

Special Cases Of Substitution

1. $\int x\sqrt{x-3} \, dx$

$u = x - 3$

$\frac{du}{dx} = 1$

$du = dx$

$x = u + 3$

$\int x u^{1/2} \, du$

$\int (u+3) u^{1/2} \, du$

$\int u^{3/2} + 3u^{1/2} \, du$

$\frac{2}{5} u^{5/2} + 2u^{3/2} + C$

$\frac{2}{5} (x-3)^{5/2} + 2(x-3)^{3/2} + C$

2. $\int \frac{x}{(x+2)^3} \, dx$

$u = x + 2$

$\frac{du}{dx} = 1$

$du = dx$

$x = u - 2$

$\int \frac{x}{u^3} \, du$

$\int (u-2) u^{-3} \, du$

$\int u^{-2} - 2u^{-3} \, du$

$-u^{-1} + u^{-2} + C$

$-\frac{1}{x+2} + \frac{1}{(x+2)^2} + C$

3. $\int \frac{x^2}{(x+2)^3} \, dx$

$u = x + 2$

$\frac{du}{dx} = 1$

$du = dx$

$x = u - 2$

$\int \frac{x^2}{u^3} \, du$

$\int (u-2)^2 u^{-3} \, du$

$\int (u^2 - 4u + 4) u^{-3} \, du$

$\int u^{-1} - 4u^{-2} + 4u^{-3} \, du$

$\ln|u| + 4u^{-1} - 2u^{-2} + C$

$\ln|x+2| + \frac{4}{x+2} - \frac{2}{(x+2)^2} + C$

4. $\int x(2x-1)^4 \, dx$

$u = 2x - 1$

$\frac{du}{dx} = 2$

$du = 2 \, dx$

$dx = \frac{du}{2}$

$x = \frac{u+1}{2}$

$\int x \cdot u^4 \cdot \frac{du}{2}$

$\frac{1}{2} \int \left(\frac{u+1}{2}\right) u^4 \, du$

$\frac{1}{4} \int u^5 + u^4 \, du$

$\frac{1}{4} \left[\frac{1}{6} u^6 + \frac{1}{5} u^5 + C \right]$

$\frac{1}{24} (2x-1)^6 + \frac{1}{40} (2x-1)^5 + C$