AP Physics C: Mechanics
1999 Scoring Guidelines

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT™, the Advanced Placement Program® (AP®), and Pacesetter®. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

1999 Physics C Solutions | **Distribution of points**

Mech. 1 (15 points)

(a) 5 points

For conservation of momentum — 1 point

$mv_0 = (m+M_0)v$

$v = \dfrac{mv_0}{m+M_0}$

For conservation of mechanical energy — 1 point

$K + U = K' + U'$ or $\dfrac{1}{2} M_{\text{total}} v^2 = M_{\text{total}} gh$

For calculating h in terms of ℓ — 1 point

$h = \ell(1 - \cos\theta)$

For substituting for M_{total} and v in the energy equation — 1 point

$$\frac{1}{2}(m + M_0)\left(\frac{mv_0}{m + M_0}\right)^2 = (m + M_0)g\ell(1 - \cos\theta)$$

For the correct answer — 1 point

$$v_0 = \frac{m+M_0}{m}\sqrt{2g\ell(1-\cos\theta)}$$

(b) 4 points

For Newton's second law (not awarded if net force was set equal to zero) — 1 point

$F_{\text{net}} = ma$

For any equation that indicated that the tension minus the weight is <u>not</u> zero — 1 point

$T - M_{\text{total}}g = M_{\text{total}}a$

For an expression for the centripetal force — 1 point

$a = \dfrac{v^2}{r}$ or $\dfrac{v^2}{\ell}$

$T - M_{\text{total}}g = M_{\text{total}}\dfrac{v^2}{\ell}$

For correctly substituting for M_{total} and v — 1 point

$$(m + M_0)\frac{1}{\ell}\left(\frac{mv_0}{m + M_0}\right)^2 = T - (m + M_0)g$$

Solving for T

$$T = (m+M_0)g(3-2\cos\theta)$$

Mech. 1 (continued)

(c) 4 points

For all of the following: — 4 points

1) A practical procedure that uses some or all of the apparatus listed and would work
2) Recognition of any assumptions that must be made
3) Indication of the proper mathematical computation using the variables measured

Two points were awarded if the description of the procedure was not complete but it would work, or if the mathematical work did not clearly specify the variables used, or any combination of the above.

No points were awarded if the procedure would not be feasible in a laboratory situation with the apparatus listed, or if the procedure was merely a repeat of that outlined in part (a).

(d) 2 points

$F_{net} = ma = -bv$

For expressing the acceleration as the time derivative of the speed, $a = \frac{dv}{dt}$ — 1 point

$$\int_{v_0}^{v} \frac{dv}{v} = \int_0^t -\frac{b}{m}\,dt$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{bt}{m}$$

$$v = v_0 e^{-bt/m}$$

$$\int_0^{\ell} ds = \int_0^t v_0 e^{-bt/m}\,dt$$

For a general expression for the length of the dart in the block as a function of time or for the expression for the total distance L — 1 point

$$\ell = \frac{mv_0}{b}\left(1 - e^{-bt/m}\right)$$

$$L = \frac{mv_0}{b}$$

Mech. 1 (continued)

(d) (continued)

Alternate Solution 1 — *Alternate points*

For indicating that work equals the change in kinetic energy — *1 point*

$$\int F\,dx = \frac{1}{2}mv_0^2$$

$$\left|F_{average}\right| = \frac{bv_0}{2}$$

$$\int_0^L \frac{bv_0}{2}\,dx = \frac{1}{2}mv_0^2$$

$$\frac{bv_0}{2}L = \frac{1}{2}mv_0^2$$

For the correct expression for the total distance L — *1 point*

$$L = \frac{mv_0}{b}$$

Alternate Solution 2 — *Alternate points*

$$\mathbf{F}_{net}\,\Delta t = \Delta\mathbf{p}$$

For the above expression — *1 point*

$$\int_0^\infty -bv\,dt = -mv_0$$

Using $v = \dfrac{ds}{dt}$ and the fact that s goes from zero to L as time goes from zero to infinity

$$\int_0^L -b\,ds = -mv_0$$

$$-bL = -mv_0$$

For the correct expression for the total distance L — *1 point*

$$L = \frac{mv_0}{b}$$

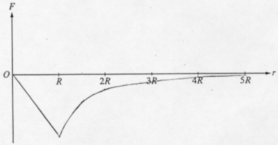
1999 Physics C Solutions **Distribution of points**

Mech. 2 (15 points)

(a) 3 points

For indicating that the equation for gravitational force is applicable	1 point
$F = -\dfrac{Gm_1m_2}{r^2}$	
For using the proper expression for the mass of the planet enclosed by the radius	1 point
$F = -\dfrac{Gm\left(\rho\frac{4}{3}\pi r^3\right)}{r^2}$	
For proper cancellation of terms to show the final result	1 point
$F = -\left(\frac{4}{3}\pi G\rho m\right)r$	

(b) 4 points



For drawing a straight line from the origin to a distance R, and not going past R	1 point
For having the maximum magnitude occur at R	1 point
For having the curve from R to $5R$ decreasing in magnitude with proper curvature and appearing to reach an asymptote	1 point
For recognizing that the force is always negative, i.e. the graph is always below the x-axis	1 point

Mech. 2 (continued)

(c) 3 points

In this and all subsequent parts, either C or $\frac{4}{3}\pi G\rho m$ could be used.

