x + 2 \]

\[ -(16x^2 - 40x + 8) \]

\[ 45x - 6 \]

Slant Asymptote: \[ y = 2x - 8 \]

2) \[ f(x) = \frac{6x^4 - 2x - 7}{2x^3 - 4x^2 + 1} \]

2x^3 - 4x^2 + 0x + 1 \[ \underline{6x^4 + 0x^3 + 0x^2 - 2x - 7} \]

\[-(6x^4 - 12x^3 + 0x^2 + 3x) \]

12x^3 + 0x^2 - 5x - 7

\[-(12x^3 - 24x^2 + 0x + 6) \]

\[ 24x^2 - 5x - 13 \]

Slant Asymptote: \[ y = 3x + 6 \]