AP Calculus AB
Continuity Practice

Name Key

tify continuity:

1. $f(x)=\begin{cases} x^2+6x+5 & x\le -2 \\ 3x+3 & -2<x<0 \\ -2+\sqrt{x} & x\ge 0 \end{cases}$

$x=-2$:

$f(-2)=-3$ $\quad \lim_{x\to -2} f(x)=-3$ $\quad \lim_{x\to -2} f(x)=f(-2)$

$\lim_{x\to -2^-} f(x)=-3$ $\quad \lim_{x\to -2^+} f(x)=-3$

$\therefore f(x)$ is continuous at $x=-2$

$x=0$:

$f(0)=-2$ $\quad \lim_{x\to 0} f(x)=$ dne

$\lim_{x\to 0^-} f(x)=3$ $\quad \lim_{x\to 0^+} f(x)=-2$

$\therefore f(x)$ is not continuous at $x=0$

2. $f(x)=\begin{cases} (x+4)^3-3 & x\le -3 \\ |x|+4 & -3<x<1 \\ -2x+5 & x\ge 1 \end{cases}$

$x=-3$:

$f(-3)=-2$ $\quad \lim_{x\to -3} f(x)=$ dne

$\lim_{x\to -3^-} f(x)=-2$ $\quad \lim_{x\to -3^+} f(x)=7$

$\therefore f(x)$ is not continuous at $x=-3$

$x=1$:

$f(1)=3$ $\quad \lim_{x\to 1} f(x)=$ dne

$\lim_{x\to 1^-} f(x)=5$ $\quad \lim_{x\to 1^+} f(x)=3$

$\therefore f(x)$ is not continuous at $x=1$

3. $f(x)=\begin{cases} -\frac{2}{3}x-2 & x\le -3 \\ \sqrt{9-x^2} & -3<x<3 \\ \sqrt{x-3} & x\ge 3 \end{cases}$

$x=-3$:

$f(-3)=0$ $\quad \lim_{x\to -3} f(x)=0$ $\quad \lim_{x\to -3} f(x)=f(-3)$

$\lim_{x\to -3^-} f(x)=0$ $\quad \lim_{x\to -3^+} f(x)=0$

$\therefore f(x)$ is continuous at $x=-3$

$x=3$:

$f(3)=0$ $\quad \lim_{x\to 3} f(x)=0$ $\quad \lim_{x\to 3} f(x)=f(3)$

$\lim_{x\to 3^-} f(x)=0$ $\quad \lim_{x\to 3^+} f(x)=0$

$\therefore f(x)$ is continuous at $x=3$

4. $f(x)=\begin{cases} -x^2-6x-2 & x\le -1 \\ -|x-1|+3 & -1<x<5 \\ \frac{2}{5}x-3 & x\ge 5 \end{cases}$

$x=-1$:

$f(-1)=3$ $\quad \lim_{x\to -1} f(x)=$ dne

$\lim_{x\to -1^-} f(x)=3$ $\quad \lim_{x\to -1^+} f(x)=1$

$\therefore f(x)$ is not continuous at $x=-1$

$x=5$:

$f(5)=-1$ $\quad \lim_{x\to 5} f(x)=-1$

$\lim_{x\to 5^-} f(x)=-1$ $\quad \lim_{x\to 5^+} f(x)=-1$

$\lim_{x\to 5} f(x)=f(5)$

$\therefore f(x)$ is continuous at $x=5$

Find variables that will make *f(x)* continuous. Justify.

1. $f(x)=\begin{cases} ax-7 & x\le 2 \\ -6+\sqrt{x-2} & x>2 \end{cases}$

$a(2)-7=-6+\sqrt{(2)-2}$

$2a-7=-6$

$2a=1$

$a=\frac{1}{2}$

$f(2)=-$

$\lim_{x\to 2} f(x)=-6$

$\lim_{x\to 2^-} f(x)=-$ $\quad \lim_{x\to 2^+} f(x)=-6$

$f(x)=\begin{cases} \frac{1}{2}x-7 & x\le 2 \\ -6+\sqrt{x-2} & x>2 \end{cases}$

$\lim_{x\to 2} f(x)=f(2)$

$\therefore f(x)$ is continuous at $x=2$

2. $f(x)=\begin{cases} ax^2-2 & x<0 \\ |x-2|-4 & x\ge 0 \end{cases}$

$a(0)^2-2=|(0)-2|-4$

$-2=-2$

all values of a make $f(x)$ continuous

3. $f(x)=\begin{cases} \frac{1}{2}x+3 & x\le -2 \\ ax^2+b & -2<x<1 \\ \ln(x)+5 & x\ge 1 \end{cases}$

