AP Calculus AB

Slope of Tangent Line-Limit Definition

Name Key

Find the slope of the tangent line at the given point.

1. $f(x) = 3x^2$ at x = -1

$\lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h}$

$\lim_{h\to 0} \frac{3(x^2+2xh+h^2) - 3x^2}{h}$

$\lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2 \cancel{-3x^2}}{h}$

$\lim_{h\to 0} \frac{6xh+3h^2}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(6x+3h)}{\cancel{h}}$

$6x+3(0)$

$f'(x) = 6x$

$f'(-1) = -6$

$f(-1) = 3$

$\lim_{x\to -1} \frac{3x^2-3}{x+1}$

$\lim_{x\to -1} \frac{3(x^2-1)}{x+1}$

$\lim_{x\to -1} \frac{3\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}$

$\lim_{x\to -1} 3(x-1)$

$f'(-1) = -6$

equation:

$y-3 = -6(x+1)$

2. $f(x) = x^2 + x$ at x = 2

$\lim_{h\to 0} \frac{(x+h)^2+(x+h)-(x^2+x)}{h}$

$\lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2 \cancel{+x}+h \cancel{-x^2} \cancel{-x}}{h}$

$\lim_{h\to 0} \frac{2xh+h^2+h}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(2x+h+1)}{\cancel{h}}$

$\lim_{h\to 0} 2x+h+1$

$2x+(0)+1$

$f'(x) = 2x+1$

$f'(2) = 2(2)+1$

$f'(2) = 5$

$f(2) = 6$

$\lim_{x\to 2} \frac{x^2+x-6}{x-2}$

$\lim_{x\to 2} \frac{(x+3)\cancel{(x-2)}}{\cancel{x-2}}$

$\lim_{x\to 2} x+3$

$f'(2) = 5$

equation:

$y-6 = 5(x-2)$

3. $f(x) = \dfrac{1}{x+1}$ at x = -5, 2, and 4

$$\lim_{h\to 0} \frac{\left(\frac{1}{(x+h)+1} - \frac{1}{x+1}\right)}{(h)} \frac{(x+1)(x+h+1)}{(x+1)(x+h+1)}$$

$$\lim_{h\to 0} \frac{\cancel{(x+1)} + \cancel{(x+h+1)}}{h(x+1)(x+h+1)}$$

$$\lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(x+1)(x+h+1)}$$

$$\lim_{h\to 0} \frac{-1}{(x+1)(x+h+1)}$$

$$\frac{-1}{(x+1)(x+(0)+1)}$$

$$f'(x) = \frac{-1}{(x+1)^2}$$

$$f'(-5) = \frac{-1}{16}$$

$$f'(2) = \frac{-1}{9}$$

$$f'(4) = \frac{-1}{25}$$

$$f(-5) = \frac{-1}{4}$$

$$\lim_{x\to -5} \frac{\left(\frac{1}{x+1} + \frac{1}{4}\right)}{(x+5)} \frac{(4(x+1))}{(4(x+1))}$$

$$\lim_{x\to -5} \frac{4+(x+1)}{(\cancel{x+5})(4(x+1))}$$

$$f'(-5) = \frac{-1}{16}$$

equation:

$$y + \frac{1}{4} = \frac{-1}{16}(x+5)$$

$$f(2) = \frac{1}{3}$$

$$\lim_{x\to 2} \frac{\left(\frac{1}{x+1} - \frac{1}{3}\right)}{(x-2)} \frac{(3(x+1))}{(3(x+1))}$$

$$\lim_{x\to 2} \frac{3+(x+1)}{(x-2)(3(x+1))}$$

$$\lim_{x\to 2} \frac{-x+2}{(x-2)(3(x+1))}$$

$$\lim_{x\to 2} \frac{-\cancel{(x-2)}}{\cancel{(x-2)}(3(x+1))}$$

$$f'(2) = \frac{-1}{9}$$

equation:

$$y - \frac{1}{3} = \frac{-1}{9}(x-2)$$

$$f(4) = \frac{1}{5}$$

$$\lim_{x\to 4} \frac{\left(\frac{1}{x+1} - \frac{1}{5}\right)}{(x-4)} \frac{(5(x+1))}{(5(x+1))}$$

$$\lim_{x\to 4} \frac{5+(x+1)}{(x-4)(5(x+1))}$$

$$\lim_{x\to 4} \frac{-x+4}{(x-4)(5(x+1))}$$

$$\lim_{x\to 4} \frac{-\cancel{(x-4)}}{\cancel{(x-4)}(5(x+1))}$$

$$f'(4) = \frac{-1}{25}$$

equation:

$$y - \frac{1}{5} = \frac{-1}{25}(x-4)$$

4. $f(x) = x^3$ at x = 3

$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$\lim_{h\to 0} \frac{(x+h)(x^2+2xh+h^2) - x^3}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^3}+2x^2h+xh^2+hx^2+2xh^2+h^3-\cancel{x^3}}{h}$$

$$\lim_{h\to 0} \frac{3x^2h+3xh^2+h^3}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$3x^2+3x(0)+(0)^2$$

$$f'(x) = 3x^2$$

$$f'(3) = 27$$

$$f(3) = 27$$

$$\lim_{x\to 3} \frac{x^3-27}{x-3}$$

$$\lim_{x\to 3} \frac{(\cancel{x-3})(x^2+3x+9)}{\cancel{x-3}}$$

$$f'(3) = 27$$

equation:

$$y - 27 = 27(x-3)$$