1. $\displaystyle\int \frac{2x^{2/3} + 5x^2 \cdot \sqrt[3]{x} - 9x}{3x^2}\, dx$

$\frac{1}{3}\int \left(2x^{2/3} + 5x^{7/3} - 9x\right)x^{-2}\, dx$

$\frac{1}{3}\int 2x^{-4/3} + 5x^{1/3} - 9x^{-1}\, dx$

$\frac{1}{3}\left[-6x^{-1/3} + \frac{15}{4}x^{4/3} - 9\ln|x| + C\right]$

$-2x^{-1/3} + \frac{5}{4}x^{4/3} - 3\ln|x| + C$

2. $\displaystyle\int \frac{\tan^2 x + 1}{\tan x}\, dx$

$\displaystyle\int \frac{\sec^2 x}{\tan x}\, dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\displaystyle\int \frac{\cancel{\sec^2 x}}{u} \cdot \frac{du}{\cancel{\sec^2 x}}$

$\int \frac{1}{u}\, du$

$\ln|u| + C$

$\ln|\tan x| + C$

3. $\int \left(4xe^{3x^2-7} + 4x^3\right) dx$

$\int 4xe^{3x^2-7}\, dx + \int 4x^3\, dx$

$u = 3x^2 - 7$

$\frac{du}{dx} = 6x$

$dx = \frac{du}{6x}$

$\int 4xe^u \cdot \frac{du}{6x}$

$\frac{2}{3}\int e^u\, du$

$\frac{2}{3}e^u$

$\frac{2}{3}e^{3x^2-7}$

$\frac{2}{3}e^{3x^2-7} + x^4 + C$

4. $\int \frac{3^{\ln(2x+1)}}{6x+3}\,dx$

$u = \ln(2x+1)$

$\frac{du}{dx} = \frac{2}{2x+1}$

$(2x+1)\,du = 2\,dx$

$dx = \frac{(2x+1)\,du}{2}$

$\int \frac{3^u}{3\cancel{(2x+1)}} \cdot \frac{\cancel{(2x+1)}\,du}{2}$

$\frac{1}{6}\int 3^u\,du$

$\frac{1}{6} \cdot \frac{3^u}{\ln 3} + C$

$\frac{1}{6} \cdot \frac{3^{\ln(2x+1)}}{\ln 3} + C$

5. $\int \sec^4(2x)\tan(2x)\,dx$

$u = \sec(2x)$

$\frac{du}{dx} = \sec(2x)\tan(2x)(2)$

$dx = \frac{du}{2\sec(2x)\tan(2x)}$

$\int u^3 \cdot \cancel{\sec(2x)\tan(2x)} \cdot \frac{du}{2\cancel{\sec(2x)\tan(2x)}}$

$\frac{1}{2}\int u^3\,du$

$\frac{1}{8}u^4 + C$

$\frac{1}{8}\sec^4(2x) + C$

6. $\int \frac{18x-15}{3x^2-5x+2}\,dx$

$u = 3x^2-5x+2$

$\frac{du}{dx} = 6x-5$

$dx = \frac{du}{6x-5}$

$\int \frac{3\cancel{(6x-5)}}{u} \cdot \frac{du}{\cancel{6x-5}}$

$3\int \frac{1}{u}\,du$

$3\ln|u| + C$

$3\ln|3x^2-5x+2| + C$

7. $\int \frac{-5e^{\sin(2x)}}{3e^{\sin(2x)}\sec(2x)}\,dx$

$u = e^{\sin(2x)}$

$\frac{du}{dx} = e^{\sin(2x)} \cdot \cos(2x) \cdot 2$

$dx = \frac{du}{2\cos(2x)e^{\sin(2x)}}$

$\frac{-5}{3}\int \frac{e^{\cancel{\sin(2x)}}\cancel{\cos(2x)}}{u} \cdot \frac{du}{2\cancel{\cos(2x)}e^{\cancel{\sin(2x)}}}$

$\frac{-5}{6}\int \frac{1}{u}\,du$

$-\frac{5}{6}\ln|u| + C$

$-\frac{5}{6}\ln|e^{\sin(2x)}| + C$

8. $\int 4x^3 - 8x^{-2} + 6x^{-1}\,dx$

$x^4 + 8x^{-1} + 6\ln|x| + C$

9. $\int \frac{2x^2+3x-35}{x-3}\,dx$

3	2	3	-35
		6	36
	2	9	1

$\int 2x + 9 + \frac{1}{x-3}\,dx$

$\frac{1}{3}x^3 + 9x + \ln|x-3| + C$

10. $\int \frac{-5e^x}{1+e^{2x}}\,dx$

$u = e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$\int \frac{-5\cancel{e^x}}{1+u^2}\cdot\frac{du}{\cancel{e^x}}$

$-5\int \frac{1}{1+u^2}\cdot du$

$-5\arctan u + C$

$-5\arctan e^x + C$

11. $\int \frac{2x}{4x^4+5}\,dx$

$u = 2x^2 \quad a = \sqrt{5}$

$\frac{du}{dx} =$

$dx = \frac{du}{4x}$

$\int \frac{2\cancel{x}}{u^2+a^2}\cdot\frac{du}{4\cancel{x}}$

$\frac{1}{2}\int \frac{1}{u^2+a^2}\,du$

$\frac{1}{2}\cdot\frac{1}{\sqrt{5}}\arctan\frac{u}{a} + C$

12. $\int \frac{5x}{\sqrt{25-2x^2}}\,dx$

$u = 25-2x^2$

$\frac{du}{dx} = -4x$

$dx = \frac{du}{-4x}$

$\int \frac{5\cancel{x}}{u^{1/2}}\cdot\frac{du}{-4\cancel{x}}$

$-\frac{5}{4}\int u^{-1/2}\,du$

$-\frac{5}{2}u^{1/2} + C$

$-\frac{5}{2}(25-2x^2)^{1/2} + C$

13. $\int -3x^2 \sqrt[3]{2x-5}\, dx$

$u = 2x-5$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$x = \frac{u+5}{2}$

$\int -3x^2 \cdot u^{1/3} \cdot \frac{du}{2}$

$-\frac{3}{2} \int x^2 \cdot u^{1/3}\, du$

$-\frac{3}{2} \int \left(\frac{u+5}{2}\right)^2 \cdot u^{1/3}\, du$

$-\frac{3}{8} \int (u^2+10u+25)\, u^{1/3}\, du$

$-\frac{3}{8} \int u^{7/3} + 10u^{4/3} + 25u^{1/3}\, du$

$-\frac{3}{8} \left[\frac{3}{10}u^{10/3} + \frac{30}{7}u^{7/3} + \frac{75}{4}u^{4/3} + C \right]$

$-\frac{9}{80}(2x-5)^{10/3} - \frac{90}{56}(2x-5)^{7/3} - \frac{225}{32}(2x-5)^{4/3} + C$