$\frac{1}{2}(-2)+3=a(-2)^2+b$

$1=4a+b$

$1=4(5-b)+b$

$1=20-4b+b$

$1=20-3b$

$-19=-3b$

$b=\frac{19}{3}$

$a(1)^2+b=\ln(1)+5$

$a+b=5$

$a=5-b$

$a=5-(\frac{19}{3})$

$a=-\frac{4}{3}$

$f(x)=\begin{cases} \frac{1}{2}x+3 & x\le -2 \\ -\frac{4}{3}x^2+\frac{19}{3} & -2<x<1 \\ \ln(x)+5 & x\ge 1 \end{cases}$

4. $f(x)=\begin{cases} x^3+9x^2+27x+32 & x\le -2 \\ ax^2+b & -2<x<0 \\ 2x-5 & x\ge 0 \end{cases}$

$(-2)^3+9(-2)^2+27(-2)+32=a(-2)^2+b$

$6=4a+b$

$6=4a+(-5)$

$9=4a$

$a=\frac{9}{4}$

$a(0)^2+b=2(0)-5$

$b=-5$

$f(x)=\begin{cases} x^3+9x^2+27x+32 & x\le -2 \\ \frac{11}{4}x^2-5 & -2<x<0 \\ 2x-5 & x\ge 0 \end{cases}$

5. $f(x)=\begin{cases} \frac{3}{7}x-5 & \text{if} \quad x\le -7 \\ ax-6 & \text{if} \quad x>-7 \end{cases}$

$\frac{3}{7}(-7)-5=a(-7)-6$

$-8=-7a-6$

$-2=-7a$

$\frac{2}{7}=a$

$f(x)=\begin{cases} \frac{3}{7}x-5 & x\le -7 \\ \frac{2}{7}x-6 & x>-7 \end{cases}$

$f(-7)=-8$

$\lim_{x\to -7} f(x)=-8$

$\lim_{x\to -7^-} f(x)=-8$ $\quad \lim_{x\to -7^+} f(x)=-8$

$\lim_{x\to -7} f(x)=f(-7)$

$\therefore$ $f(x)$ is continuous at $x=-7$

6. $f(x)=\begin{cases} 3x+8 & \text{if} \quad x<-3 \\ ax^2+5x-4 & \text{if} \quad x\ge -3 \end{cases}$

$3(-3)+8=a(-3)^2+5(-3)-4$

$-1=9a-19$

$18=9a$

$a=2$

$f(x)=\begin{cases} 3x+8 & x<-3 \\ 2x^2+5x-4 & x\ge -3 \end{cases}$

$f(-3)=-1$

$\lim_{x\to -3} f(x)=-1$

$\lim_{x\to -3^-} f(x)=-1$ $\quad \lim_{x\to -3^+} f(x)=-1$

$\lim_{x\to -3} f(x)=f(-3)$

$\therefore$ $f(x)$ is continuous at $x=-3$

3. (justification)

$$f(x) = \begin{cases} \frac{1}{2}x+3 & x \le -2 \\ -\frac{4}{3}x^2+\frac{19}{3} & -2 < x < 1 \\ \ln x + 5 & x \ge 1 \end{cases}$$

$x = -2$:

$f(-2) = 1$ $\quad \lim_{x\to -2} f(x) = 1$ $\quad \lim_{x\to -2} f(x) = f(-2)$

$\lim_{x\to -2^-} f(x) = 1$ $\quad \lim_{x\to -2^+} f(x) = 1$

$\therefore f(x)$ is continuous at $x = -2$

$x = 1$:

$f(1) = 5$ $\quad \lim_{x\to 1} f(x) = 5$ $\quad \lim_{x\to 1} f(x) = f(1)$

$\lim_{x\to 1^-} f(x) = 5$ $\quad \lim_{x\to 1^+} f(x) = 5$

$\therefore f(x)$ is continuous at $x = 1$

4. (justification)

$$f(x) = \begin{cases} x^3+9x^2+27x+32 & x \le -2 \\ \frac{11}{4}x^2-5 & -2 < x < 0 \\ 2x-5 & x \ge 0 \end{cases}$$

$x = -2$:

$f(-2) = 6$ $\quad \lim_{x\to -2} f(x) = 6$ $\quad \lim_{x\to -2} f(x) = f(-2)$

$\lim_{x\to -2^-} f(x) = 6$ $\quad \lim_{x\to -2^+} f(x) = 6$

$\therefore f(x)$ is continuous at $x = -2$

$x = 0$:

$f(0) = -5$ $\quad \lim_{x\to 0} f(x) = -5$ $\quad \lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0^-} f(x) = -5$ $\quad \lim_{x\to 0^+} f(x) = -5$

$\therefore f(x)$ is continuous at $x = 0$