For indicating the integral that needs to be calculated to determine the work — 1 point

$$W = \int F\, dr = \int -Cr\, dr$$

For using the proper limits on the integral (R to zero, not r) — 1 point

$$W = \int_R^0 -Cr\, dr$$

$$W = -C\frac{r^2}{2}\Big|_R^0$$

For the correct answer — 1 point

$$W = \frac{CR^2}{2}$$

Alternate Solution	*Alternate points*
For recognition that the work is the area under the curve, which is triangular	*1 point*
For using the corrects limits (zero to R)	*1 point*
For the correct answer	*1 point*

$$W = \frac{CR^2}{2}$$

(d) 2 points

For using conservation of energy or work-energy relationship — 1 point

$$\Delta K = \Delta U = W$$

$$\frac{1}{2}mv^2 = \frac{CR^2}{2}$$

For the correct answer — 1 point

$$v = \sqrt{\frac{CR^2}{m}}$$

An alternate solution indicating the potential energy as that of a harmonic oscillator also received full credit.

(e) 2 points

For indicating that the ball will move from the center to the surface of the planet — 1 point

For indicating that the ball will stop at the surface, return to the center, and continue oscillating in this manner, with no damping — 1 point

Describing the motion as simple harmonic oscillation with no damping earned full credit

Mech. 2 (continued)

(f) 1 point

For showing a proper application of Newton's first law 1 point

$F = ma$

$Cr = m\dfrac{d^2r}{dt^2}$

Alternately, one could relate the time to the period of oscillation, $T = 2\pi\sqrt{\dfrac{3}{4\pi G\rho}}$,

i.e. the time is one-fourth this period. The above equation was required; a more general form was not acceptable.

1999 Physics C Solutions

Distribution of points

Mech. 3 (15 points)

(a) 5 points

For indicating that the net torque is zero, or that the clockwise and counterclockwise torques are equal	1 point
For a correct expression for the torque exerted by the rod	1 point
$\tau_{rod} = mgR\sin\theta_0$	
For a correct expression for the torque exerted by the block	1 point
$\tau_{block} = 2mg(2R)\sin\theta_0 = 4mgR\sin\theta_0$	
For a correct expression for the torque exerted by the string	1 point
$\tau_{string} = TR$	
For adding the counterclockwise torques and setting the sum equal to the clockwise torque (this point not awarded for just one torque)	1 point

$TR = 4mgR\sin\theta_0 + mgR\sin\theta_0$

$T = 5mg\sin\theta_0$

Only four points could be earned if the wrong trigonometric function was used.
Only three points could be earned if no trigonometric function was used.

(b)

i. 4 points

For indicating that the rotational inertia is the sum of the inertias of the disk, rod, and block	1 point
For calculating the total rotational inertia	1 point

$$I = I_{disk} + I_{rod} + I_{block}$$

$$= \frac{3}{2}mR^2 + \frac{4}{3}mR^2 + 2m(2R)^2$$

$$= \frac{65}{6}mR^2$$

$$\alpha = \tau_{net}/I$$

For substituting the value of torque from part (a) — 1 point

$$\alpha = \frac{5mgR\sin\theta_0}{\frac{65}{6}mR^2}$$

For an answer consistent with the values use for torque and rotational inertia — 1 point

$$\alpha = \frac{6g\sin\theta_0}{13R}$$

Distribution of points

Mech. 3 (continued)

(b) (continued)

ii. 1 point

Expressing the linear acceleration in terms of the angular acceleration

$a = \alpha r$

For substituting the value of α and the correct radius, $2R$ — 1 point

$$a = \frac{12g\sin\theta_0}{13}$$

(c) 5 points

For indicating that energy is conserved — 1 point

For indicating that the potential energy of two bodies (the rod and the block) changes — 1 point

$\Delta U = mgh_{rod} + mgh_{block}$

For the correct expressions for these two potential energies — 1 point

$\Delta U = mgR\cos\theta_0 + 2mg(2R)\cos\theta_0$

For indicating the correct kinetic energy when the rod is horizontal — 1 point

$$K = \frac{1}{2}I\omega^2$$

Equating the kinetic and potential energies, and solving for the angular speed

$$\frac{1}{2}\left(\frac{65}{6}mR^2\right)\omega^2 = mgR\cos\theta_0 + 4mgR\cos\theta_0$$

$$\omega = \sqrt{\frac{12g\cos\theta_0}{13R}}$$

For using the relationship between linear and angular speed, and substituting ω and the correct radius, $2R$ — 1 point

$v = \omega r$

$$v = \left(\sqrt{\frac{12g\cos\theta_0}{13R}}\right)(2R) = 4\sqrt{\frac{3gR\cos\theta_0}{13}}$$

Alternate methods of solution included use of the following proper integrations

$$\omega^2 = 2\int_{\theta_0}^{\pi/2} \alpha \, d\theta$$

$$K = \int_{\theta_0}^{\pi/2} \tau \, d\theta$$