AP Calculus AB

Optimization Worksheet #1

Name Key

1. The sum of a number and 3 times another number is 12. Their product is a maximum. Find the numbers and the max product.

given:

$x + 3y = 12$

$x = 12 - 3y$

$x = 12 - 3(2)$

$x = 6$

max/min:

$P_{max} = xy$

$P = (12-3y)y$

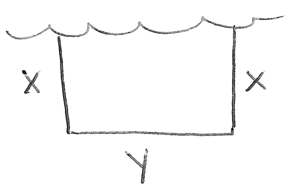
$P_{max} = 12y - 3y^2$

$P' = 12 - 6y$

$0 = 6(2-y)$

$y = 2$

2. A rectangular piece of land is bordered on one side by a river. The other 3 sides are to be enclosed by 300 feet of fencing. What is the maximum area that can be enclosed?



$2x + y = 300$

$y = 300 - 2x$

$y = 300 - 2(75)$

$y = 150$ ft.

$A_{max} = xy$

$A_{max} = x(300-2x)$

$A_{max} = 300x - 2x^2$

$A' = 300 - 4x$

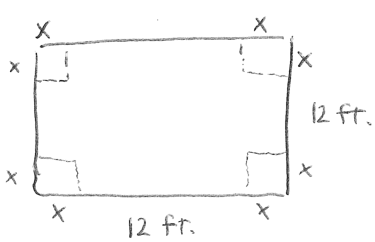
$0 = 4(75 - x)$

$x = 75$ ft

$A_{max} = (75)(150)$

$A_{max} = 11250$ ft^2

3. An open rectangular box is made by cutting 4 congruent squares from the corners of a square piece of cardboard and folding the sides up. The cardboard is 12 feet on each side. What are the dimensions of the box with the maximum volume?



$l = 12 - 2x$

$w = 12 - 2x$

$h = x$

$l = 12 - 2(2) = 8$ ft.

$w = 12 - 2(2) = 8$ ft.

$h = 2$ ft

$V_{max} = lwh$

$V_{max} = (12-2x)(12-2x)(x)$

$V_{max} = (144 - 48x + 4x^2)x$

$V_{max} = 144x - 48x^2 + 4x^3$

$V' = 144 - 96x + 12x^2$

$0 = 12(x^2 - 8x + 12)$

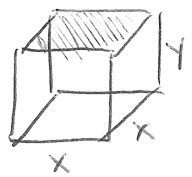
$0 = 12(x-6)(x-2)$

$x = 6$ $\quad$ $x = 2$

↑

extraneous

4. An open box (no lid) with a square base has a volume of 4 cubic feet. What dimensions will minimize the surface area?



$V = x \cdot x \cdot y$

$4 = x^2 y$

$y = \frac{4}{x^2}$

$y = \frac{4}{(2)^2} = 1$ ft.

$SA_{min} = x^2 + 4xy$

$SA_{min} = x^2 + 4x\left(\frac{4}{x^2}\right)$

$SA_{min} = x^2 + 16x^{-1}$

$SA' = 2x - 16x^{-2}$

$0 = 2x - \frac{16}{x^2}$

$\frac{16}{x^2} = 2x$

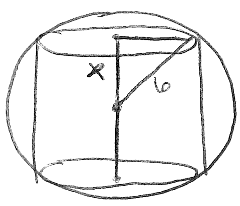
$16 = 2x^3$

$\sqrt[3]{8} = \sqrt[3]{x^3}$

$x = 2$ ft.

2 ft. x 2 ft. x 1 ft.

5. A sphere has a radius of 6 feet. What are the dimensions of the cylinder with the greatest volume that can be inscribed in the sphere? What is the maximum volume of this cylinder?



$r = \sqrt{36 - x^2}$

$h = 2x$

$r = \sqrt{36 - 12}$

$r = \sqrt{24}$ ft.

$h = 2(2\sqrt{3})$

$h = 4\sqrt{3}$ ft.

$V_{max} = \pi r^2 h$

$V_{max} = \pi\left(\sqrt{36 - x^2}\right)^2 (2x)$

$V_{max} = 2\pi x(36 - x^2)$

$V_{max} = 72\pi x - 2\pi x^3$

$V' = 72\pi - 6\pi x^2$

$0 = 6\pi(12 - x^2)$

$x^2 = 12$

$x = \pm 2\sqrt{3}$

$V_{max} = \pi(\sqrt{24})^2(4\sqrt{3})$

$V_{max} = (24\pi)(4\sqrt{3})$

$V_{max} = 96\pi\sqrt{3}$ ft.^{3}