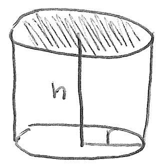
AP Calculus AB
Optimization Worksheet #2

Name Key

1. An open cylinder (has a bottom, but no top) has a volume of 8π cubic feet. What dimensions minimize the surface area?



$8\pi = \pi r^2 h$

$h = \frac{8}{r^2}$

$h = \frac{8}{(2)^2} = 2$ ft.

$SA_{min} = \pi r^2 + 2\pi r h$

$SA = \pi r^2 + 2\pi r\left(\frac{8}{r^2}\right)$

$SA = \pi r^2 + 16\pi r^{-1}$

$SA' = 2\pi r - 16\pi r^{-2}$

$0 = 2\pi r - \frac{16\pi}{r^2}$

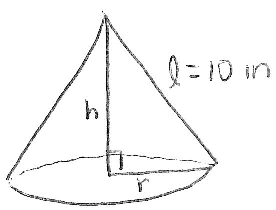
$\frac{16\pi}{r^2} = 2\pi r$

$16\pi = 2\pi r^3$

$8 = r^3$

$r = 2$ ft.

2. Find the maximum volume of a cone with a slant height of 10 inches.



$r = \sqrt{100 - h^2}$

$r = \sqrt{100 - \left(\frac{10\sqrt{3}}{3}\right)^2}$

$r = \sqrt{100 - \frac{100}{3}}$

$r = \sqrt{\frac{200}{3}}$ in

$V_{max} = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\sqrt{100 - h^2}\right)^2 h$

$V = \frac{\pi}{3} h (100 - h^2)$

$V = \frac{100\pi}{3} h - \frac{\pi}{3} h^3$

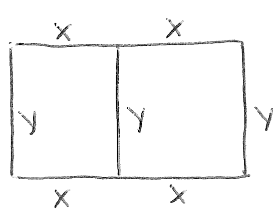
$V' = \frac{100\pi}{3} - \pi h^2$

$0 = \frac{100\pi}{3} - \pi h^2$

$\pi h^2 = \frac{100\pi}{3}$

$h = \frac{10\sqrt{3}}{3}$ in

3. A 384 square meter plot of land is to be enclosed by a fence and divided into two equal parts by another fence parallel to one pair of sides. What dimensions of the outer rectangle will minimize the amount of fence used?



$384 = 2xy$

$x = \frac{192}{y}$

$x = \frac{192}{(16)} = 12$ m

$F_{min} = 4x + 3y$

$F = 4\left(\frac{192}{y}\right) + 3y$

$F = 768y^{-1} + 3y$

$F' = -768y^{-2} + 3$

$0 = -\frac{768}{y^2} + 3$

$\frac{768}{y^2} = 3$

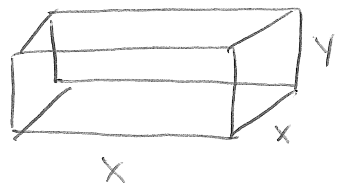
$768 = 3y^2$

$256 = y^2$

$y = 16$ m

24m × 16m

4. A closed rectangular box with a square base has a volume of 252 cubic feet. The wood for the bottom costs $5.00 per square foot and the top costs $2.00 per square foot. The sides costs $3.00 per square foot. What dimensions will minimize the cost of the wood?



$252 = x^2 y$

$y = \frac{252}{x^2}$

$y = \frac{252}{(6)^2} = 7$ ft.

$C_{min} = 5x^2 + 2x^2 + 3(4xy)$

$C_{min} = 7x^2 + 12xy$

$C = 7x^2 + 12x\left(\frac{252}{x^2}\right)$

$C = 7x^2 + 3024x^{-1}$

$C' = 14x - 3024x^{-2}$

$0 = 14x - \frac{3024}{x^2}$

$\frac{3024}{x^2} = 14x$

$3024 = 14x^3$

$216 = x^3$

$x = 6$ ft.

6 ft. × 6 ft. × 7 ft.