The Six Trigonometric Functions

1. sine
2. cosine
3. tangent
4. cosecant
5. secant
6. cotangent

Relationship to one another:

\[
\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta} = \frac{\sin \theta}{\cos \theta} \\
\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}
\]

**S**h**h**C**h**T**a

Right Triangle Definitions of Trigonometric Functions

\[
\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}} \\
\csc \theta = \frac{\text{hyp}}{\text{opp}} \quad \sec \theta = \frac{\text{hyp}}{\text{adj}} \quad \cot \theta = \frac{\text{adj}}{\text{opp}}
\]
Examples:

1. Find the values of the six trigonometric functions of \( \theta \) given the following:

\[
\begin{align*}
\sin \theta &= \frac{12}{13} \\
\cos \theta &= \frac{5}{13} \\
\tan \theta &= \frac{12}{5} \\
\csc \theta &= \frac{13}{12} \\
\sec \theta &= \frac{13}{5} \\
\cot \theta &= \frac{5}{12}
\end{align*}
\]

2. Given that \( \sin \theta = \frac{3}{4} \) and lies in the first quadrant, find the remaining five trigonometric functions of \( \theta \).

\[
\begin{align*}
\sin \theta &= \frac{3}{4} \\
\cos \theta &= \frac{\sqrt{7}}{4} \\
\tan \theta &= \frac{3\sqrt{7}}{7} \\
\csc \theta &= \frac{4}{3} \\
\sec \theta &= \frac{4\sqrt{7}}{7}
\end{align*}
\]

3. If \( \cos \theta = \frac{\sqrt{2}}{2} \) and lies in quadrant IV. Find the exact values of the remaining 5 trig functions.

\[
\begin{align*}
\sin \theta &= -\frac{\sqrt{2}}{2} \\
\csc \theta &= -\frac{2}{\sqrt{2}} = -\sqrt{2} \\
\sec \theta &= \frac{2\cdot\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \sqrt{2} \\
\tan \theta &= -1 \\
\cot \theta &= -1
\end{align*}
\]

4. Find the exact values of the six trig functions in standard position at the point \((-3, -2)\).

\[
\begin{align*}
\sin \theta &= \frac{2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13} \\
\cos \theta &= \frac{3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13} \\
\tan \theta &= \frac{2}{3} \\
\csc \theta &= \frac{\sqrt{13}}{2} \\
\sec \theta &= \frac{\sqrt{13}}{3} \\
\cot \theta &= \frac{3}{2}
\end{align*}
\]
Special Angles

• It is very important to be able to find the trigonometric values at the \( \theta \) values of \( 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \) and \( 2\pi \) because they lie on the axis of the unit circle.
• There are three other important angles that we are responsible for knowing the trigonometric values of. They are \( \frac{\pi}{3}, \frac{\pi}{4}, \) and \( \frac{\pi}{6} \)

Here is an easy way to remember the sine and cosine values for each of these special values:

\[
\begin{align*}
\cos \frac{\pi}{3} &= \frac{1}{2} & \sin \frac{\pi}{3} &= \frac{\sqrt{3}}{2} & \tan \frac{\pi}{3} &= \sqrt{3} \\
\cos \frac{\pi}{4} &= \frac{\sqrt{2}}{2} & \sin \frac{\pi}{4} &= \frac{\sqrt{2}}{2} & \tan \frac{\pi}{4} &= 1 \\
\cos \frac{\pi}{6} &= \frac{\sqrt{3}}{2} & \sin \frac{\pi}{6} &= \frac{1}{2} & \tan \frac{\pi}{6} &= \frac{\sqrt{3}}{3}
\end{align*}
\]

Since every other function can be written as sine and cosine values, you can now find the other four trigonometric values for each of these special angles.

Calculator Tips:

• You can check your answers with the calculator, but you need to be able to come up with these values without one.

• You might notice that there are no cosecant, secant, or cotangent buttons on the calculator. To enter them in, make sure to use their relationships that they have with other trigonometric functions.

Examples:
Use a calculator to evaluate the following: Round to at least 3 decimal places when necessary.

1. \( \cos 20^\circ \) \hspace{1cm} 2. \( \tan \frac{4\pi}{7} \) \hspace{1cm} 3. \( \csc \frac{\pi}{5} \) \hspace{1cm} 4. \( \cot 215^\circ \) 
   \( .9397 \) \hspace{1cm} -4.381 \hspace{1cm} 1.701 \hspace{1cm} 1.428

5. \( \sin 18^\circ \) \hspace{1cm} 6. \( \sec \frac{\pi}{2} \) \hspace{1cm} 7. \( \cot 270^\circ \) \hspace{1cm} 8. \( \tan(-135)^\circ \) 
   \( .309 \) \hspace{1cm} \text{undefined} \hspace{1cm} \text{undefined} \hspace{1cm} 1