Holes and Vertical Asymptotes

The graph of a rational function will have a hole in the location of any x value that causes the denominator to equal zero and is in a factor that can be cancelled.

The graph of a rational function will have a vertical asymptote in the location of any value of x which causes the denominator to equal zero and it is not in a factor that can be cancelled.

1) $f(x) = \dfrac{3x+12}{x^2-16}$, $x \neq \pm 4$

$f(x) = \dfrac{3(x+4)}{(x-4)(x+4)}$ The $(x+4)$ factor cancels.

There is a hole in the graph at x = -4 because –4 causes the denominator to equal zero and that factor can be cancelled.

There is a vertical asymptote in the graph at x = 4 because 4 causes the denominator to equal zero and that factor cannot be cancelled.

Horizontal Asymptotes

The graph of a rational function will have horizontal asymptotes whenever the leading power in the numerator is less than or equal to the leading power in the denominator.

Rules:

1) If the numerator power is less than the denominator power, the horizontal asymptote is y = 0 (the x-axis)

Examples:

1) $f(x) = \dfrac{x+7}{x^2+3}$ H.A. y = 0

2) If the numerator power is equal to the denominator power, the horizontal asymptote is y = the fraction formed by the leading coefficients.

Examples:

2) $f(x) = \dfrac{4x^3-7x+1}{6x^3+5}$ H.A. y = $\dfrac{4}{6} \Rightarrow y = \dfrac{2}{3}$

3) $f(x) = \dfrac{8x^2+3}{5-2x^2} = \dfrac{8x^2+3}{-2x^2+5}$ H.A. y = $\dfrac{8}{-2} \Rightarrow y = -4$

Slant Asymptotes: The graph of a rational function will have a slant asymptote whenever the leading numerator power is exactly one more than the leading denominator power.

Finding the slant asymptote

Procedure:

1) Divide the numerator by denominator.
2) Ignore the remainder.
3) Set the answer equal to y.

Examples:

1) $f(x) = \dfrac{4x^3 - 6x^2 + 3x + 2}{2x^2 + 5x - 1}$

$$
\begin{array}{r}
2x - 8 \\
2x^2 + 5x - 1 \overline{)\, 4x^3 - 6x^2 + 3x + 2} \\
\underline{-(4x^3 + 10x^2 - 2x)} \\
-16x^2 + 5x + 2 \\
\underline{-(16x^2 - 40x + 8)} \\
45x - 6
\end{array}
$$

Slant Asymptote: y = 2x – 8

2) $f(x) = \dfrac{6x^4 - 2x - 7}{2x^3 - 4x^2 + 1}$

$$
\begin{array}{r}
3x + 6 \\
2x^3 - 4x^2 + 0x + 1 \overline{)\, 6x^4 + 0x^3 + 0x^2 - 2x - 7} \\
\underline{\textbf{-(}6x^4 - 12x^3 + 0x^2 + 3x)} \\
12x^3 + 0x^2 - 5x - 7 \\
\underline{-(12x^3 - 24x^2 + 0x + 6)} \\
24x^2 - 5x - 13
\end{array}
$$

Slant Asymptote: y = 3x + 6